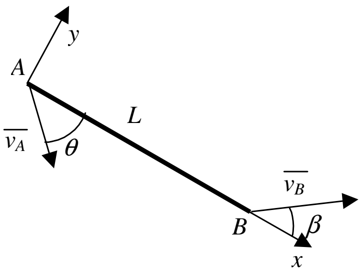
Simulations pour les exercices 2, 3 et 5 disponible sur :
http://www.ulb.ac.be/polytech/smana/Seance01CinematiqueCIR.htm

1.



$$\overline{v}_B = \overline{v}_A + \overline{\omega}_{AB} \times \overline{AB} \Rightarrow \begin{cases} v_B \cos\beta = v_A \cos\theta \\ v_B \sin\beta = -v_A \sin\theta + \omega_{AB} L \end{cases}$$

$$\omega_{AB} = \frac{1}{L}\left(v_B \sqrt{1-\left(\frac{v_A}{v_B}\right)^2 \cos^2\theta} + v_A \sin\theta \right)$$

2. $$\overline{AG} = \left[\frac{L}{2}\cos\theta + \frac{3L}{8}\cos(\theta+\alpha)\right]\overline{1}_x + \left[\frac{L}{2}\sin\theta + \frac{3L}{8}\sin(\theta+\alpha)\right]\overline{1}_y$$

Par dérivation des coordonnées dans le repère Axy

$$\overline{v}_G = \frac{d\overline{AG}}{dt} = \left[-\frac{L}{2}\sin\theta\dot\theta - \frac{3L}{8}\sin(\theta+\alpha)\dot\theta\right]\overline{1}_x + \left[\frac{L}{2}\cos\theta\dot\theta + \frac{3L}{8}\cos(\theta+\alpha)\dot\theta\right]\overline{1}_y$$

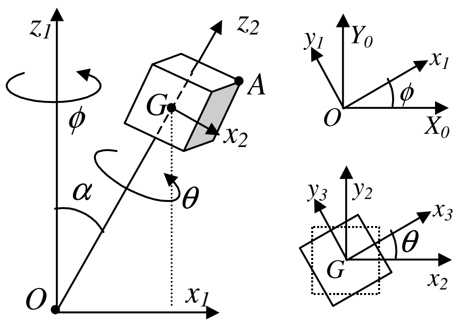
$$\overline{a}_G = \frac{d\overline{v}_G}{dt} = \left[-\frac{L}{2}\cos\theta\dot\theta^2 - \frac{3L}{8}\cos(\theta+\alpha)\dot\theta^2 - \frac{L}{2}\sin\theta\ddot\theta - \frac{3L}{8}\sin(\theta+\alpha)\ddot\theta\right]\overline{1}_x$$
$$+\left[-\frac{L}{2}\sin\theta\dot\theta^2 - \frac{3L}{8}\sin(\theta+\alpha)\dot\theta^2 + \frac{L}{2}\cos\theta\ddot\theta + \frac{3L}{8}\cos(\theta+\alpha)\ddot\theta\right]\overline{1}_y$$

Par la méthode des distribution des vitesse dans le repère Axy $(\overline{\omega}_{AG} = \dot\theta\overline{1}_z)$

$$\overline{v}_G = \overline{v}_A + \overline{\omega}_{AG} \times \overline{AG} = \dot\theta\overline{1}_z \times \left(\left[\frac{L}{2}\cos\theta + \frac{3L}{8}\cos(\theta+\alpha)\right]\overline{1}_x + \left[\frac{L}{2}\sin\theta + \frac{3L}{8}\sin(\theta+\alpha)\right]\overline{1}_y\right)$$

$$\overline{a}_G = \overline{a}_A + \overline{\omega}_{AG} \times \left(\overline{\omega}_{AG} \times \overline{AG}\right) + \overline{\varepsilon}_{AG} \times \overline{AG} \quad \text{avec} \quad \overline{\varepsilon}_{AG} = \ddot\theta\overline{1}_z$$

3



R_0 : Axe $OX_0Y_0Z_0$ fixe.
R_1 : Axe $Ox_1y_1z_1$ tourne autour de l'axe $Z_0 = z_1$ (ϕ)
R_2 : Axe $Gx_2y_2z_2$ incliné d'un angle α par rapport aux axes $Ox_1y_1z_1$, avec z_2 lié à la tige, tourne autour de l'axe z_1 (ϕ)
R_3 : Axe $Gx_3y_3z_3$ liés au cube et tourne autour de z_1 (ϕ) et z_2 (θ)
=> les vecteurs de Darboux :
$\overline{\omega}_{R_1/R_0} = \dot\phi\overline{1}_{z_1}$; $\overline{\omega}_{R_2/R_0} = \dot\phi\overline{1}_{z_1}$; $\overline{\omega}_{R_3/R_0} = \dot\phi\overline{1}_{z_1} + \dot\theta\overline{1}_{z_2}$

a. Par la formule des distribution des vitesses exprimé dans le repère $Ox_2y_2z_2$

