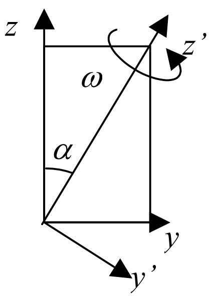
1. O = point appartenant à l'axe de rotation fixe.

	$\overline{\mathfrak{R}} = m\overline{v}_G$	$\overline{M}_O = \overline{\overline{I}}_O.\overline{\omega}$	$T = \frac{1}{2}\overline{\omega}.\overline{\overline{I}}_O.\omega$
1	$\overline{\mathfrak{R}} = -ma\omega\overline{1}_x$	$\overline{M}_O = -P_{xz}\omega\overline{1}_x - P_{yz}\omega\overline{1}_y + I_z\omega\overline{1}_z$ $= -mab\omega\overline{1}_y + \frac{4}{3}ma^2\omega\overline{1}_z$	$T = \frac{1}{2}I_z\omega^2$ $= \frac{2}{3}ma^2\omega^2$
2	$\overline{\mathfrak{R}} = 0$	$\overline{M}_O = -P_{xz}\omega\overline{1}_x - P_{yz}\omega\overline{1}_y + I_z\omega\overline{1}_z$ $= \frac{ma^2}{3}\omega\overline{1}_z$	$T = \frac{1}{2}I_z\omega^2$ $= \frac{1}{6}ma^2\omega^2$
3	$\overline{\mathfrak{R}} = -mb\omega\overline{1}_y$ $+ ma\omega\overline{1}_z$	$\overline{M}_O = +I_x\omega\overline{1}_x - P_{xy}\omega\overline{1}_y - P_{xz}\omega\overline{1}_z$ $= \frac{4}{3}m(a^2+b^2)\omega\overline{1}_x$	$T = \frac{1}{2}I_x\omega^2$ $= \frac{2}{3}m(a^2+b^2)\omega^2$
4	$\overline{\mathfrak{R}} = 0$	$\overline{M}_O = \cancel{-P_{xy}\omega_y\overline{1}_x} + I_y\omega_y\overline{1}_y - P_{yz}\omega_y\overline{1}_z$ $\cancel{-P_{xz}\omega_z\overline{1}_x} - P_{yz}\omega_z\overline{1}_y + I_z\omega_z\overline{1}_z$ $\overline{M}_O = +\frac{4}{3}mb^2\omega\frac{a}{\sqrt{a^2+b^2}}\overline{1}_y - mab\omega\frac{b}{\sqrt{a^2+b^2}}\overline{1}_y$ $+\frac{4}{3}ma^2\omega\frac{b}{\sqrt{a^2+b^2}}\overline{1}_z - mab\omega\frac{a}{\sqrt{a^2+b^2}}\overline{1}_z$ ou $\overline{M}_O = \cancel{-P_{x'z'}\omega_{z'}\overline{1}_{x'}} - P_{y'z'}\omega\overline{1}_{y'} + I_{z'}\omega\overline{1}_{z'}$ $= \frac{1}{3}mab\frac{b^2-a^2}{a^2+b^2}\omega\overline{1}_{y'} + \frac{2}{3}\frac{ma^2b^2}{a^2+b^2}\omega\overline{1}_{z'}$ On aurait pu faire : $\overline{M}_O = \overline{M}_G$	$T = \frac{1}{2}I_{z'}\omega^2$ $= \frac{1}{3}m\frac{a^2b^2}{a^2+b^2}\omega^2$



(4)

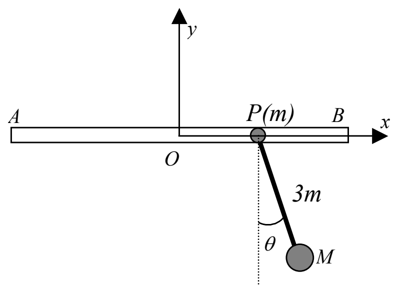
$$I_{z'} = \int y'^2 dm = \int (\cos\alpha y - \sin\alpha z)^2 dm = \cos^2\alpha I_z + \sin^2\alpha I_y - 2\sin\alpha\cos\alpha P_{yz}$$

$$= \frac{b^2}{a^2+b^2}.\frac{4}{3}ma^2 + \frac{a^2}{a^2+b^2}.\frac{4}{3}mb^2 - 2\frac{ab}{a^2+b^2}.mab = \frac{2}{3}\frac{ma^2b^2}{a^2+b^2}$$

$$P_{y'z'} = \int y'z'dm = \int\left(\cos\alpha y - \sin\alpha z\right)\left(\sin\alpha y + \cos\alpha z\right)dm = \cos\alpha\sin\alpha\left(I_z - I_y\right) + \left(\cos^2\alpha - \sin^2\alpha\right)P_{yz}$$

$$= \frac{ab}{a^2+b^2}\left(\frac{4}{3}ma^2 - \frac{4}{3}mb^2\right) + \left(\frac{b^2}{a^2+b^2} - \frac{a^2}{a^2+b^2}\right)mab = \frac{1}{3}\frac{ma^3b}{a^2+b^2} - \frac{1}{3}\frac{mab^3}{a^2+b^2}$$

2.



