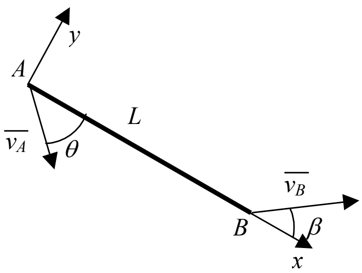
Animation en ligne : http://www.ulb.ac.be/polytech/smana/TP1_Ex2_Ex5.htm

1.



$$\overline{v}_B = \overline{v}_A + \overline{\omega}_{AB} \times \overline{AB} \Rightarrow \begin{cases} v_B \cos\beta = v_A \cos\theta \\ v_B \sin\beta = -v_A \sin\theta + \omega_{AB} L \end{cases}$$

$$\omega_{AB} = \frac{1}{L}\left(v_B \sqrt{1-\left(\frac{v_A}{v_B}\right)^2 \cos^2\theta} + v_A \sin\theta \right)$$

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2. $$\overline{AG} = \left[\frac{L}{2}\cos\theta + \frac{3L}{8}\cos(\theta+\alpha)\right]\overline{1}_x + \left[\frac{L}{2}\sin\theta + \frac{3L}{8}\sin(\theta+\alpha)\right]\overline{1}_y$$

Par dérivation des coordonnées dans le repère *Axy*

$$\overline{v}_G = \frac{d\overline{AG}}{dt} = \left[-\frac{L}{2}\sin\theta\dot{\theta} - \frac{3L}{8}\sin(\theta+\alpha)\dot{\theta}\right]\overline{1}_x + \left[\frac{L}{2}\cos\theta\dot{\theta} + \frac{3L}{8}\cos(\theta+\alpha)\dot{\theta}\right]\overline{1}_y$$

$$\overline{a}_G = \frac{d\overline{v}_G}{dt} = \left[-\frac{L}{2}\cos\theta\dot{\theta}^2 - \frac{3L}{8}\cos(\theta+\alpha)\dot{\theta}^2 - \frac{L}{2}\sin\theta\ddot{\theta} - \frac{3L}{8}\sin(\theta+\alpha)\ddot{\theta}\right]\overline{1}_x$$
$$+\left[-\frac{L}{2}\sin\theta\dot{\theta}^2 - \frac{3L}{8}\sin(\theta+\alpha)\dot{\theta}^2 + \frac{L}{2}\cos\theta\ddot{\theta} + \frac{3L}{8}\cos(\theta+\alpha)\ddot{\theta}\right]\overline{1}_y$$

Par la méthode des distribution des vitesse dans le repère *Axy* $(\overline{\omega}_{AG} = \dot{\theta}\overline{1}_z)$

$$\overline{v}_G = \overline{v}_A + \overline{\omega}_{AG} \times \overline{AG} = \dot{\theta}\overline{1}_z \times \left(\left[\frac{L}{2}\cos\theta + \frac{3L}{8}\cos(\theta+\alpha)\right]\overline{1}_x + \left[\frac{L}{2}\sin\theta + \frac{3L}{8}\sin(\theta+\alpha)\right]\overline{1}_y\right)$$

$$\overline{a}_G = \overline{a}_A + \overline{\omega}_{AG} \times \left(\overline{\omega}_{AG} \times \overline{AG}\right) + \overline{\varepsilon}_{AG} \times \overline{AG} \quad \text{avec} \quad \overline{\varepsilon}_{AG} = \ddot{\theta}\overline{1}_z$$

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3. $\overline{\omega}_{AB} = 6 \text{ rad/sec } \overline{1}_z \Rightarrow \overline{\varepsilon}_{AB} = 0$; $\overline{\omega}_{BC} = \omega_{BC}\overline{1}_z \Rightarrow \overline{\varepsilon}_{BC} = \varepsilon_{BC}\overline{1}_z$; $\overline{\omega}_{CD} = -\omega_{CD}\overline{1}_z \Rightarrow \overline{\varepsilon}_{CD} = -\varepsilon_{CD}\overline{1}_z$

$$\overline{v}_B = \overline{v}_A + \overline{\omega}_{AB} \times \overline{AB} = \omega_{AB} L_{AB}\left(\frac{1}{2}\overline{1}_x - \frac{\sqrt{3}}{2}\overline{1}_y\right)$$