$$\overline{\omega}_{Tige} = \dot\phi\overline{1}_{z_1} \quad \text{et} \quad \overline{\omega}_{Carré} = \dot\phi\overline{1}_{z_1} + \dot\theta\overline{1}_{z_2} = -\dot\phi\sin\alpha\overline{1}_{x_2} + \left(\dot\theta + \dot\phi\cos\alpha\right)\overline{1}_{z_2}$$

$$\overline{v}_G = \overline{v}_O + \overline{\omega}_{Tige} \times \overline{OG} = 0 + L\dot\phi\sin\alpha\overline{1}_y = L\dot\phi\sin\alpha\overline{1}_{y'}$$

$$\overline{GA} = \frac{d}{2}\left(\overline{1}_{x_3} + \overline{1}_{y_3} + \overline{1}_{z_3}\right) = \frac{d}{2}\left((\cos\theta - \sin\theta)\overline{1}_{x_2} + (\sin\theta + \cos\theta)\overline{1}_{y_2} + \overline{1}_{z_2}\right)$$

$$\overline{v}_A = \overline{v}_G + \overline{\omega}_{Carré} \times \overline{GA} = \begin{cases} -\frac{d}{2}\left(\left(\dot\theta + \dot\phi\cos\alpha\right)(\sin\theta + \cos\theta)\right)\overline{1}_{x_2} \\ +\left(\left(L + \frac{d}{2}\right)\dot\phi\sin\alpha + \frac{d}{2}\left(\dot\theta + \dot\phi\cos\alpha\right)(\cos\theta - \sin\theta)\right)\overline{1}_{y_2} \\ -\frac{d}{2}\dot\phi\sin\alpha(\sin\theta + \cos\theta)\overline{1}_{z_2} \end{cases}$$

b. <u>Par dérivation des coordonnées exprimées dans le repère $Ox_3y_3z_3$</u>

$$\overline{OA}=\frac{d}{2}\overline{1}_{x_3}+\frac{d}{2}\overline{1}_{y_3}+\left(\frac{d}{2}+L\right)\overline{1}_{z_3} \quad \text{et} \quad \overline{\omega}_{R_3/R_0}=-\dot{\phi}\sin\alpha\cos\theta\,\overline{1}_{x_3}+\dot{\phi}\sin\alpha\sin\theta\,\overline{1}_{y_3}+\left(\dot{\theta}+\dot{\phi}\cos\alpha\right)\overline{1}_{z_3}$$

$$\overline{v}_A=\frac{d\overline{OA}}{dt}=\underbrace{\left.\frac{d\overline{OA}}{dt}\right|_{x_3y_3z_3}}_{=0}+\overline{\omega}_{R_3/R_0}\times\overline{OA}=\left\{\begin{array}{l}\left(\left(\frac{d}{2}+L\right)\dot{\phi}\sin\alpha\sin\theta-\frac{d}{2}\left(\dot{\theta}+\dot{\phi}\cos\alpha\right)\right)\overline{1}_{x_3}\\ +\left(\left(\dot{\theta}+\dot{\phi}\cos\alpha\right)\frac{d}{2}+\left(\frac{d}{2}+L\right)\dot{\phi}\sin\alpha\cos\theta\right)\overline{1}_{y_3}\\ -\frac{d}{2}\dot{\phi}\sin\alpha\left(\cos\theta+\sin\theta\right)\overline{1}_{z_3}\end{array}\right.$$

b'. <u>Par dérivation des coordonnées exprimées dans le repère $Ox_2y_2z_2$</u>

$$\overline{OA}=\frac{d}{2}\left(\cos\theta-\sin\theta\right)\overline{1}_{x_2}+\frac{d}{2}\left(\sin\theta+\cos\theta\right)\overline{1}_{y_2}+\left(\frac{d}{2}+L\right)\overline{1}_{z_2} \quad \text{et} \quad \overline{\omega}_{R_2/R_0}=-\dot{\phi}\sin\alpha\,\overline{1}_{x_2}+\dot{\phi}\cos\alpha\,\overline{1}_{z_2}$$

$$\overline{v}_A=\frac{d\overline{OA}}{dt}=\left.\frac{d\overline{OA}}{dt}\right|_{R_2}+\overline{\omega}_{R_2/R_0}\times\overline{OA}=\left\{\begin{array}{l}-\frac{d}{2}\left(\sin\theta+\cos\theta\right)\left(\dot{\theta}+\dot{\phi}\cos\alpha\right)\overline{1}_{x_2}\\ +\left(\left(\dot{\theta}+\dot{\phi}\cos\alpha\right)\frac{d}{2}\left(\cos\theta-\sin\theta\right)+\dot{\phi}\sin\alpha\left(\frac{d}{2}+L\right)\right)\overline{1}_{y_2}\\ -\left(\dot{\phi}\sin\alpha\frac{d}{2}\left(\sin\theta+\cos\theta\right)\right)\overline{1}_{z_2}\end{array}\right.$$

