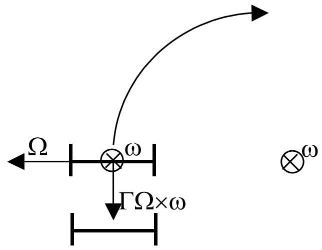
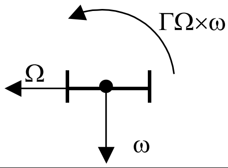
1. Gyrostat : $\Gamma\overline{\Omega}\times\overline{\omega}$
les roues droites de la voiture ont tendance à décoller.

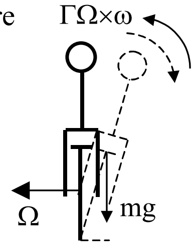
Vue de haut



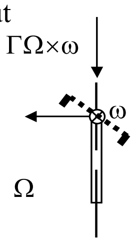
Vue de derrière



2.a Vue de derrière



Vue de haut



Si le cycliste se penche vers la droite, la force de pesanteur génère un moment qui a tendance à faire basculer le cycliste. Pour ne pas tomber, il doit générer un moment opposé qui le ramène en position verticale. En tournant son guidon vers la droite (du côté où il penche), il crée un effet gyroscopique qui le ramène dans sa position verticale.

2.b Roue = 60 cm de diamètre ;
$\Gamma = 0{,}6.(0{,}3)^2$

$v = \Omega R \quad \Rightarrow \quad 8 = \Omega.0{,}3 \quad \Rightarrow \quad \Omega = 8/0{,}3 \;\; rad/s$

$\overline{m}_{e,C} = \overline{CG}\times m\overline{g} = 0{,}03.80.9{,}81$

$\Gamma\overline{\Omega}\times\overline{\omega} = 0{,}6.(0{,}3)^2 \dfrac{8}{0{,}3}\omega$

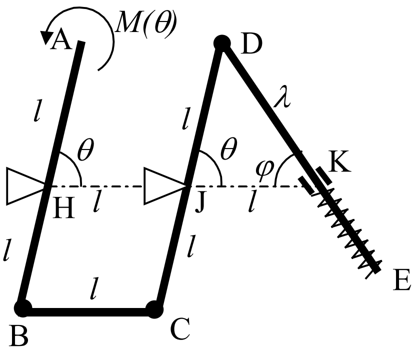
Pour retrouver l'équilibre :

$$\frac{\mathrm{d}}{\mathrm{dt}}\overline{M}_C = \mathrm{m}_{e,C} + \Gamma\overline{\Omega}\times\overline{\omega}$$

$$\Rightarrow \quad 0 = 0{,}03.80.9{,}81 - 0{,}6.(0{,}3)^2 \frac{8}{0{,}3}\omega$$

$$\omega = 16{,}35 \;\; \mathrm{rad/s}$$

3.1



3 Paramètre de position : $\theta,\ \varphi,\ \lambda$

2 relations : $\varphi = \dfrac{\pi}{2} - \dfrac{\theta}{2} \Rightarrow \dot{\varphi} = -\dfrac{\dot{\theta}}{2}$

$\lambda = 2L\sin\dfrac{\theta}{2}$ ou $L\sin\theta = \lambda\sin\varphi$

=> 1 ddl.

<u>Force</u> : 2 réactions en H, 2 réactions en J, 1 réaction en K perpendiculaire à DE, Force de rappel en E, poids des 4 tiges.

<u>Equation de mouvement : Lagrange.</u>
Les réactions en H, J et K ne travaillent pas car les points sont fixes ou le déplacement se fait perpendiculaire <u>aux forces</u>.
Le couple extérieur appliqué en A ne dérive pas d'un potentiel => Application du théorème de Lagrange généralisé.

Energie cinétique : $T = \displaystyle\sum_{i=1}^{4}\left(\frac{1}{2}mv_{G_i}^2 + \frac{1}{2}\overline{\omega}.\overline{\overline{I}}_{G_i}.\overline{\omega}\right)$

$$T = \left(\frac{1}{2}2m\frac{(2l)^2}{12}\dot{\theta}^2\right)_1 + \left(\frac{m}{2}\left(l\dot{\theta}\right)^2\right)_2 + \left(\frac{1}{2}2m\frac{(2l)^2}{12}\dot{\theta}^2\right)_3 + \left(\frac{2m}{2}\frac{5}{4}l^2\dot{\theta}^2 - \frac{2m}{2}l^2\dot{\theta}^2\sin\frac{\theta}{2} + \frac{1}{2}2m\frac{(2l)^2}{12}\frac{\dot{\theta}^2}{4}\right)_4$$