$$\left.\begin{aligned} \overline{v}_{C\in BC} &= \overline{v}_B + \overline{\omega}_{BC} \times \overline{BC} = \omega_{AB} L_{AB}\left(\frac{1}{2}\overline{1}_x - \frac{\sqrt{3}}{2}\overline{1}_y\right) + \omega_{BC} L_{BC}\overline{1}_y \\ \overline{v}_{C\in CD} &= \overline{v}_D + \overline{\omega}_{CD} \times \overline{DC} = \omega_{CD} L_{CD}\left(\frac{\sqrt{3}}{2}\overline{1}_x + \frac{1}{2}\overline{1}_y\right) \end{aligned}\right\} \Rightarrow \begin{cases} \omega_{CD} = \frac{1}{\sqrt{3}}\frac{L_{AB}}{L_{CD}}\omega_{AB};\ \omega_{BC} = 2\frac{\sqrt{3}}{3}\frac{L_{AB}}{L_{BC}}\omega_{AB} \\ \overline{\omega}_{CD} = -2{,}165\ \text{s}^{-1}\,\overline{1}_z;\quad \overline{\omega}_{BC} = 5{,}773\ \text{s}^{-1}\,\overline{1}_z \end{cases}$$

$$\overline{a}_B = \overline{a}_A + \overline{\omega}_{AB} \times \left(\overline{\omega}_{AB} \times \overline{AB}\right) + \overline{\varepsilon}_{AB} \times \overline{AB} = \overline{\omega}_{AB} \times \left(\overline{\omega}_{AB} \times \overline{AB}\right) = \omega^2{}_{AB} L_{AB}\left(\frac{\sqrt{3}}{2}\overline{1}_x + \frac{1}{2}\overline{1}_y\right)$$

$$\overline{a}_{C\in BC} = \overline{a}_B + \overline{\omega}_{BC} \times \left(\overline{\omega}_{BC} \times \overline{BC}\right) + \overline{\varepsilon}_{BC} \times \overline{BC} = \omega^2{}_{AB} L_{AB}\left(\frac{\sqrt{3}}{2}\overline{1}_x + \frac{1}{2}\overline{1}_y\right) - \omega^2{}_{BC} L_{BC}\overline{1}_x + \varepsilon_{BC} L_{BC}\overline{1}_y$$

$$\overline{a}_{C\in CD} = \overline{a}_D + \overline{\omega}_{DC} \times \left(\overline{\omega}_{DC} \times \overline{DC}\right) + \overline{\varepsilon}_{DC} \times \overline{DC} = \omega^2{}_{CD} L_{CD}\left(\frac{1}{2}\overline{1}_x - \frac{\sqrt{3}}{2}\overline{1}_y\right) + L_{CD}\varepsilon_{DC}\left(\frac{\sqrt{3}}{2}\overline{1}_x + \frac{1}{2}\overline{1}_y\right)$$

$$\begin{cases} \varepsilon_{DC} = \left(\dfrac{\sqrt{3}}{2} \omega^2{}_{AB} L_{AB} - \omega^2{}_{BC} L_{BC} - \dfrac{1}{2} \omega^2{}_{CD} L_{CD} \right) \dfrac{2\sqrt{3}}{3} \dfrac{1}{L_{CD}} \\ \varepsilon_{BC} = \dfrac{1}{L_{BC}} \left(-\dfrac{1}{2} \omega^2{}_{AB} L_{AB} - \dfrac{\sqrt{3}}{2} \omega^2{}_{CD} L_{CD} + \dfrac{1}{2} L_{CD} \varepsilon_{DC} \right) \end{cases}$$

$$\begin{cases} \varepsilon_{DC} = \left(\dfrac{\sqrt{3}}{2} \dfrac{1}{L_{AB}} - \dfrac{4}{3} \dfrac{1}{L_{BC}} - \dfrac{1}{6} \dfrac{1}{L_{CD}} \right) \dfrac{2\sqrt{3}}{3} \dfrac{L^2{}_{AB}}{L_{CD}} \omega^2{}_{AB} \Rightarrow \overline{\varepsilon}_{DC} = 9,073 \; s^{-2} \, \overline{1}_z \\ \varepsilon_{BC} = \dfrac{1}{L_{BC}} \left(-\dfrac{1}{2} \omega^2{}_{AB} L_{AB} - \dfrac{\sqrt{3}}{2} \left(\dfrac{1}{\sqrt{3}} \dfrac{L_{AB}}{L_{CD}} \omega_{AB} \right)^2 L_{CD} + \dfrac{1}{2} L_{CD} \varepsilon_{DC} \right) \Rightarrow \overline{\varepsilon}_{BC} = -26,4618 \; s^{-2} \, \overline{1}_z \end{cases}$$