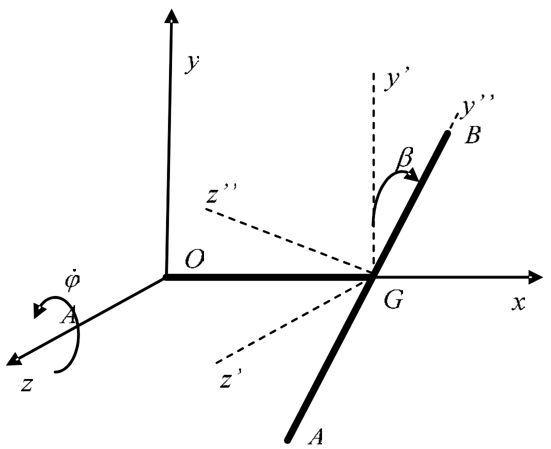
$$\overline{\mathfrak{R}} = \sum_i m\overline{v}_{G_i} = m\overline{v}_P\overline{1}_x + 3m\overline{v}_{G_{tige}} + M\overline{v}_{G_{balle}}$$

$$\text{avec}\begin{cases} x = a\cos\omega t; \\ \overline{OG} = \left(a\cos\omega t + \frac{a}{2}\sin\theta\right)\overline{1}_x - \frac{a}{2}\cos\theta\overline{1}_y\ ; \\ \overline{OG'} = \left(a\cos\omega t + \left(a+R\right)\sin\theta\right)\overline{1}_x - \left(a+R\right)\cos\theta\overline{1}_y \end{cases}$$

$$\overline{\Re}=\left[-ma\sin\omega t\,.\omega\overline{1}_x\right]+\left[3m\left(\left(-a\sin\omega t.\omega+\frac{a}{2}\cos\theta\dot{\theta}\right)\overline{1}_x+\frac{a}{2}\sin\theta\dot{\theta}\overline{1}_y\right)\right]$$
$$+\left[M\left(\left(-a\sin\omega t.\omega+(a+R)\cos\theta\dot{\theta}\right)\overline{1}_x+(a+R)\sin\theta\dot{\theta}\overline{1}_y\right)\right]$$
$$\Rightarrow \overline{\Re}=(4m+M)\left[\left(-\omega a\sin\omega t+a\dot{\theta}cos\theta\right)\overline{1}_x+A\dot{\theta}\sin\theta\overline{1}_y\right]\text{ avec }A=\frac{3m\frac{a}{2}+M(a+R)}{4m+M}$$

$$T=\sum_i T_i=\left[\frac{1}{2}mv_{G_m}^2+\underbrace{\frac{1}{2}\overline{\omega}.\overline{\overline{I}}_{G_m}.\overline{\omega}}_{=0(\text{masse ponct})}\right]+\left[\frac{1}{2}(3m)v_{G_{tige}}^2+\frac{1}{2}\overline{\omega}_{tige}.\overline{\overline{I}}_{G_{tige}}.\overline{\omega}_{tige}\right]+\left[\frac{1}{2}Mv_{G_{balle}}^2+\frac{1}{2}\overline{\omega}_{balle}.\overline{\overline{I}}_{G_{balle}}.\overline{\omega}_{balle}\right]$$
$$T=\left[\frac{1}{2}m\dot{x}^2\right]+\left[\frac{1}{2}3mv_{G_{tige}}^2+\frac{1}{2}\frac{3ma}{12}\dot{\theta}^2\right]+\left[\frac{1}{2}Mv_{G_{balle}}^2+\frac{1}{2}\frac{2}{5}MR^2\dot{\theta}^2\right]$$
$$\Rightarrow T=\left(2m+\frac{M}{2}\right)a^2\omega^2\sin^2\omega t+\frac{1}{2}\left[ma^2+M(a+R)^2+\frac{2}{5}MR^2\right]\dot{\theta}^2$$
$$-\left[\frac{3}{2}ma^2+Ma(a+R)\right]\omega\dot{\theta}\sin\omega t\cos\theta$$
$$\overline{M}_O=\sum_i\overline{M}_{O,i}=\underbrace{\left[\overline{OP}\times m\overline{v}_P\right]}_{=0(\text{pas de rot.})}+\left[\overline{OG}_{Tige}\times m\overline{v}_{G_{tige}}+\overline{M}_G\right]+\left[\overline{OG'}_{balle}\times m\overline{v}_{G'_{balle}}+\overline{M}_{G'_{balle}}\right]$$
$$\overline{OG}_{Tige}\times m\overline{v}_{G_{tige}}=3m\left(\frac{a^2}{4}\dot{\theta}+\frac{a^2}{2}\left(\cos\omega t\sin\theta\dot{\theta}-\cos\theta\sin\omega t.\omega\right)\right)\overline{1}_z\Rightarrow\overline{M}_{G_{tige}}=\overline{\overline{I}}_G.\overline{\omega}=\frac{3ma^2}{12}\dot{\theta}\overline{1}_z$$
$$\overline{OG'}_{balle}\times m\overline{v}_{G_{balle}}=M(a+R)\left((a+R)\dot{\theta}+a\left(\cos\omega t\sin\theta\dot{\theta}-\cos\theta\sin\omega t.\omega\right)\right)\overline{1}_z\Rightarrow\overline{M}_{G_{balle}}=\overline{\overline{I}}_{G'}.\overline{\omega}=\frac{2MR^2}{5}\dot{\theta}\overline{1}_z$$
$$\Rightarrow\overline{M}_O=\left(\left(M(a+R)^2+\frac{2MR^2}{5}+ma^2\right)\dot{\theta}+\left[3m\frac{a^2}{2}+Ma(a+R)\right]\left(\cos\omega t\sin\theta\dot{\theta}-\cos\theta\sin\omega t.\omega\right)\right)\overline{1}_z$$