$$\Rightarrow T = \left(\frac{5}{2} - \sin\frac{\theta}{2}\right)ml^2\dot{\theta}^2$$

$$
\text{avec}\quad\begin{cases}
\overline{JG_4} = \left(L\cos\theta + L\cos\varphi\right)\bar{1}_x + \left(L\sin\theta - L\sin\varphi\right)\bar{1}_y \\
\bar{v}_{G_4} = \bar{v}_D + \left(-\dot{\varphi}\bar{1}_z\right)\times\overline{DG_4} = L\dot{\theta}\left(-\sin\theta\,\bar{1}_x + \cos\theta\,\bar{1}_y\right) + \dfrac{\dot{\theta}}{2}L\left(\cos\dfrac{\theta}{2}\bar{1}_x + \sin\dfrac{\theta}{2}\bar{1}_y\right) = \dfrac{d\left(\overline{JG_4}\right)}{dt} \\
v_{G_4}^2 = L^2\dot{\theta}^2 + \dfrac{\dot{\theta}^2}{4}L^2 + L^2\dot{\theta}^2\left(-\sin\theta\cos\dfrac{\theta}{2} + \cos\theta\sin\dfrac{\theta}{2}\right) = \dfrac{5}{4}L^2\dot{\theta}^2 + L^2\dot{\theta}^2\sin\left(-\dfrac{\theta}{2}\right)
\end{cases}
$$

$$
Q_\theta = \sum \bar{F}_h \frac{\partial\bar{\varphi}_h}{\partial q_\theta} = -mg\,\bar{1}_y.\frac{\partial\overline{OG_2}}{\partial\theta} - 2mg\,\bar{1}_y.\frac{\partial\overline{JG_4}}{\partial\theta} + \bar{F}_r.\frac{\partial\overline{JE}}{\partial\theta} + M(\theta)
$$

$$
\bar{F}_r = -k\left(L-L_0\right)\bar{1}_{DE} = k2L\sin\frac{\theta}{2}\bar{1}_{DE} = k2L\sin\frac{\theta}{2}\left(\sin\frac{\theta}{2}\bar{1}_x - \cos\frac{\theta}{2}\bar{1}_x\right)
$$

$$
\text{avec}\quad\begin{cases}
\overline{OG_2} = \left(-L\cos\theta + \dfrac{L}{2}\right)\bar{1}_x - L\sin\theta\,\bar{1}_y \Rightarrow \delta\overline{OG_2} = \left(L\sin\theta\,\bar{1}_x - L\cos\theta\,\bar{1}_y\right)\delta\theta = \dfrac{\partial\overline{OG_2}}{\partial\theta}\delta\theta \\
\overline{JG_4} \Rightarrow \delta\overline{JG_4} = \left(\left(-L\sin\theta + \dfrac{\dot{\theta}}{2}L\cos\dfrac{\theta}{2}\right)\bar{1}_x + \left(L\cos\theta + \dfrac{\dot{\theta}}{2}L\sin\dfrac{\theta}{2}\right)\bar{1}_y\right)\delta\theta = \dfrac{\partial\overline{JG_4}}{\partial\theta}\delta\theta \\
\overline{JE} = \left(L\cos\theta + 2L\cos\varphi\right)\bar{1}_x + \left(L\sin\theta - 2L\sin\varphi\right)\bar{1}_y = L\left(\cos\theta + 2\sin\dfrac{\theta}{2}\right)\bar{1}_x + L\left(\sin\theta - 2\cos\dfrac{\theta}{2}\right)\bar{1}_y \\
\Rightarrow \delta\overline{JE} = \left(L\left(-\sin\theta + \cos\dfrac{\theta}{2}\right)\bar{1}_x + L\left(\cos\theta + \sin\dfrac{\theta}{2}\right)\bar{1}_y\right)\delta\theta = \dfrac{\partial\overline{JE}}{\partial\theta}\delta\theta
\end{cases}
$$