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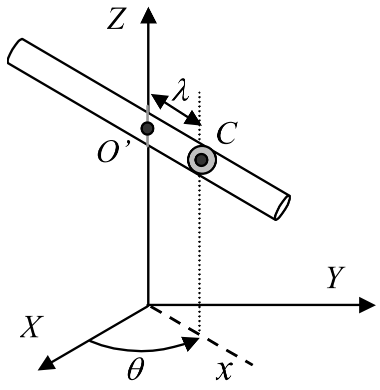
4. $$\begin{cases} \overline{1}_x = \cos\theta \, \overline{1}_X + \sin\theta \, \overline{1}_Y \\ \overline{1}_y = -\sin\theta \, \overline{1}_X + \cos\theta \, \overline{1}_Y \quad \text{et} \quad \overline{\omega} = \omega \, \overline{1}_z \\ \overline{1}_z = \overline{1}_z \end{cases}$$



$$\overline{v}_C = \overline{v}_{C-rel} + \overline{v}_{C-entr} \quad \text{avec} \quad \begin{cases} \overline{v}_{C-rel} = \left. \dfrac{d\overline{O'C}}{dt} \right|_{rel} = \dot{\lambda} \, \overline{1}_x \\ \overline{v}_{C-entr} = \overline{v}_{O'} + \overline{\omega} \times \overline{O'C} = 0 + \lambda\omega \, \overline{1}_y \end{cases}$$

$$\overline{a}_C = \overline{a}_{C-rel} + \overline{a}_{C-entr} + \overline{a}_{C-cor}$$

$$\text{avec} \quad \begin{cases} \overline{a}_{C-rel} = \ddot{\lambda} \, \overline{1}_x \\ \overline{a}_{C-cor} = 2\overline{\omega} \times \overline{v}_{C-rel} = 2\omega \, \overline{1}_z \times \dot{\lambda} \, \overline{1}_x = 2\omega\dot{\lambda} \, \overline{1}_y \\ \overline{a}_{C-entr} = \overline{a}_{O'} + \overline{\omega}_T \times \left(\overline{\omega}_T \times \overline{O'C} \right) + \overline{\varepsilon}_T \times \overline{O'C} = 0 - \lambda\omega^2 \, \overline{1}_x + 0 \end{cases}$$

$$\Rightarrow \overline{a}_C = \ddot{\lambda} \, \overline{1}_x - \lambda\omega^2 \, \overline{1}_x + 2\omega\dot{\lambda} \, \overline{1}_y$$

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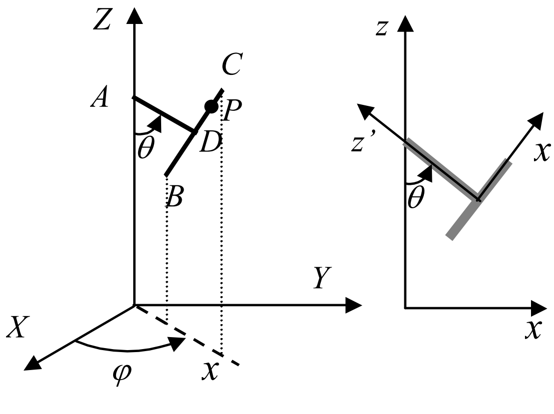
5. $$\begin{cases} \overline{1}_{x'} = \cos\theta \, \overline{1}_x + \sin\theta \, \overline{1}_z \\ \overline{1}_{y'} = \overline{1}_y \\ \overline{1}_{z'} = -\sin\theta \, \overline{1}_x + \cos\theta \, \overline{1}_z \end{cases} \quad \text{et} \quad \begin{cases} \overline{1}_x = \cos\theta \, \overline{1}_{x'} - \sin\theta \, \overline{1}_{z'} \\ \overline{1}_y = \overline{1}_{y'} \\ \overline{1}_z = \sin\theta \, \overline{1}_{x'} + \cos\theta \, \overline{1}_{z'} \end{cases}$$