c. $\overline{\omega}_{Cube}=-\dot{\phi}\sin\alpha\,\overline{1}_{x_2}+\left(\dot{\theta}+\dot{\phi}\cos\alpha\right)\overline{1}_{z_2}$

$$\overline{\varepsilon}_{Cube}=\left.\frac{d\overline{\omega}_{Cube}}{dt}\right|_{rel\text{ dans }R_2}+\overline{\omega}_{R_2/R_0}\times\overline{\omega}_{Cube} \quad \text{avec} \quad \overline{\omega}_{R_2/R_0}=-\dot{\phi}\sin\alpha\,\overline{1}_{x_2}+\dot{\phi}\cos\alpha\,\overline{1}_{z_2}$$

$$\overline{\varepsilon}_{Cube}=-\ddot{\phi}\sin\alpha\,\overline{1}_{x_2}+\left(\ddot{\theta}+\ddot{\phi}\cos\alpha\right)\overline{1}_{z_2}-\cos\alpha\sin\alpha\dot{\phi}^2\,\overline{1}_{y_2}+\left(\sin\alpha\dot{\phi}\dot{\theta}+\sin\alpha\cos\alpha\dot{\phi}^2\right)\overline{1}_{y_2}$$

$$\overline{\varepsilon}_{Cube}=-\ddot{\phi}\sin\alpha\,\overline{1}_{x_2}+\sin\alpha\dot{\phi}\dot{\theta}\,\overline{1}_{y_2}+\left(\ddot{\theta}+\ddot{\phi}\cos\alpha\right)\overline{1}_{z_2}$$

d. $\overline{\omega}_{Cube}=-\sin\alpha\cos\theta\dot{\phi}\,\overline{1}_{x_3}+\sin\alpha\sin\theta\dot{\phi}\,\overline{1}_{y_3}+\left(\dot{\theta}+\cos\alpha\dot{\phi}\right)\overline{1}_{z_3}$ avec $\overline{\omega}_{R_3/R_0}=\overline{\omega}_{Cube}$

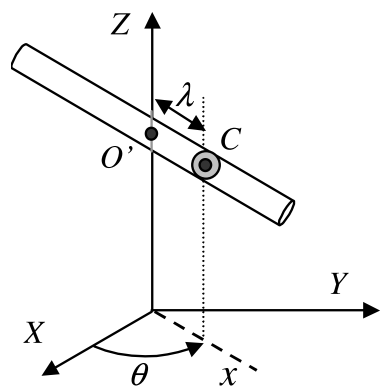
$$\overline{\varepsilon}_{Cube}=\left.\frac{d\overline{\omega}_{Cube}}{dt}\right|_{rel\text{ dans }R_3}+\underbrace{\overline{\omega}_{R_3/R_0}\times\overline{\omega}_{Cube}}_{=0}$$

$$\overline{\varepsilon}_{Cube}=-\sin\alpha\cos\theta\ddot{\phi}\,\overline{1}_{x_3}+\dot{\phi}\sin\alpha\sin\theta\dot{\theta}\,\overline{1}_{x_3}+\sin\alpha\sin\theta\ddot{\phi}\,\overline{1}_{y_3}+\sin\alpha\cos\theta\dot{\theta}\dot{\phi}\,\overline{1}_{y_3}+\left(\ddot{\theta}+\ddot{\phi}\cos\alpha\right)\overline{1}_{z_3}$$

$$\overline{\varepsilon}_{Cube}=\left(\sin\alpha\sin\theta\dot{\phi}\dot{\theta}-\sin\alpha\cos\theta\ddot{\phi}\right)\overline{1}_{x_3}+\left(\sin\alpha\cos\theta\dot{\phi}\dot{\theta}+\sin\alpha\sin\theta\ddot{\phi}\right)\overline{1}_{y_3}+\left(\ddot{\theta}+\ddot{\phi}\cos\alpha\right)\overline{1}_{z_3}$$

4.

$$\left\{\begin{array}{l}\overline{1}_x=\cos\theta\,\overline{1}_X+\sin\theta\,\overline{1}_Y\\ \overline{1}_y=-\sin\theta\,\overline{1}_X+\cos\theta\,\overline{1}_Y\\ \overline{1}_z=\overline{1}_z\end{array}\right. \quad \text{et} \quad \overline{\omega}=\omega\overline{1}_z$$



$$\overline{v}_C=\overline{v}_{C-rel}+\overline{v}_{C-entr} \quad \text{avec} \quad \left\{\begin{array}{l}\overline{v}_{C-rel}=\left.\frac{d\overline{O'C}}{dt}\right|_{rel}=\dot{\lambda}\overline{1}_x\\ \overline{v}_{C-entr}=\overline{v}_{O'}+\overline{\omega}\times\overline{O'C}=0+\lambda\omega\overline{1}_y\end{array}\right.$$