Autre méthode, par le calcul du potentiel :

$$
Q_\theta = Q_{\theta(F\ \text{dérivant de}\ V)} + Q_{\theta(F\ \text{ne dérivant de}\ V)} \quad \text{avec} \quad Q_{\theta(F\ \text{ne dérivant de}\ V)} = M(\theta)
$$

$$
Q_{\theta(F\ \text{dérivant de}\ V)} = -\frac{\partial V}{\partial\theta} \quad \text{avec} \quad V = \frac{k}{2}\left(L-L_0\right)^2 - mgL\sin\theta + 2mg\left(L\sin\theta - L\sin\varphi\right)
$$

$$
Q_\theta = M - kL^2\sin\theta + mgL\cos\theta - 2mg\left(L\cos\theta + \frac{L}{2}\sin\frac{\theta}{2}\right) = M - kL^2\sin\theta - mgL\cos\theta - mgL\sin\frac{\theta}{2}
$$

$$
\Rightarrow \begin{cases}
\dfrac{d}{dt}\dfrac{\partial T}{\partial\dot{\theta}} - \dfrac{\partial T}{\partial\theta} = Q_\theta \\
\left(5 - 2\sin\dfrac{\theta}{2}\right)\ddot{\theta} - \dfrac{1}{2}\cos\dfrac{\theta}{2}\dot{\theta}^2 = \dfrac{M}{ml^2} - \dfrac{k}{m}\sin\theta - \dfrac{g}{L}\cos\theta - \dfrac{g}{L}\sin\dfrac{\theta}{2}
\end{cases}
$$

3.2.a $\dfrac{d}{dt}\bar{M}_D = \sum \bar{m}_{e,D} + 2m\bar{v}_G\times\bar{v}_D$ pour la tige DE seule

$$
\bar{M}_D = \bar{\bar{I}}_D.\bar{\omega} + 2m\overline{DG}\times\bar{v}_D = \left(\frac{2}{3}ml^2\dot{\theta} - 4ml^2\sin\frac{\theta}{2}\dot{\theta}\right)\bar{1}_z
$$

$$
\bar{m}_{e,D} = -2mgl\sin\frac{\theta}{2}\bar{1}_z + 2l\sin\frac{\theta}{2}R_K\,\bar{1}_z
$$

$$
\Rightarrow\ ml^2\ddot{\theta}\left(\frac{2}{3} - 4\sin\frac{\theta}{2}\right) - ml^2\dot{\theta}^2\cos\frac{\theta}{2} = -2mgl\sin\frac{\theta}{2} + 2l\sin\frac{\theta}{2}R_K + ml^2\dot{\theta}^2\cos\frac{\theta}{2}
$$

$$
\Rightarrow R_K = \frac{ml}{2\sin\frac{\theta}{2}}\left(\left(\frac{2}{3} - 4\sin\frac{\theta}{2}\right)\ddot{\theta} - 2\cos\frac{\theta}{2}\dot{\theta}^2\right) + mg
$$

3.2.b $\frac{d}{dt}\overline{M}_B = \sum \overline{m}_{e,B}$ pour la tige AB seule

$$\overline{M}_B = \overline{\overline{I}}_B.\overline{\omega} + 2m\overline{BG}\times\overline{v}_B = \frac{2m(2L)^2}{3}\dot{\theta}\,\overline{1}_z - 2mL^2\dot{\theta}\left(\cos^2\theta + \sin^2\theta\right)\overline{1}_z = \frac{2mL^2}{3}\dot{\theta}\overline{1}_z$$

$$=> \frac{2mL^2}{3}\ddot{\theta} = M - 2mgL\cos\theta + Y_H L\cos\theta - X_H L\sin\theta$$

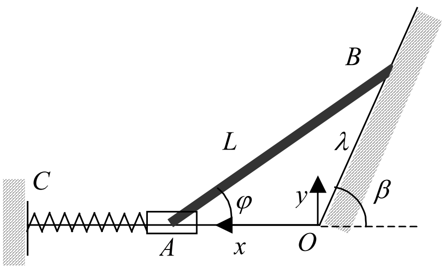
$\frac{d}{dt}\overline{M}_C = \sum \overline{m}_{e,C}$ pour les tiges AB et BC avec $\overline{M}_C = \overline{M}_{C,AB} + \overline{M}_{C,BC}$

$$\overline{M}_{C,BC} = \overline{\overline{I}}_C.\overline{\omega} + m\overline{CG_2}\times\overline{v}_C = 0 + m\frac{L}{2}.L\dot{\theta}.\cos\theta\overline{1}_z \quad ; \quad \overline{M}_{C,AB} = \overline{M}_{G,AB} + \overline{CG_1}\times\overline{R} = \frac{2m(2L)^2}{3}\dot{\theta}\overline{1}_z$$

$$=> mL^2\left(\frac{8}{3} + \frac{1}{2}\cos\theta\right)\ddot{\theta} - m\frac{L^2}{2}\sin\theta\dot{\theta}^2 = M + 2mg\left(L - L\cos\theta\right) + mg\frac{L}{2} - Y_H\left(L - L\cos\theta\right) - X_H L\sin\theta$$

2 équations, 2 inconnues

4.