3.



dans les axes principaux $Gy"z"$ lié à la tige :

$\overline{\omega}=-\dot{\beta}\overline{1}_x+\dot{\varphi}\overline{1}_z=-\dot{\beta}\overline{1}_{x"}+\dot{\varphi}\cos\beta\overline{1}_{z"}-\dot{\varphi}\sin\beta\overline{1}_{y"}$

<u>1. Dans les axes Oxyz</u>

$$\overline{M}_{O_{(S1+S2)}}=\overline{\overline{I}}_{O_{(S1+S2)}}.\overline{\omega}=\left[\begin{pmatrix}0 & / & 0\\0 & & 0\\0 & / & \frac{m_1.L_1^2}{3}\end{pmatrix}+\begin{pmatrix}\frac{m_2.L_2^2}{12} & / & 0\\0 & & \frac{m_2.L_2^2}{12}\cos\beta\sin\beta\\0 & / & \frac{m_2.L_2^2}{12}\cos^2\beta\end{pmatrix}+\begin{pmatrix}0 & / & 0\\0 & & 0\\0 & / & m_2.L_1^2\end{pmatrix}\right]\begin{pmatrix}-\dot{\beta}\\0\\\dot{\varphi}\end{pmatrix}$$

$$\Rightarrow\boxed{\overline{M}_O=-\frac{m_2.L_2^2}{12}\dot{\beta}\overline{1}_x+\frac{m_2.L_2^2}{12}\frac{\sin2\beta}{2}\dot{\varphi}\overline{1}_y+\left(\left(\frac{m_1.L_1^2}{3}+m_2.L_1^2\right)+\frac{m_2.L_2^2}{12}\cos^2\beta\right)\dot{\varphi}\overline{1}_z}$$

$$\frac{d\overline{M_O}}{dt}=\left.\frac{d\overline{M_O}}{dt}\right|_{xyz}+\bar{\omega}_{xyz/XYZ}\times\overline{M_O}=$$

$$\frac{d\overline{M_O}}{dt}=-\frac{m_2.L_2^{\,2}}{12}\ddot{\beta}\overline{1}_x+\frac{m_2.L_2^{\,2}}{12}\frac{\sin 2\beta}{2}\ddot{\varphi}\overline{1}_y+\left(\left(\frac{m_1.L_1^{\,2}}{3}+m_2.L_1^{\,2}\right)+\frac{m_2.L_2^{\,2}}{12}\cos^2\beta\right)\ddot{\varphi}\overline{1}_z$$

$$+\frac{m_2.L_2^{\,2}}{12}\cos 2\beta\dot{\beta}\dot{\varphi}\overline{1}_y+\left(-\frac{m_2.L_2^{\,2}}{12}2\cos\beta\sin\beta\right)\dot{\varphi}\dot{\beta}\overline{1}_z\underbrace{-\frac{m_2.L_2^{\,2}}{12}\dot{\beta}\dot{\varphi}\overline{1}_y-\frac{m_2.L_2^{\,2}}{12}\frac{\sin 2\beta}{2}\dot{\varphi}^2\overline{1}_x}_{\bar{\omega}_{xyz/XYZ}\times\overline{M_O}}$$