$$\overline{a}_C=\overline{a}_{C-rel}+\overline{a}_{C-entr}+\overline{a}_{C-cor}$$

$$\text{avec} \quad \left\{\begin{array}{l}\overline{a}_{C-rel}=\ddot{\lambda}\overline{1}_x\\ \overline{a}_{C-cor}=2\overline{\omega}\times\overline{v}_{C-rel}=2\omega\overline{1}_z\times\dot{\lambda}\overline{1}_x=2\omega\dot{\lambda}\overline{1}_y\\ \overline{a}_{C-entr}=\overline{a}_{O'}+\overline{\omega}_T\times\left(\overline{\omega}_T\times\overline{O'C}\right)+\overline{\varepsilon}_T\times\overline{O'C}=0-\lambda\omega^2\overline{1}_x+0\end{array}\right.$$

$$\Rightarrow \overline{a}_C=\ddot{\lambda}\overline{1}_x-\lambda\omega^2\overline{1}_x+2\omega\dot{\lambda}\overline{1}_y$$

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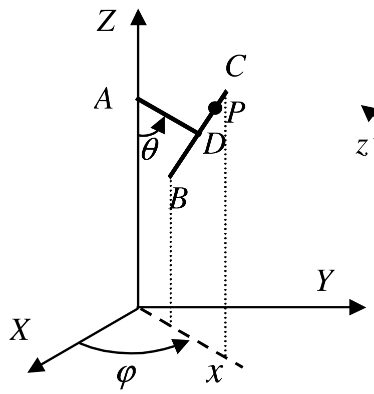
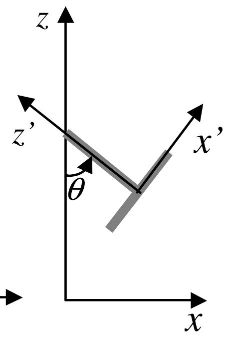
5. $\begin{cases}\overline{1}_{x'}=\cos\theta\,\overline{1}_x+\sin\theta\,\overline{1}_z\\ \overline{1}_{y'}=\overline{1}_y\\ \overline{1}_{z'}=-\sin\theta\,\overline{1}_x+\cos\theta\,\overline{1}_z\end{cases}$ et $\begin{cases}\overline{1}_x=\cos\theta\,\overline{1}_{x'}-\sin\theta\,\overline{1}_{z'}\\ \overline{1}_y=\overline{1}_{y'}\\ \overline{1}_z=\sin\theta\,\overline{1}_{x'}+\cos\theta\,\overline{1}_{z'}\end{cases}$





Cinématique (formule de distribution des vitesses) avec les vecteurs exprimés dans le repère *x'y'z'* :

a. $\overline{\omega}_T=\sin\theta\dot{\phi}\,\overline{1}_{x'}-\dot{\theta}\,\overline{1}_{y'}+\cos\theta\dot{\phi}\,\overline{1}_{z'}$ et $\overline{\omega}_{x'y'z'/XYZ}=\overline{\omega}_T=\sin\theta\dot{\phi}\,\overline{1}_{x'}-\dot{\theta}\,\overline{1}_{y'}+\cos\theta\dot{\phi}\,\overline{1}_{z'}$

$$\Rightarrow \overline{\varepsilon}_T=\left.\frac{d\overline{\omega}_T}{dt}\right|_{rel}+\overline{\omega}_{x'y'z'/XYZ}\times\overline{\omega}_T=\left(\cos\theta\dot{\theta}\dot{\phi}+\sin\theta\ddot{\phi}\right)\overline{1}_{x'}-\ddot{\theta}\,\overline{1}_{y'}+\left(\cos\theta\ddot{\phi}-\sin\theta\dot{\theta}\dot{\phi}\right)\overline{1}_{z'}$$

$$\overline{AD}=-L\,\overline{1}_{z'}$$

$$\overline{v}_D=\overline{v}_A+\overline{\omega}_T\times\overline{AD}=0+L\dot{\theta}\,\overline{1}_{x'}+L\sin\theta\dot{\phi}\,\overline{1}_{y'}$$

$$\overline{a}_D=\overline{a}_A+\overline{\omega}_T\times\left(\overline{\omega}_T\times\overline{AD}\right)+\overline{\varepsilon}_T\times\overline{AD}=L\left(\ddot{\theta}-\sin\theta\cos\theta\dot{\phi}^2\right)\overline{1}_{x'}+L\left(2\cos\theta\dot{\theta}\dot{\phi}+\sin\theta\ddot{\phi}\right)\overline{1}_{y'}+\left(L\sin^2\theta\dot{\phi}^2+L\dot{\theta}^2\right)\overline{1}_{z'}$$

b. $\overline{AP}=\lambda\,\overline{1}_{x'}-L\,\overline{1}_{z'}$

$$\overline{v}_{P_{entr}}=\overline{v}_D+\overline{\omega}_T\times\overline{DP}=L\dot{\theta}\,\overline{1}_{x'}+\left(L\sin\theta\dot{\phi}+\lambda\cos\theta\dot{\phi}\right)\overline{1}_{y'}+\lambda\dot{\theta}\,\overline{1}_{z'}\quad\text{et}\quad\overline{v}_{P_{rel}}=\dot{\lambda}\,\overline{1}_{x'}$$

$$\Rightarrow \overline{v}_P=\left(L\dot{\theta}+\dot{\lambda}\right)\overline{1}_{x'}+\left(L\sin\theta\dot{\phi}+\lambda\cos\theta\dot{\phi}\right)\overline{1}_{y'}+\lambda\dot{\theta}\,\overline{1}_{z'}$$