$L\sin\phi = \lambda\sin\beta$

$$\overline{OG} = \left(\frac{L}{2}\cos\phi - \lambda\cos\beta\right)\overline{1}_x + \frac{L}{2}\sin\phi\overline{1}_y$$

$$\overline{OG} = \left(\frac{L}{2}\cos\phi - L\frac{\sin\phi\cos\beta}{\sin\beta}\right)\overline{1}_x + \frac{L}{2}\sin\phi\overline{1}_y$$

$$\overline{OG} = \left(\frac{L}{2}\cos\phi - L\cos\phi + L\frac{\sin(\beta-\phi)}{\sin\beta}\right)\overline{1}_x + \frac{L}{2}\sin\phi\overline{1}_y$$

$$=> \overline{OG} = \left(L\frac{\sin(\beta-\phi)}{\sin\beta} - \frac{L}{2}\cos\phi\right)\overline{1}_x + \frac{L}{2}\sin\phi\overline{1}_y$$

4.1

$$\overline{v}_G = \left(L\frac{\cos(\beta-\phi)(-\dot{\phi})}{\sin\beta} + \frac{L}{2}\sin\phi\dot{\phi}\right)\overline{1}_x + \left(\frac{L}{2}\cos\phi\dot{\phi}\right)\overline{1}_y$$

$$T = \left(\frac{1}{2}mv_{G_i}^2 + \frac{1}{2}\overline{\omega}.\overline{\overline{I}}_{G_i}.\overline{\omega}\right) = \frac{1}{2}m\left(L^2\frac{\cos^2(\beta-\phi)\dot{\phi}^2}{\sin^2\beta} + \frac{L^2}{4}\dot{\phi}^2 - L^2\frac{\cos(\beta-\phi)}{\sin\beta}\sin\phi\dot{\phi}^2\right) + \frac{1}{2}m\frac{L^2}{12}\dot{\phi}^2$$

$$=> T = \frac{1}{2}m\left(\frac{\cos^2(\beta-\phi)}{\sin^2\beta} - \frac{\cos(\beta-\phi)}{\sin\beta}\sin\phi\right)L^2\dot{\phi}^2 + \frac{1}{2}m\frac{L^2}{3}\dot{\phi}^2$$

$$\delta\tau = \sum_i \overline{F}_i.\delta\overline{OP}_i = \sum_i \overline{F}_i.\sum_j \frac{\partial\overline{\varphi}_i}{\partial q_j}\delta q_j = \sum_j\left(\sum_i \overline{F}_i.\frac{\partial\overline{\varphi}_i}{\partial q_j}\right)\delta q_j = \sum_j Q_{q_j}\delta q_j => Q_{q_j} = \sum_i \overline{F}_i.\frac{\partial\overline{\varphi}_i}{\partial q_j}$$

$$\overline{OG} = \left(\frac{L}{2}\cos\phi - \lambda\cos\beta\right)\overline{1}_x + \frac{L}{2}\sin\phi\overline{1}_y => \delta\overline{OG}\Big|_y = \frac{L}{2}\cos\phi\delta\phi \text{ et } \overline{F}_G = -mg\overline{1}_y$$

$$\overline{OA} = L\left(\frac{\sin\beta\cos\phi - \sin\phi\cos\beta}{\sin\beta}\right)\overline{1}_x = L\left(\frac{\sin(\beta-\phi)}{\sin\beta}\right)\overline{1}_x$$

$$=> \delta\overline{OA} = -L\left(\frac{\cos(\beta-\phi)}{\sin\beta}\right)\delta\phi \text{ et } \overline{F}_A = k\left(L - \left(L\cos\phi - \lambda\cos\beta\right)\right)\overline{1}_x$$

$$\overline{OB} = -\lambda\cos\beta\overline{1}_x + \lambda\sin\beta\overline{1}_y = -L\frac{\sin\phi}{\sin\beta}\cos\beta\overline{1}_x + L\sin\phi\overline{1}_y = \lambda\overline{1}_v = L\frac{\sin\phi}{\sin\beta}\overline{1}_v$$

$$=> \delta\overline{OB} = L\frac{\cos\phi}{\sin\beta}\delta\phi\overline{1}_v \text{ et } \overline{F}_B = N_B\overline{1}_u - T_B\frac{\dot{\phi}}{|\dot{\phi}|}\overline{1}_v \text{ avec } T_B = fN_B\frac{\dot{\phi}}{|\dot{\phi}|}$$

$$Q_\phi = \sum_i \overline{F}_i.\frac{\partial\overline{\varphi}_i}{\partial q_\phi} => Q_\phi = -mg\frac{L}{2}\cos\phi - \frac{kL^2}{\sin^2\beta}\cos(\beta-\phi)\left(\sin\beta - \sin(\beta-\phi)\right) - fN_B\frac{\dot{\phi}}{|\dot{\phi}|}L\frac{\cos\phi}{\sin\beta}$$