Cinématique (formule de distribution des vitesses) avec les vecteurs exprimés dans le repère $x'y'z'$:

a. $\overline{\omega}_T = \sin\theta\dot{\phi} \, \overline{1}_{x'} - \dot{\theta} \, \overline{1}_{y'} + \cos\theta\dot{\phi} \, \overline{1}_{z'}$ et $\overline{\omega}_{x'y'z'/XYZ} = \overline{\omega}_T = \sin\theta\dot{\phi} \, \overline{1}_{x'} - \dot{\theta} \, \overline{1}_{y'} + \cos\theta\dot{\phi} \, \overline{1}_{z'}$

$$\Rightarrow \overline{\varepsilon}_T = \left. \frac{d\overline{\omega}_T}{dt} \right|_{rel} + \overline{\omega}_{x'y'z'/XYZ} \times \overline{\omega}_T = \left(\cos\theta\dot{\theta}\dot{\phi} + \sin\theta\ddot{\phi} \right) \overline{1}_{x'} - \ddot{\theta} \, \overline{1}_{y'} + \left(\cos\theta\ddot{\phi} - \sin\theta\dot{\theta}\dot{\phi} \right) \overline{1}_{z'}$$

$$\overline{AD} = -L \, \overline{1}_{z'}$$

$$\overline{v}_D = \overline{v}_A + \overline{\omega}_T \times \overline{AD} = 0 + L\dot{\theta} \, \overline{1}_{x'} + L\sin\theta\dot{\phi} \, \overline{1}_{y'}$$

$$\overline{a}_D = \overline{a}_A + \overline{\omega}_T \times \left(\overline{\omega}_T \times \overline{AD} \right) + \overline{\varepsilon}_T \times \overline{AD} = L\left(\ddot{\theta} - \sin\theta\cos\theta\dot{\phi}^2 \right) \overline{1}_{x'} + L\left(2\cos\theta\dot{\theta}\dot{\phi} + \sin\theta\ddot{\phi} \right) \overline{1}_{y'} + \left(L\sin^2\theta\dot{\phi}^2 + L\dot{\theta}^2 \right) \overline{1}_{z'}$$

b. $\overline{AP} = \lambda \, \overline{1}_{x'} - L \, \overline{1}_{z'}$

$$\overline{v}_{P_{entr}} = \overline{v}_D + \overline{\omega}_T \times \overline{DP} = L\dot{\theta} \, \overline{1}_{x'} + \left(L\sin\theta\dot{\phi} + \lambda\cos\theta\dot{\phi} \right) \overline{1}_{y'} + \lambda\dot{\theta} \, \overline{1}_{z'} \quad \text{et} \quad \overline{v}_{P_{rel}} = \dot{\lambda} \, \overline{1}_{x'}$$

$$\Rightarrow \overline{v}_P = \left(L\dot{\theta} + \dot{\lambda} \right) \overline{1}_{x'} + \left(L\sin\theta\dot{\phi} + \lambda\cos\theta\dot{\phi} \right) \overline{1}_{y'} + \lambda\dot{\theta} \, \overline{1}_{z'}$$

Cinématique (formule de distribution des vitesses) avec les vecteurs exprimés dans le repère xyz :

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a. $\overline{\omega}_T = -\dot{\theta}\overline{1}_y + \dot{\phi}\overline{1}_z$; $\overline{\omega}_{xyz/XYZ} = \dot{\phi}\overline{1}_z$ => $\overline{\varepsilon}_T = \left.\dfrac{d\overline{\omega}_2}{dt}\right|_{rel} + \overline{\omega}_{xyz/XYZ} \times \overline{\omega}_T = \dot{\theta}\dot{\phi}\overline{1}_x - \ddot{\theta}\overline{1}_y + \ddot{\phi}\overline{1}_z$;

$\overline{AD} = L\sin\theta\overline{1}_x - L\cos\theta\overline{1}_z$

$\overline{v}_D = \overline{v}_A + \overline{\omega}_T \times \overline{AD} = L\cos\theta\dot{\theta}\overline{1}_x + L\sin\theta\dot{\phi}\overline{1}_y + L\sin\theta\dot{\theta}\overline{1}_z$

$$\overline{a}_D = \overline{a}_A + \overline{\omega}_T \times \left(\overline{\omega}_T \times \overline{AD}\right) + \overline{\varepsilon}_T \times \overline{AD} = \left|\begin{array}{l} \left(L\cos\theta\ddot{\theta} - L\sin\theta\dot{\phi}^2 - L\sin\theta\dot{\theta}^2\right)\overline{1}_x \\ +\left(2L\cos\theta\dot{\theta}\dot{\phi} + L\sin\theta\ddot{\phi}\right)\overline{1}_y \\ +\left(L\cos\theta\dot{\theta}^2 + L\sin\theta\ddot{\theta}\right)\overline{1}_z \end{array}\right.$$

b. $\overline{AP} = \left(L\sin\theta + \lambda\cos\theta\right)\overline{1}_x + \left(-L\cos\theta + \lambda\sin\theta\right)\overline{1}_z$