Cinématique (formule de distribution des vitesses) avec les vecteurs exprimés dans le repère *xyz* :

a. $\overline{\omega}_T=-\dot{\theta}\,\overline{1}_y+\dot{\phi}\,\overline{1}_z$; $\overline{\omega}_{xyz/XYZ}=\dot{\phi}\,\overline{1}_z$ => $\overline{\varepsilon}_T=\left.\frac{d\overline{\omega}_2}{dt}\right|_{rel}+\overline{\omega}_{xyz/XYZ}\times\overline{\omega}_T=\dot{\theta}\dot{\phi}\,\overline{1}_x-\ddot{\theta}\,\overline{1}_y+\ddot{\phi}\,\overline{1}_z$;

$$\overline{AD}=L\sin\theta\,\overline{1}_x-L\cos\theta\,\overline{1}_z$$

$$\overline{v}_D=\overline{v}_A+\overline{\omega}_T\times\overline{AD}=L\cos\theta\dot{\theta}\,\overline{1}_x+L\sin\theta\dot{\phi}\,\overline{1}_y+L\sin\theta\dot{\theta}\,\overline{1}_z$$

$$\overline{a}_D=\overline{a}_A+\overline{\omega}_T\times\left(\overline{\omega}_T\times\overline{AD}\right)+\overline{\varepsilon}_T\times\overline{AD}=\begin{cases}\left(L\cos\theta\ddot{\theta}-L\sin\theta\dot{\phi}^2-L\sin\theta\dot{\theta}^2\right)\overline{1}_x\\ +\left(2L\cos\theta\dot{\theta}\dot{\phi}+L\sin\theta\ddot{\phi}\right)\overline{1}_y\\ +\left(L\cos\theta\dot{\theta}^2+L\sin\theta\ddot{\theta}\right)\overline{1}_z\end{cases}$$

b. $\overline{AP}=\left(L\sin\theta+\lambda\cos\theta\right)\overline{1}_x+\left(-L\cos\theta+\lambda\sin\theta\right)\overline{1}_z$

$$\overline{v}_{P_{entr}}=\overline{v}_A+\overline{\omega}_{xyz/XYZ}\times\overline{AP}=\left(L\sin\theta+\lambda\cos\theta\right)\dot{\phi}\,\overline{1}_y$$

$$\overline{v}_{P_{rel}}=\left.\frac{d\overline{AP}}{dt}\right|_{rel-xyz}=\left(L\cos\theta\dot{\theta}+\dot{\lambda}\cos\theta-\lambda\sin\theta\dot{\theta}\right)\overline{1}_x+\left(L\sin\theta\dot{\theta}+\dot{\lambda}\sin\theta+\lambda\cos\theta\dot{\theta}\right)\overline{1}_z$$

$$\Rightarrow \overline{v}_P=\left(L\cos\theta\dot{\theta}-\lambda\sin\theta\dot{\theta}+\dot{\lambda}\cos\theta\right)\overline{1}_x+\left(L\sin\theta\dot{\phi}+\lambda\cos\theta\dot{\phi}\right)\overline{1}_y+\left(L\sin\theta\dot{\theta}+\lambda\cos\theta\dot{\theta}+\dot{\lambda}\sin\theta\right)\overline{1}_z$$

Par dérivation des vecteurs dans le repère *x'y'z'* :

a. $\overline{\omega}_{x'y'z'/XYZ}=\sin\theta\dot{\phi}\,\overline{1}_{x'}-\dot{\theta}\,\overline{1}_{y'}+\cos\theta\dot{\phi}\,\overline{1}_{z'}$