2. Dans les axes Oxyz tournant avec les 2 tiges. (S=S1+S2)

$$\overline{M_O}_{(S1+S2)}=\overline{\overline{I}}_{O(S1+S2)}.\bar{\omega}=\left[\begin{pmatrix}0&0&0\\0&\frac{m_1.L_1^{\,2}}{3}&0\\0&0&\frac{m_1.L_1^{\,2}}{3}\end{pmatrix}+\begin{pmatrix}\frac{m_2.L_2^{\,2}}{12}&0&0\\0&0&0\\0&0&\frac{m_2.L_2^{\,2}}{12}\end{pmatrix}+\begin{pmatrix}0&0&0\\0&m_2.L_1^{\,2}&0\\0&0&m_2.L_1^{\,2}\end{pmatrix}\right]\begin{pmatrix}-\dot{\beta}\\-\dot{\varphi}\sin\beta\\\dot{\varphi}\cos\beta\end{pmatrix}$$

$$\boxed{\overline{M_O}=-\frac{m_2.L_2^{\,2}}{12}\dot{\beta}\overline{1}_{x''}-\left(\frac{m_1.L_1^{\,2}}{3}+m_2.L_1^{\,2}\right)\dot{\varphi}\sin\beta\overline{1}_{y''}+\left(\frac{m_1.L_1^{\,2}}{3}+\frac{m_2.L_2^{\,2}}{12}+m_2.L_1^{\,2}\right)\dot{\varphi}\cos\beta\overline{1}_{z''}}$$

$$\frac{d\overline{M_O}}{dt}=\left.\frac{d\overline{M_O}}{dt}\right|_{x''y''z''}+\bar{\omega}_{x''y''z''/XYZ}\times\overline{M_O}=$$

$$-\frac{m_2.L_2^{\,2}}{12}\ddot{\beta}\overline{1}_{x''}-\left(\frac{m_1.L_1^{\,2}}{3}+m_2.L_1^{\,2}\right)\ddot{\varphi}\sin\beta\overline{1}_{y''}-\left(\frac{m_1.L_1^{\,2}}{3}+m_2.L_1^{\,2}\right)\dot{\varphi}\cos\beta\dot{\beta}\overline{1}_{y''}$$

$$+\left(\frac{m_1.L_1^{\,2}}{3}+\frac{m_2.L_2^{\,2}}{12}+m_2.L_1^{\,2}\right)\ddot{\varphi}\cos\beta\overline{1}_{z''}-\left(\frac{m_1.L_1^{\,2}}{3}+\frac{m_2.L_2^{\,2}}{12}+m_2.L_1^{\,2}\right)\dot{\varphi}\sin\beta\dot{\beta}\overline{1}_{z''}$$

$$-\frac{m_2.L_2^{\,2}}{12}\dot{\beta}\left(\dot{\varphi}\cos\beta\overline{1}_{y''}+\dot{\varphi}\sin\beta\overline{1}_{z''}\right)-\left(\frac{m_1.L_1^{\,2}}{3}+m_2.L_1^{\,2}\right)\dot{\varphi}\sin\beta\left(-\dot{\beta}\overline{1}_{z''}-\dot{\varphi}\cos\beta\overline{1}_{x''}\right)$$

$$+\left(\frac{m_1.L_1^{\,2}}{3}+\frac{m_2.L_2^{\,2}}{12}+m_2.L_1^{\,2}\right)\dot{\varphi}\cos\beta\left(\dot{\beta}\overline{1}_{y''}-\dot{\varphi}\sin\beta\overline{1}_{x''}\right)$$

3. en considérant séparément les deux solides.

$$\overline{M_O}=\overline{M_O}_{(S1)}+\overline{M_O}_{(S2)}$$

$$\overline{M_O}_{(S1)}=\overline{\overline{I}}_O.\bar{\omega}=\begin{pmatrix}/&/&0\\/&/&0\\/&/&\frac{m_1.L_1^{\,2}}{3}\end{pmatrix}\begin{pmatrix}0\\0\\\dot{\varphi}\end{pmatrix}=\frac{m_1.L_1^{\,2}}{3}\dot{\varphi}\overline{1}_z$$

$$\overline{M_O}_{(S2)}=\overline{M_G}+\overline{OG}\times\overline{R}=\overline{\overline{I}}_G.\bar{\omega}_2+m_2\left(L_1\overline{1}_x\right)\times L_1\dot{\varphi}\overline{1}_y$$

$$\overline{\overline{I}}_G.\bar{\omega}_2=\begin{pmatrix}\frac{m_2.L_2^{\,2}}{12}&0&0\\0&0&0\\0&0&\frac{m_2.L_2^{\,2}}{12}\end{pmatrix}\begin{pmatrix}-\dot{\beta}\\-\dot{\varphi}\sin\beta\\\dot{\varphi}\cos\beta\end{pmatrix}=-\frac{m_2.L_2^{\,2}}{12}\dot{\beta}\overline{1}_{x''}+\frac{m_2.L_2^{\,2}}{12}\dot{\varphi}\cos\beta\overline{1}_{z''}$$