$\overline{v}_{P_{entr}} = \overline{v}_A + \overline{\omega}_{xyz/XYZ} \times \overline{AP} = \left(L\sin\theta + \lambda\cos\theta\right)\dot{\phi}\overline{1}_y$

$\overline{v}_{P_{rel}} = \left.\dfrac{d\overline{AP}}{dt}\right|_{rel-xyz} = \left(L\cos\theta\dot{\theta} + \dot{\lambda}\cos\theta - \lambda\sin\theta\dot{\theta}\right)\overline{1}_x + \left(L\sin\theta\dot{\theta} + \dot{\lambda}\sin\theta + \lambda\cos\theta\dot{\theta}\right)\overline{1}_z$

=> $\overline{v}_P = \left(L\cos\theta\dot{\theta} - \lambda\sin\theta\dot{\theta} + \dot{\lambda}\cos\theta\right)\overline{1}_x + \left(L\sin\theta\dot{\phi} + \lambda\cos\theta\dot{\phi}\right)\overline{1}_y + \left(L\sin\theta\dot{\theta} + \lambda\cos\theta\dot{\theta} + \dot{\lambda}\sin\theta\right)\overline{1}_z$

Par dérivation des vecteurs dans le repère *x'y'z'* :

a. $\overline{\omega}_{x'y'z'/XYZ} = \sin\theta\dot{\phi}\overline{1}_{x'} - \dot{\theta}\overline{1}_{y'} + \cos\theta\dot{\phi}\overline{1}_{z'}$

$\overline{AD} = -L\overline{1}_{z'}$ => A fixe : $\overline{v}_D = \underbrace{\left.\dfrac{d\overline{AD}}{dt}\right|_{rel}}_{\overline{v}_{D_{rel}}} + \underbrace{\overline{\omega}_{x'y'z'/XYZ} \times \overline{AD}}_{\overline{v}_{D_{entr}}} = 0 + L\sin\theta\dot{\phi}\overline{1}_{y'} + L\dot{\theta}\overline{1}_{x'}$

$\overline{a}_D = \left.\dfrac{d\,\overline{v}_D}{dt}\right|_{rel} + \overline{\omega}_{x'y'z'/XYZ} \times \overline{v}_D = L\left(\ddot{\theta} - \sin\theta\cos\theta\dot{\phi}^2\right)\overline{1}_{x'} + L\left(2\cos\theta\dot{\theta}\dot{\phi} + \sin\theta\ddot{\phi}\right)\overline{1}_{y'} + L\left(\sin^2\theta\dot{\phi}^2 + \dot{\theta}^2\right)\overline{1}_{z'}$

$\overline{AP} = \lambda\overline{1}_{x'} - L\overline{1}_{z'}$ => $\overline{v}_P = \left.\dfrac{d\overline{AP}}{dt}\right|_{rel} + \overline{\omega}_{x'y'z'/XYZ} \times \overline{AP} = \left(L\dot{\theta} + \dot{\lambda}\right)\overline{1}_{x'} + \left(\lambda\cos\theta\dot{\phi} + L\sin\theta\dot{\phi}\right)\overline{1}_{y'} + \lambda\dot{\theta}\overline{1}_{z'}$

$\overline{a}_P = \left.\dfrac{d\overline{v}_P}{dt}\right|_{rel} + \overline{\omega}_{x'y'z'/XYZ} \times \overline{v}_P =$

$$\left\{\begin{array}{l} \left(L\ddot{\theta} + \ddot{\lambda} - \left(\lambda\cos^2\theta + L\cos\theta\sin\theta\right)\dot{\phi}^2 - \lambda\dot{\theta}^2\right)\overline{1}_{x'} \\ +\left(2\dot{\lambda}\cos\theta\dot{\phi} + \left(-\lambda sin\theta + 2L\cos\theta\right)\dot{\theta}\dot{\phi} + \left(\lambda\cos\theta + L\sin\theta\right)\ddot{\phi} - \lambda\sin\theta\dot{\theta}\dot{\phi}\right)\overline{1}_{y'} \\ +\left(\lambda\ddot{\theta} + 2\dot{\lambda}\dot{\theta} + L\dot{\theta}^2 + \left(\lambda\sin\theta\cos\theta + L\sin^2\theta\right)\dot{\phi}^2\right)\overline{1}_{z'} \end{array}\right.$$