$\overline{AD}=-L\,\overline{1}_{z'}$ => A fixe : $\overline{v}_D=\underbrace{\left.\frac{d\overline{AD}}{dt}\right|_{rel}}_{\overline{v}_{D_{rel}}}+\underbrace{\overline{\omega}_{x'y'z'/XYZ}\times\overline{AD}}_{\overline{v}_{D_{entr}}}=0+L\sin\theta\dot{\phi}\,\overline{1}_{y'}+L\dot{\theta}\,\overline{1}_{x'}$

$$\overline{a}_D=\left.\frac{d\,\overline{v}_D}{dt}\right|_{rel}+\overline{\omega}_{x'y'z'/XYZ}\times\overline{v}_D=L\left(\ddot{\theta}-\sin\theta\cos\theta\dot{\phi}^2\right)\overline{1}_{x'}+L\left(2\cos\theta\dot{\theta}\dot{\phi}+\sin\theta\ddot{\phi}\right)\overline{1}_{y'}+L\left(\sin^2\theta\dot{\phi}^2+\dot{\theta}^2\right)\overline{1}_{z'}$$

$$\overline{AP} = \lambda \overline{1}_{x'} - L\overline{1}_{z'} \Rightarrow \overline{v}_P = \left.\frac{d\overline{AP}}{dt}\right|_{rel} + \overline{\omega}_{x'y'z'/XYZ} \times \overline{AP} = \left(L\dot{\theta} + \dot{\lambda}\right)\overline{1}_{x'} + \left(\lambda\cos\theta\dot{\phi} + L\sin\theta\dot{\phi}\right)\overline{1}_{y'} + \lambda\dot{\theta}\overline{1}_{z'}$$

$$\overline{a}_P = \left.\frac{d\overline{v}_P}{dt}\right|_{rel} + \overline{\omega}_{x'y'z'/XYZ} \times \overline{v}_P =$$

$$\begin{cases} \left(L\ddot{\theta} + \ddot{\lambda} - \left(\lambda\cos^2\theta + L\cos\theta\sin\theta\right)\dot{\phi}^2 - \lambda\dot{\theta}^2\right)\overline{1}_{x'} \\ +\left(2\dot{\lambda}\cos\theta\dot{\phi} + \left(-\lambda sin\theta + 2L\cos\theta\right)\dot{\theta}\dot{\phi} + \left(\lambda\cos\theta + L\sin\theta\right)\ddot{\phi} - \lambda\sin\theta\dot{\theta}\dot{\phi}\right)\overline{1}_{y'} \\ +\left(\lambda\ddot{\theta} + 2\dot{\lambda}\dot{\theta} + L\dot{\theta}^2 + \left(\lambda\sin\theta\cos\theta + L\sin^2\theta\right)\dot{\phi}^2\right)\overline{1}_{z'} \end{cases}$$

Par dérivation dans les axes *xyz*: $\overline{\omega}_{xyz/XYZ} = \dot{\phi}\overline{1}_z$

$$\overline{AD} = L\sin\theta\overline{1}_x - L\cos\theta\overline{1}_z \Rightarrow \overline{v}_D = \left.\frac{d\overline{AD}}{dt}\right|_{rel} + \overline{\omega}_{xyz/XYZ} \times \overline{AD} = L\cos\dot{\theta}\overline{1}_x + L\sin\theta\dot{\phi}\overline{1}_y + L\sin\theta\dot{\theta}\overline{1}_z$$

<u>Par dérivation des vecteurs dans le repère *xyz* :</u>

$$\overline{\omega}_{xyz/XYZ} = \dot{\phi}\overline{1}_z$$

$$\overline{AD} = L\sin\theta\overline{1}_x - L\cos\theta\overline{1}_z \Rightarrow \overline{v}_D = \left.\frac{d\overline{AD}}{dt}\right|_{rel} + \overline{\omega}_{xyz/XYZ} \times \overline{AD} = L\cos\theta\dot{\theta}\overline{1}_x + L\sin\theta\dot{\phi}\overline{1}_y + L\sin\theta\dot{\theta}\overline{1}_z$$

$$\overline{a}_D = \left.\frac{d\,\overline{v}_D}{dt}\right|_{rel} + \overline{\omega}_{xyz/XYZ} \times \overline{v}_D = \begin{cases} \left(L\cos\theta\ddot{\theta} - L\sin\theta\dot{\theta}^2 + L\sin\theta\dot{\phi}^2\right)\overline{1}_x \\ +\left(2L\cos\theta\dot{\theta}\dot{\phi} + L\sin\theta\ddot{\phi}\right)\overline{1}_y \\ +\left(L\cos\theta\dot{\phi}^2 + L\sin\theta\ddot{\theta}\right)\overline{1}_z \end{cases}$$

b. $\overline{AP} = \left(\lambda\cos\theta + L\sin\theta\right)\overline{1}_x + \left(\lambda\sin\theta - L\cos\theta\right)\overline{1}_z$

$$\overline{v}_P = \left.\frac{d\overline{AP}}{dt}\right|_{rel} + \overline{\omega}_{xyz/XYZ} \times \overline{AP} = \begin{cases} \left(-\lambda\sin\theta\dot{\theta} + \dot{\lambda}\cos\theta + L\cos\theta\dot{\theta}\right)\overline{1}_x \\ +\left(\lambda\cos\theta\dot{\phi} + L\sin\theta\dot{\phi}\right)\overline{1}_y \\ +\left(\lambda\cos\theta\dot{\theta} + \dot{\lambda}\sin\theta + L\sin\theta\dot{\theta}\right)\overline{1}_z \end{cases}$$