$$\Rightarrow \overline{\overline{I}}_G.\bar{\omega}_2=-\frac{m_2.L_2^{\,2}}{12}\dot{\beta}\overline{1}_x+\frac{m_2.L_2^{\,2}}{12}\dot{\varphi}\cos\beta\sin\beta\overline{1}_y+\frac{m_2.L_2^{\,2}}{12}\dot{\varphi}\cos\beta\cos\beta\overline{1}_z$$

$$\overline{M_O}_{(S2)}=-\frac{m_2.L_2^{\,2}}{12}\dot{\beta}\overline{1}_x+\frac{m_2.L_2^{\,2}}{24}\dot{\varphi}\sin 2\beta\overline{1}_y+\left(\frac{m_2.L_2^{\,2}}{12}\dot{\varphi}\cos^2\beta+m_2L_1^{\,2}\dot{\varphi}\right)\overline{1}_z$$

$$\Rightarrow \overline{M_O}=-\frac{m_2.L_2^{\,2}}{12}\dot{\beta}\overline{1}_x+\frac{m_2.L_2^{\,2}}{24}\dot{\varphi}\sin 2\beta\overline{1}_y+\left(\frac{m_2.L_2^{\,2}}{12}\dot{\varphi}\cos^2\beta+m_2L_1^{\,2}\dot{\varphi}+\frac{m_1.L_1^{\,2}}{3}\dot{\varphi}\right)\overline{1}_z$$

4.1

$$M = k\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} d\theta \int_0^H zdz \int_0^{\frac{R}{H}z} rdr = k\frac{\left(\frac{R}{H}\right)^2}{2}\frac{(H)^4}{4}\pi = k\pi\frac{R^2H^2}{8}$$

$$I_z = k\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} d\theta \int_0^H zdz \int_0^{\frac{R}{h}z} r^3dr = k\frac{\left(\frac{R}{H}\right)^4}{4}\frac{(H)^6}{6}\pi = \frac{k\pi}{24}H^2R^4 = \frac{MR^2}{3}$$

$$I_{xy} = k\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} d\theta \int_0^H zdz \int_0^{\frac{R}{H}z} \left(z^2\right)rdr = k\pi\frac{\left(\frac{R}{H}\right)^2}{2}\frac{H^6}{6} = \frac{2MH^2}{3}$$

$$I_{yz} = k\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} d\theta \int_0^H zdz \int_0^{\frac{R}{H}z} \left(r\cos\theta\right)^2 rdr = k\frac{\pi}{2}\frac{\left(\frac{R}{H}\right)^4}{4}\frac{H^6}{6} = \frac{MR^2}{6} = I_{xz}$$

$$\Rightarrow I_x = I_{xy} + I_{xz} = \frac{MR^2}{6} + \frac{2MH^2}{3} = I_y;$$

$P_{xy} = P_{yz} = 0$ grace à l'intégration de y sur un domaine symétrique

$$P_{zx} = k\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} d\theta \int_0^H zdz \int_0^{\frac{R}{H}z} z\left(r\cos\theta\right)rdr = k2\frac{\left(\frac{R}{H}\right)^3}{3}\frac{H^6}{6} = \frac{8M}{9\pi}RH$$

4.2 Trouvons les points P appartenant à l'axe *z* où les moments d'inertie (I_{xP} , I_{yP} , I_{zP}) sont égaux dans le repère $Px_Py_Pz_P$

$$\underbrace{\frac{MR^2}{6} + \frac{2MH^2}{3}}_{I_x} - M\underbrace{\left(\frac{3H}{4}\right)^2}_{d(\text{axe } x,\text{axe } x_G)} + M\underbrace{d^2}_{d(\text{axe } x_G,\text{axe } x_P)^2} = \frac{MR^2}{3}$$

$\Rightarrow$ 2 racines réelles $d = \pm\sqrt{\frac{R^2}{6} - \left(\frac{5}{48}\right)H^2}$ à condition que $\frac{\text{H}}{\text{R}} < \sqrt{\frac{8}{5}}$

Il faut en plus que en ces points le produit d'inertie $P_{xp\,z}$ soit nul aussi pour qu'on soit bien dans des axes principaux (ce qui ne semble pas être le cas)
=>Il n'existe pas de point de l'axe *z* qui annule le produit d'inertie dans un système d'axe parallèle à *xyz*, car le décalage de l'axe en *x* est nul =>