Par dérivation dans les axes *xyz*: $\overline{\omega}_{xyz/XYZ} = \dot{\phi}\overline{1}_z$

$\overline{AD} = L\sin\theta\overline{1}_x - L\cos\theta\overline{1}_z$ => $\overline{v}_D = \left.\dfrac{d\overline{AD}}{dt}\right|_{rel} + \overline{\omega}_{xyz/XYZ} \times \overline{AD} = L\cos\dot{\theta}\overline{1}_x + L\sin\theta\dot{\phi}\overline{1}_y + L\sin\theta\dot{\theta}\overline{1}_z$

Par dérivation des vecteurs dans le repère *xyz* :

$\overline{\omega}_{xyz/XYZ} = \dot{\phi}\overline{1}_z$

$\overline{AD} = L\sin\theta\overline{1}_x - L\cos\theta\overline{1}_z$ => $\overline{v}_D = \left.\dfrac{d\overline{AD}}{dt}\right|_{rel} + \overline{\omega}_{xyz/XYZ} \times \overline{AD} = L\cos\theta\dot{\theta}\overline{1}_x + L\sin\theta\dot{\phi}\overline{1}_y + L\sin\theta\dot{\theta}\overline{1}_z$

$$\overline{a}_D = \left.\dfrac{d\,\overline{v}_D}{dt}\right|_{rel} + \overline{\omega}_{xyz/XYZ} \times \overline{v}_D = \left\{\begin{array}{l} \left(L\cos\theta\ddot{\theta} - L\sin\theta\dot{\theta}^2 + L\sin\theta\dot{\phi}^2\right)\overline{1}_x \\ +\left(2L\cos\theta\dot{\theta}\dot{\phi} + L\sin\theta\ddot{\phi}\right)\overline{1}_y \\ +\left(L\cos\theta\dot{\phi}^2 + L\sin\theta\ddot{\theta}\right)\overline{1}_z \end{array}\right.$$

b. $\overline{AP}=\left(\lambda\cos\theta+L\sin\theta\right)\overline{1}_x+\left(\lambda\sin\theta-L\cos\theta\right)\overline{1}_z$

$$\overline{v}_P=\left.\frac{d\overline{AP}}{dt}\right|_{rel}+\overline{\omega}_{xyz/XYZ}\times\overline{AP}=\begin{cases}\left(-\lambda\sin\theta\dot\theta+\dot\lambda\cos\theta+L\cos\theta\dot\theta\right)\overline{1}_x\\+\left(\lambda\cos\theta\dot\phi+L\sin\theta\dot\phi\right)\overline{1}_y\\+\left(\lambda\cos\theta\dot\theta+\dot\lambda\sin\theta+L\sin\theta\dot\theta\right)\overline{1}_z\end{cases}$$

$$\overline{a}_P=\left.\frac{d\,\overline{v}_P}{dt}\right|_{rel}+\overline{\omega}_{xyz/XYZ}\times\overline{v}_P$$

$$=\begin{cases}\left(\ddot\lambda\cos\theta-2\dot\lambda\sin\theta\dot\theta+\left(L\cos\theta-\lambda\sin\theta\right)\ddot\theta-\left(\lambda\cos\theta+L\sin\theta\right)\dot\theta^2-\left(\lambda\cos\theta+L\sin\theta\right)\dot\phi^2\right)\overline{1}_x\\+\left(\left(\lambda\cos\theta+L\sin\theta\right)\ddot\phi+2\dot\lambda\cos\theta\dot\phi+\left(2L\cos\theta-2\lambda\sin\theta\right)\dot\theta\dot\phi\right)\overline{1}_y\\+\left(2\dot\lambda\cos\theta\dot\theta+\ddot\lambda\sin\theta+\left(L\cos\theta-\lambda\sin\theta\right)\dot\theta^2+\left(\lambda\cos\theta+L\sin\theta\right)\ddot\theta\right)\overline{1}_z\end{cases}$$