$$\overline{a}_P = \left.\frac{d\,\overline{v}_P}{dt}\right|_{rel} + \overline{\omega}_{xyz/XYZ} \times \overline{v}_P$$

$$= \begin{cases} \left(\ddot{\lambda}\cos\theta - 2\dot{\lambda}\sin\theta\dot{\theta} + \left(L\cos\theta - \lambda\sin\theta\right)\ddot{\theta} - \left(\lambda\cos\theta + L\sin\theta\right)\dot{\theta}^2 - \left(\lambda\cos\theta + L\sin\theta\right)\dot{\phi}^2\right)\overline{1}_x \\ +\left(\left(\lambda\cos\theta + L\sin\theta\right)\ddot{\phi} + 2\dot{\lambda}\cos\theta\dot{\phi} + \left(2L\cos\theta - 2\lambda\sin\theta\right)\dot{\theta}\dot{\phi}\right)\overline{1}_y \\ +\left(2\dot{\lambda}\cos\theta\dot{\theta} + \ddot{\lambda}\sin\theta + \left(L\cos\theta - \lambda\sin\theta\right)\dot{\theta}^2 + \left(\lambda\cos\theta + L\sin\theta\right)\ddot{\theta}\right)\overline{1}_z \end{cases}$$

<u>Utilisation des formules du mouvement relatif dans le repère *x'y'z'* :</u>

D = origine du repère mobile ($\overline{\omega}_{x'y'z'/XYZ} = \sin\theta\dot{\phi}\overline{1}_{x'} - \dot{\theta}\overline{1}_{y'} + \cos\theta\dot{\phi}\overline{1}_{z'} = \overline{\omega}_T$)

$\Rightarrow$ $\overline{\varepsilon}_T = \left(\cos\theta\dot{\theta}\dot{\phi} + \sin\theta\ddot{\phi}\right)\overline{1}_{x'} - \ddot{\theta}\overline{1}_{y'} + \left(\cos\theta\ddot{\phi} - \sin\theta\dot{\theta}\dot{\phi}\right)\overline{1}_{z'}$

b.

$$\overline{v}_P = \overline{v}_{P-rel} + \overline{v}_{P-entr} \quad \text{avec} \quad \begin{cases} \overline{v}_{P-rel} = \left.\frac{d\overline{DP}}{dt}\right|_{rel-x'y'z'} = \dot{\lambda}\overline{1}_{x'} \\ \overline{v}_{P-entr} = \overline{v}_D + \overline{\omega}_T \times \overline{DP} = L\dot{\theta}\overline{1}_{x'} + \left(L\sin\theta\dot{\phi} + \lambda\cos\theta\dot{\phi}\right)\overline{1}_{y'} + \lambda\dot{\theta}\overline{1}_{z'} \end{cases}$$

$\overline{a}_P = \overline{a}_{P-rel} + \overline{a}_{P-entr} + \overline{a}_{P-cor}$ avec

$\overline{a}_{P-rel} = \ddot{\lambda}\overline{1}_{x'}$

$\overline{a}_{P-cor} = 2\overline{\omega}_{x'y'z'/XYZ} \times \overline{v}_{rel} = 2\left(\sin\theta\dot{\phi}\overline{1}_{x'} - \dot{\theta}\overline{1}_{y'} + \cos\theta\dot{\phi}\overline{1}_{z'}\right) \times \dot{\lambda}\overline{1}_{x'} = +2\dot{\lambda}\cos\theta\dot{\phi}\overline{1}_{y'} + 2\dot{\theta}\dot{\lambda}\overline{1}_{z'}$

$$\bar{a}_{P-entr} = \bar{a}_D + \bar{\omega}_T \times \left(\bar{\omega}_T \times \overline{DP}\right) + \bar{\varepsilon}_T \times \overline{DP} \quad \text{avec } \overline{DP} = \lambda \bar{1}_{x'}$$

$$\bar{a}_{P-entr} = \begin{cases} L\left(\ddot{\theta} - \sin\theta\cos\theta\dot{\phi}^2\right)\bar{1}_{x'} + L\left(2\cos\theta\dot{\theta}\dot{\phi} + \sin\theta\ddot{\phi}\right)\bar{1}_{y'} + \left(L\sin^2\theta\dot{\phi}^2 + L\dot{\theta}^2\right)\bar{1}_{z'} \\ +\left(-\lambda\sin\theta\dot{\theta}\dot{\phi}\bar{1}_{y'} - \lambda\dot{\theta}^2\bar{1}_{x'}\right) + \left(\lambda\cos\theta\sin\theta\dot{\phi}^2\bar{1}_{z'} - \lambda\cos^2\theta\dot{\phi}^2\bar{1}_{x'}\right) \\ +\lambda\ddot{\theta}\bar{1}_{z'} + \lambda\left(\cos\theta\ddot{\phi} - \sin\theta\dot{\theta}\dot{\phi}\right)\bar{1}_{y'} \end{cases}$$