4.3 L'angle alpha est l'angle entre l'axe *z* et l'axe *z'*. Par la formule de changement d'axe ($\bar{1}_{z'} = \left(0; \sin\alpha; \cos\alpha\right)$)

$$I_{z'} = I_z\cos^2\alpha + I_y\sin^2\alpha \quad \text{où} \quad \cos^2\alpha = \frac{H^2}{H^2+R^2} \quad \text{et} \quad \sin^2\alpha = \frac{R^2}{H^2+R^2}$$

$$\Rightarrow I_{z'} = \frac{MR^2}{3}\frac{H^2}{H^2+R^2} + \left(\frac{MR^2}{6} + \frac{2MH^2}{3}\right)\frac{R^2}{H^2+R^2} = \frac{R^2}{H^2+R^2}\left(MH^2 + \frac{MR^2}{6}\right)$$

4.4 $\overline{M_O} = \overline{\overline{I}}_O.\overline{\omega}$ avec $\overline{\omega} = \omega\overline{1}_{z'}$ $\Rightarrow \overline{M_O} = -P_{x'z'}.\omega\overline{1}_{x'} - P_{y'z'}.\omega\overline{1}_{y'} + I_{z'}\omega\overline{1}_{z'}$

$$P_{y'z'} = \int(\cos\alpha\, y - \sin\alpha\, z)(\sin\alpha\, y + \cos\alpha\, z)\,dm = \cos\alpha\sin\alpha\left(I_z - I_y\right) + \left(\cos^2\alpha - \sin^2\alpha\right)P_{yz}$$

$$= \frac{HR}{H^2+R^2}\left(\frac{MR^2}{3} - \left(\frac{MR^2}{6} + \frac{2MH^2}{3}\right)\right) + \left(\frac{H^2-R^2}{H^2+R^2}\right)\underbrace{P_{yz}}_{\substack{dom.sym \\ \text{en } y \Rightarrow 0}} = \frac{HR}{H^2+R^2}\left(\frac{MR^2}{6} - \frac{2MH^2}{3}\right)$$

$$P_{x'z'} = \int x(\sin\alpha\, y + \cos\alpha\, z)\,dm = \sin\alpha P_{xy} + \cos\alpha P_{xz} = \frac{H}{\sqrt{H^2+R^2}}\frac{8MRH}{9\pi}$$

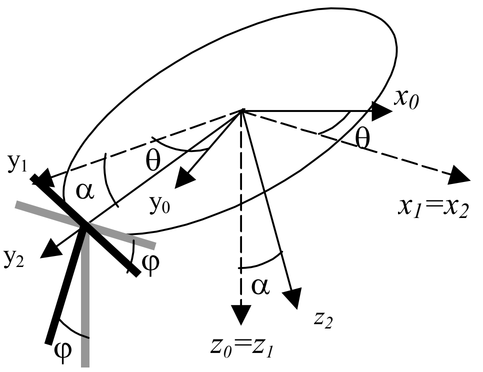
$$\Rightarrow \overline{M_O} = -\frac{H\omega}{\sqrt{H^2+R^2}}\frac{4MRH}{9\pi}\overline{1}_{x'} + \frac{HR}{H^2+R^2}\left(\frac{MH^2}{3}\right)\omega\overline{1}_{y'} + \frac{R^2}{H^2+R^2}\left(\frac{MH^2}{2} + \frac{MR^2}{6}\right)\omega\overline{1}_{z'}$$

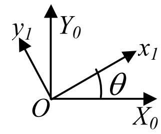
ou

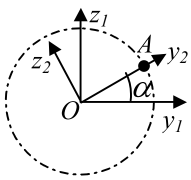
$$\overline{M_O} = -P_{xz}\omega\cos\alpha\,\overline{1}_x + I_y\omega\sin\alpha\,\overline{1}_y + I_z\omega\cos\alpha\,\overline{1}_z$$

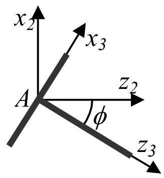
$$\overline{M_O} = -\frac{4MRH}{9\pi}\frac{H\omega}{\sqrt{H^2+R^2}}\overline{1}_x + \left(\frac{MR^2}{6} + \frac{MH^2}{3}\right)\frac{R\omega}{\sqrt{H^2+R^2}}\overline{1}_y + \frac{MR^2}{6}\frac{H\omega}{\sqrt{H^2+R^2}}\overline{1}_z$$

5.