Utilisation des formules du mouvement relatif dans le repère *x'y'z'* :

D = origine du repère mobile ($\overline{\omega}_{x'y'z'/XYZ}=\sin\theta\dot\phi\overline{1}_{x'}-\dot\theta\overline{1}_{y'}+\cos\theta\dot\phi\overline{1}_{z'}=\overline{\omega}_T$)

=> $\overline{\varepsilon}_T=\left(\cos\theta\dot\theta\dot\phi+\sin\theta\ddot\phi\right)\overline{1}_{x'}-\ddot\theta\overline{1}_{y'}+\left(\cos\theta\ddot\phi-\sin\theta\dot\theta\dot\phi\right)\overline{1}_{z'}$

b.

$$\overline{v}_P=\overline{v}_{P-rel}+\overline{v}_{P-entr}\quad\text{avec}\quad\begin{cases}\overline{v}_{P-rel}=\left.\dfrac{d\overline{DP}}{dt}\right|_{rel-x'y'z'}=\dot\lambda\overline{1}_{x'}\\\overline{v}_{P-entr}=\overline{v}_D+\overline{\omega}_T\times\overline{DP}=L\dot\theta\overline{1}_{x'}+\left(L\sin\theta\dot\phi+\lambda\cos\theta\dot\phi\right)\overline{1}_{y'}+\lambda\dot\theta\overline{1}_{z'}\end{cases}$$

$\overline{a}_P=\overline{a}_{P-rel}+\overline{a}_{P-entr}+\overline{a}_{P-cor}$ avec

$\overline{a}_{P-rel}=\ddot\lambda\overline{1}_{x'}$

$\overline{a}_{P-cor}=2\overline{\omega}_{x'y'z'/XYZ}\times\overline{v}_{rel}=2\left(\sin\theta\dot\phi\overline{1}_{x'}-\dot\theta\overline{1}_{y'}+\cos\theta\dot\phi\overline{1}_{z'}\right)\times\dot\lambda\overline{1}_{x'}=+2\dot\lambda\cos\theta\dot\phi\overline{1}_{y'}+2\dot\theta\dot\lambda\overline{1}_{z'}$

$\overline{a}_{P-entr}=\overline{a}_D+\overline{\omega}_T\times\left(\overline{\omega}_T\times\overline{DP}\right)+\overline{\varepsilon}_T\times\overline{DP}$ avec $\overline{DP}=\lambda\overline{1}_{x'}$

$$\overline{a}_{P-entr}=\begin{cases}L\left(\ddot\theta-\sin\theta\cos\theta\dot\phi^2\right)\overline{1}_{x'}+L\left(2\cos\theta\dot\theta\dot\phi+\sin\theta\ddot\phi\right)\overline{1}_{y'}+\left(L\sin^2\theta\dot\phi^2+L\dot\theta^2\right)\overline{1}_{z'}\\+\left(-\lambda\sin\theta\dot\theta\dot\phi\overline{1}_{y'}-\lambda\dot\theta^2\overline{1}_{x'}\right)+\left(\lambda\cos\theta\sin\theta\dot\phi^2\overline{1}_{z'}-\lambda\cos^2\theta\dot\phi^2\overline{1}_{x'}\right)\\+\lambda\ddot\theta\overline{1}_{z'}+\lambda\left(\cos\theta\ddot\phi-\sin\theta\dot\theta\dot\phi\right)\overline{1}_{y'}\end{cases}$$

$$\overline{a}_P=\begin{cases}\left(\ddot\lambda+L\ddot\theta-\lambda\dot\theta^2-\left(L\sin\theta\cos\theta+\lambda\cos^2\theta\right)\dot\phi^2\right)\overline{1}_{x'}\\+\left(2\dot\lambda\cos\theta\dot\phi+\left(2L\cos\theta-\lambda\sin\theta\right)\dot\theta\dot\phi+\left(L\sin\theta+\lambda\cos\theta\right)\ddot\phi-\lambda\sin\theta\dot\theta\dot\phi\right)\overline{1}_{y'}\\+\left(2\dot\theta\dot\lambda+\left(\lambda\cos\theta\sin\theta+L\sin^2\theta\right)\dot\phi^2+L\dot\theta^2+\lambda\ddot\theta\right)\overline{1}_{z'}\end{cases}$$