$$\bar{a}_P = \begin{cases} \left(\ddot{\lambda} + L\ddot{\theta} - \lambda\dot{\theta}^2 - \left(L\sin\theta\cos\theta + \lambda\cos^2\theta\right)\dot{\phi}^2\right)\bar{1}_{x'} \\ +\left(2\dot{\lambda}\cos\theta\dot{\phi} + (2L\cos\theta - \lambda\sin\theta)\dot{\theta}\dot{\phi} + (L\sin\theta + \lambda\cos\theta)\ddot{\phi} - \lambda\sin\theta\dot{\theta}\dot{\phi}\right)\bar{1}_{y'} \\ +\left(2\dot{\theta}\dot{\lambda} + \left(\lambda\cos\theta\sin\theta + L\sin^2\theta\right)\dot{\phi}^2 + L\dot{\theta}^2 + \lambda\ddot{\theta}\right)\bar{1}_{z'} \end{cases}$$

Par l'utilisation des formules du mouvement relatif dans le repère *xyz* :

A = origine du repère mobile xyz $\left(\bar{\omega}_1 = \bar{\omega}_{xyz/XYZ} = \dot{\phi}\bar{1}_z\right)$

$$\overline{AP} = (L\sin\theta + \lambda\cos\theta)\bar{1}_x + (\lambda\sin\theta - L\cos\theta)\bar{1}_z$$

$$\bar{v}_P = \bar{v}_{P-rel} + \bar{v}_{P-entr} \quad \text{avec} \begin{cases} \bar{v}_{P-rel} = \left.\dfrac{d\overline{AP}}{dt}\right|_{rel} = \left(L\cos\theta\dot{\theta} + \dot{\lambda}\cos\theta - \lambda\sin\theta\dot{\theta}\right)\bar{1}_x + \left(\dot{\lambda}\sin\theta + \lambda\cos\theta\dot{\theta} + L\sin\theta\dot{\theta}\right)\bar{1}_z \\ \bar{v}_{P-entr} = \bar{v}_A + \bar{\omega}_1 \times \overline{AP} = (L\sin\theta + \lambda\cos\theta)\dot{\phi}\bar{1}_y \end{cases}$$

$\bar{a}_P = \bar{a}_{P-rel} + \bar{a}_{P-entr} + \bar{a}_{P-cor}$ avec

$$\begin{cases} \bar{a}_{P-rel} = \begin{cases} \left(+\ddot{\lambda}\cos\theta - 2\dot{\lambda}\sin\theta\dot{\theta} - (L\sin\theta + \lambda\cos\theta)\dot{\theta}^2 + (L\cos\theta - \lambda\sin\theta)\ddot{\theta}\right)\bar{1}_x \\ +\left(\ddot{\lambda}\sin\theta + 2\dot{\lambda}\cos\theta\dot{\theta} + (L\cos\theta - \lambda\sin\theta)\dot{\theta}^2 + (\lambda\cos\theta + L\sin\theta)\ddot{\theta}\right)\bar{1}_z \end{cases} \\ \bar{a}_{P-cor} = 2\bar{\omega}_{xyz/XYZ} \times \bar{v}_{P-rel} = 2\left((L\cos\theta - \lambda\sin\theta)\dot{\theta}\dot{\phi} + \dot{\lambda}\cos\theta\dot{\phi}\right)\bar{1}_y \\ \bar{a}_{P-entr} = \bar{a}_A + \bar{\omega}_1 \times \left(\bar{\omega}_1 \times \overline{AP}\right) + \bar{\varepsilon}_1 \times \overline{AP} = -(L\sin\theta + \lambda\cos\theta)\dot{\phi}^2\bar{1}_x + (L\sin\theta + \lambda\cos\theta)\ddot{\phi}\bar{1}_y \end{cases}$$

$$\bar{a}_P = \begin{cases} \left(+\ddot{\lambda}\cos\theta - 2\dot{\lambda}\sin\theta\dot{\theta} - (L\sin\theta + \lambda\cos\theta)\dot{\theta}^2 - (L\sin\theta + \lambda\cos\theta)\dot{\phi}^2 + (L\cos\theta - \lambda\sin\theta)\ddot{\theta}\right)\bar{1}_x \\ \left(+2\dot{\lambda}\cos\theta\dot{\phi} + (2L\cos\theta - 2\lambda\sin\theta)\dot{\theta}\dot{\phi} + (L\sin\theta + \lambda\cos\theta)\ddot{\phi}\right)\bar{1}_y \\ +\left(\ddot{\lambda}\sin\theta + 2\dot{\lambda}\cos\theta\dot{\theta} + (L\cos\theta - \lambda\sin\theta)\dot{\theta}^2 + (\lambda\cos\theta + L\sin\theta)\ddot{\theta}\right)\bar{1}_z \end{cases}$$

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Les énoncés et les corrigés sont accessibles et mis à jour sont sur le site de méca :
http://beams.ulb.ac.be/beams/teaching/meca200/tps.html