=> les vecteurs de Darboux :

$\overline{\omega}_{R_1/R_0} = \dot{\theta}\overline{1}_{z_1}$; $\overline{\omega}_{R_2/R_0} = \overline{\omega}_{R_2/R_1} + \overline{\omega}_{R_1/R_0} = \dot{\theta}\overline{1}_{z_1}$

$\overline{\omega}_{R_3/R_0} = \overline{\omega}_{R_3/R_2} + \overline{\omega}_{R_2/R_0} = \dot{\theta}\overline{1}_{z_1} - \dot{\phi}\overline{1}_{y_2}$

=> les vecteurs vitesses angulaires des solides :

$\overline{\omega}_{Cercle} = \overline{\omega}_{R_1/R_0} = \dot{\theta}\overline{1}_{z_1}$ et $\overline{\omega}_T = \overline{\omega}_{R_3/R_0} = \dot{\theta}\overline{1}_{z_1} - \dot{\phi}\overline{1}_{y_2}$

R_0 : Axe $OX_0Y_0Z_0$ fixe. ; R_1 : Axe $Ox_1y_1z_1$ tourne autour de l'axe $Z_0 = z_1$ (θ)

R_2 : Axe $Ox_2y_2z_2$ incliné d'un angle α par rapport aux axes $Ox_1y_1z_1$, avec z_2 lié à la tige, R_2 tourne autour de l'axe z_1 (ϕ)

R_3 : Axe $Gx_3y_3z_3$ liés au cube et tourne autour de z_1 (ϕ) et z_2 ()

$T = T^{(C)} + T^{(BC)} + T^{(AD)}$ et $\overline{M}_O = \overline{M}_O^{(C)} + \overline{M}_O^{(BC)} + \overline{M}_O^{(AD)}$

1. Dans les axes principaux du cercle (Repère R_2):

$$T^{(C)} = \frac{1}{2}\overline{\omega}.\overline{\overline{I}}_G.\overline{\omega} = \frac{1}{2}\left(Ap^2 + Bq^2 + Cr^2\right) \text{ avec } \begin{cases} A = B = \dfrac{MR^2}{2} ; C = MR^2 \\ \overline{\omega}_T = \dot{\theta}\overline{1}_{z_1} = \sin\alpha\dot{\theta}\overline{1}_{y_2} + \cos\alpha\dot{\theta}\overline{1}_{z_2} \end{cases}$$

$$\Rightarrow T^{(T)} = \frac{MR^2}{4}\dot{\theta}^2\left(1 + \cos^2\alpha\right)$$

$\overline{M}_O = \overline{\overline{I}}_O.\overline{\omega} = Ap\overline{1}_{x_2} + Bq\overline{1}_{y_2} + Cr\overline{1}_{z_2}$ $\overline{M}_O^{(T)}$ => $\overline{M}_O =$

2. Dans les axes principaux du T (des tiges soudées) (Repère R_3)

$$T^{(BC)}=\frac{1}{2}mv_G^2+\frac{1}{2}\bar{\omega}.\bar{\bar{I}}_G.\bar{\omega}\underset{G=A}{=}\frac{1}{2}Mv_A^2+\frac{1}{2}\left(A'p'^2+B'q'^2+C'r'^2\right)$$

$$\text{avec}\begin{cases}\overline{OA}=R\bar{1}_{y_2}\Rightarrow\bar{v}_A=R\dfrac{d\bar{1}_{y_2}}{dt}=R\bar{\omega}_{R_2/R_0}\times\bar{1}_{y_2}=-R\dot{\theta}\cos\alpha\,\bar{1}_{x_2}\ \text{avec}\ \bar{\omega}_{R_2/R_0}=\dot{\theta}\bar{1}_{z_1}=\dot{\theta}\left(\sin\alpha\,\bar{1}_{y_2}+\cos\alpha\,\bar{1}_{z_2}\right)\\ \bar{\omega}_T=\left(\sin\alpha\dot{\theta}-\dot{\phi}\right)\bar{1}_{y_2}+\cos\alpha\dot{\theta}\bar{1}_{z_2}=\left(\sin\alpha\dot{\theta}-\dot{\phi}\right)\bar{1}_{y_3}+\cos\alpha\dot{\theta}\left(\sin\phi\,\bar{1}_{x_3}+\cos\phi\,\bar{1}_{z_3}\right)=p'\bar{1}_{x_3}+q'\bar{1}_{y_3}+r'\bar{1}_{z_3}\\ \Rightarrow p'=\sin\phi\cos\alpha\dot{\theta};\ q'=\sin\alpha\dot{\theta}-\dot{\phi};\ r'=\cos\phi\cos\alpha\dot{\theta}\quad\text{et}\quad A'=0;\ B'=C'=I_{x_{3,A}}=\dfrac{MR^2}{12}\end{cases}$$

$$T^{(BC)}=\frac{1}{2}MR^2\dot{\theta}^2\cos^2\alpha+\frac{1}{2}\left(\frac{MR^2}{12}\left(\sin\alpha\dot{\theta}-\dot{\phi}\right)^2+\frac{MR^2}{12}\cos^2\phi\cos^2\alpha\dot{\theta}^2\right)$$