Par l'utilisation des formules du mouvement relatif dans le repère *xyz* :

A = origine du repère mobile xyz $(\overline{\omega}_1 = \overline{\omega}_{xyz/XYZ} = \dot{\phi}\,\overline{1}_z)$

$$\overline{AP} = \left(L\sin\theta + \lambda\cos\theta\right)\overline{1}_x + \left(\lambda\sin\theta - L\cos\theta\right)\overline{1}_z$$

$$\overline{v}_P = \overline{v}_{P-rel} + \overline{v}_{P-entr} \quad \text{avec} \begin{cases} \overline{v}_{P-rel} = \left.\dfrac{d\,\overline{AP}}{dt}\right|_{rel} = \left(L\cos\theta\dot{\theta} + \dot{\lambda}\cos\theta - \lambda\sin\theta\dot{\theta}\right)\overline{1}_x + \left(\dot{\lambda}\sin\theta + \lambda\cos\theta\dot{\theta} + L\sin\theta\dot{\theta}\right)\overline{1}_z \\ \overline{v}_{P-entr} = \overline{v}_A + \overline{\omega}_1 \times \overline{AP} = \left(L\sin\theta + \lambda\cos\theta\right)\dot{\phi}\,\overline{1}_y \end{cases}$$

$$\overline{a}_P = \overline{a}_{P-rel} + \overline{a}_{P-entr} + \overline{a}_{P-cor} \quad \text{avec}$$

$$\begin{cases} \overline{a}_{P-rel} = \begin{cases} \left(+\ddot{\lambda}\cos\theta - 2\dot{\lambda}\sin\theta\dot{\theta} - \left(L\sin\theta + \lambda\cos\theta\right)\dot{\theta}^2 + \left(L\cos\theta - \lambda\sin\theta\right)\ddot{\theta}\right)\overline{1}_x \\ +\left(\ddot{\lambda}\sin\theta + 2\dot{\lambda}\cos\theta\dot{\theta} + \left(L\cos\theta - \lambda\sin\theta\right)\dot{\theta}^2 + \left(\lambda\cos\theta + L\sin\theta\right)\ddot{\theta}\right)\overline{1}_z \end{cases} \\ \overline{a}_{P-cor} = 2\overline{\omega}_{xyz/XYZ} \times \overline{v}_{P-rel} = 2\left(\left(L\cos\theta - \lambda\sin\theta\right)\dot{\theta}\dot{\phi} + \dot{\lambda}\cos\theta\dot{\phi}\right)\overline{1}_y \\ \overline{a}_{P-entr} = \overline{a}_A + \overline{\omega}_1 \times \left(\overline{\omega}_1 \times \overline{AP}\right) + \overline{\varepsilon}_1 \times \overline{AP} = -\left(L\sin\theta + \lambda\cos\theta\right)\dot{\phi}^2\,\overline{1}_x + \left(L\sin\theta + \lambda\cos\theta\right)\ddot{\phi}\,\overline{1}_y \end{cases}$$

$$\overline{a}_P = \begin{cases} \left(+\ddot{\lambda}\cos\theta - 2\dot{\lambda}\sin\theta\dot{\theta} - \left(L\sin\theta + \lambda\cos\theta\right)\dot{\theta}^2 - \left(L\sin\theta + \lambda\cos\theta\right)\dot{\phi}^2 + \left(L\cos\theta - \lambda\sin\theta\right)\ddot{\theta}\right)\overline{1}_x \\ \left(+2\dot{\lambda}\cos\theta\dot{\phi} + \left(2L\cos\theta - 2\lambda\sin\theta\right)\dot{\theta}\dot{\phi} + \left(L\sin\theta + \lambda\cos\theta\right)\ddot{\phi}\right)\overline{1}_y \\ +\left(\ddot{\lambda}\sin\theta + 2\dot{\lambda}\cos\theta\dot{\theta} + \left(L\cos\theta - \lambda\sin\theta\right)\dot{\theta}^2 + \left(\lambda\cos\theta + L\sin\theta\right)\ddot{\theta}\right)\overline{1}_z \end{cases}$$

Pour les problèmes relatifs au Tps et aux laboratoires, contactez Emmanuelle.Vin@ulb.ac.be
Les énoncés et les corrigés sont accessibles et mis à jour sont sur le site de méca :
http://beams.ulb.ac.be/beams/teaching/meca200/tps.html