$\bar{M}_O^{(BC)}=\bar{M}_A+\overline{OA}\times\bar{R}$ avec $\bar{M}_A=A'p'\bar{1}_{x_3}+B'q'\bar{1}_{y_3}+C'r'\bar{1}_{z_3}$ car A=Centre de masse G

$$\bar{M}_O^{(BC)}=-\frac{MR^2}{12}\left(\sin\alpha\dot{\theta}-\dot{\phi}\right)\bar{1}_{y_3}+\frac{MR^2}{12}\cos\phi\cos\alpha\dot{\theta}\bar{1}_{z_3}+-MR^2\dot{\theta}\cos\alpha\left(\cos\theta\,\bar{1}_{x_3}\right)$$

3. Dans les axes principaux du T (des tiges soudées) (Repère R_3)

$$T^{(AD)}=\frac{1}{2}mv_E^2+\frac{1}{2}\bar{\omega}.\bar{\bar{I}}_E.\bar{\omega}=\frac{1}{2}Mv_E^2+\frac{1}{2}\left(A''p'^2+B''q'^2+C''r'^2\right)\quad\text{avec } E=\text{milieu de } AD$$

$$\bar{1}_{z_3}=\cos\psi\,\bar{1}_{z_2}-\sin\psi\,\bar{1}_{x_2};\ \bar{1}_{x_2}=\cos\psi\,\bar{1}_{x_3}-\sin\psi\,\bar{1}_{z_3};\ \bar{1}_{y_2}=\bar{1}_{y_3};\ \bar{1}_{z_2}=\cos\psi\,\bar{1}_{z_3}+\sin\psi\,\bar{1}_{x_3};$$

$$\text{avec}\begin{cases}\bar{v}_E=\bar{v}_A+\bar{\omega}_{AD}\times\overline{AE}=-R\cos\alpha\dot{\theta}\bar{1}_{x_2}+\left[\left(\dot{\psi}+\sin\alpha\dot{\theta}\right)\bar{1}_{y_2}+\cos\alpha\dot{\theta}\bar{1}_{z_2}\right]\times\dfrac{R}{2}\underbrace{\left(\cos\psi\,\bar{1}_{z_2}-\sin\psi\,\bar{1}_{x_2}\right)}_{\bar{1}_{z_3}}\\ \bar{v}_E=-R\cos\alpha\cos\psi\dot{\theta}\bar{1}_{x_3}+R\cos\alpha\sin\psi\dot{\theta}\bar{1}_{z_3}+\left[\left(\dot{\psi}+\sin\alpha\dot{\theta}\right)\bar{1}_{y_3}+\cos\alpha\dot{\theta}\cos\psi\,\bar{1}_{z_3}+\cos\alpha\dot{\theta}\sin\psi\,\bar{1}_{x_3}\right]\times\dfrac{R}{2}\bar{1}_{z_3}\\ \bar{v}_E=+\left(\dfrac{R}{2}\left(\dot{\psi}+\sin\alpha\dot{\theta}\right)-R\cos\alpha\cos\psi\dot{\theta}\right)\bar{1}_{x_3}-\dfrac{R}{2}\cos\alpha\sin\psi\dot{\theta}\bar{1}_{y_3}+R\cos\alpha\sin\psi\dot{\theta}\bar{1}_{z_3}\\ A''=B''=\dfrac{MR^2}{12};\ C''=0\end{cases}$$

$$T^{(AD)}=\frac{MR^2}{2}\dot{\theta}^2\cos^2\alpha+MR^2\dot{\theta}\cos\alpha\cos\psi\left(\dot{\theta}\sin\alpha-\dot{\psi}\right)+\frac{MR^2}{6}\left(\dot{\theta}\sin\alpha-\dot{\psi}\right)^2+\dot{\theta}^2\cos^2\alpha\sin^2\psi$$

$\bar{M}_O^{(AD)}=\bar{M}_E+\overline{OE}\times\bar{R}$ avec $\bar{M}_E=A''p'\bar{1}_{x_3}+B''q'\bar{1}_{y_3}+C''r'\bar{1}_{z_3}$

$$\bar{M}_O^{(AD)}=\left[\frac{1}{3}MR^2\dot{\theta}\cos\alpha\sin\psi+\frac{MR^2}{2}\left(\dot{\theta}\sin\alpha-\dot{\psi}\right)\sin\psi\right]\bar{1}_{x_2}$$

$$+\left[\frac{1}{3}MR^2\left(\dot{\theta}\sin\alpha-\dot{\psi}\right)-\frac{MR^2}{2}\dot{\theta}\cos\alpha\cos\psi\right]\bar{1}_{y_2}$$

$$+\left[\frac{1}{3}MR^2\dot{\theta}\cos\alpha\sin^2\psi+MR^2\dot{\theta}\cos\alpha-\frac{MR^2}{2}\left(\dot{\theta}\sin\alpha-\dot{\psi}\right)\cos\psi\right]\bar{1}_{z_2}$$