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{\phi}}\right)-\left(\frac{\partial T}{\partial \phi}\right)=Q_\phi$$

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{\phi}}\right)-\left(\frac{\partial T}{\partial \phi}\right)=\frac{d}{dt}\left(m\left(\frac{\cos^2(\beta-\phi)}{\sin^2\beta}-\frac{\cos(\beta-\phi)}{\sin\beta}\sin\phi\right)L^2\dot{\phi}+m\frac{L^2}{3}\dot{\phi}\right)$$

$$-\left(m\left(\frac{\cos(\beta-\phi)\sin(\beta-\phi)}{\sin^2\beta}-\frac{1}{2}\frac{\sin(\beta-\phi)}{\sin\beta}\sin\phi-\frac{1}{2}\frac{\cos(\beta-\phi)}{\sin\beta}\cos\phi\right)L^2\dot{\phi}^2\right)$$

$$=\left(\begin{array}{l} m\left(\dfrac{\cos^2(\beta-\phi)}{\sin^2\beta}-\dfrac{\cos(\beta-\phi)}{\sin\beta}\sin\phi\right)L^2\ddot{\phi}+m\dfrac{L^2}{3}\ddot{\phi} \\ +m\left(\dfrac{2\cos(\beta-\phi)\sin(\beta-\phi)}{\sin^2\beta}-\dfrac{\sin(\beta-\phi)}{\sin\beta}\sin\phi-\dfrac{\cos(\beta-\phi)}{\sin\beta}\cos\phi\right)L^2\dot{\phi}^2 \end{array}\right)$$

$$-\left(m\left(\frac{\cos(\beta-\phi)\sin(\beta-\phi)}{\sin^2\beta}-\frac{1}{2}\frac{\sin(\beta-\phi)}{\sin\beta}\sin\phi-\frac{1}{2}\frac{\cos(\beta-\phi)}{\sin\beta}\cos\phi\right)L^2\dot{\phi}^2\right)$$

$$=m\left(\frac{1}{3}+\frac{\cos^2(\beta-\phi)}{\sin^2\beta}-\frac{\cos(\beta-\phi)}{\sin\beta}\sin\phi\right)L^2\ddot{\phi}+m\left(\frac{\cos(\beta-\phi)\sin(\beta-\phi)}{\sin^2\beta}\right)L^2\dot{\phi}^2$$

$$+m\left(\frac{1}{2}\frac{\sin(\beta-\phi)}{\sin\beta}\sin\phi+\frac{1}{2}\frac{\cos(\beta-\phi)}{\sin\beta}\cos\phi\right)L^2\dot{\phi}^2$$

$$\Rightarrow \frac{d}{dt}\left(\frac{\partial T}{\partial \dot{\phi}}\right)-\left(\frac{\partial T}{\partial \phi}\right)=Q_\phi$$

$$m\left(\frac{1}{3}+\frac{\cos^2(\beta-\phi)}{\sin^2\beta}-\frac{\cos(\beta-\phi)}{\sin\beta}\sin\phi\right)L^2\ddot{\phi}+m\left(\frac{\cos(\beta-\phi)\sin(\beta-\phi)}{\sin^2\beta}+\frac{\cos(\beta-2\phi)}{2\sin\beta}\right)L^2\dot{\phi}^2$$

$$=-mg\frac{L}{2}\cos\phi-\frac{kL^2}{\sin^2\beta}\cos(\beta-\phi)\left(\sin\beta-\sin(\beta-\phi)\right)-fN_B\frac{\dot{\phi}}{|\dot{\phi}|}L\frac{\cos\phi}{\sin\beta}$$

Théorème de la résultante cinétique :

$$\left.\frac{d}{dt}\overline{R}\right|_x=\left.\overline{F}\right|_x$$

$$\frac{d}{dt}m\left(L\frac{\cos(\beta-\phi)(-\dot{\phi})}{\sin\beta}+\frac{L}{2}\sin\phi\dot{\phi}\right)=k\left(L-\left(L\cos\phi-\lambda\cos\beta\right)\right)+fN_B\cos\beta+N_B\sin\beta$$

$$mL\left(\frac{1}{2}\sin\phi-\frac{\cos(\beta-\phi)}{\sin\beta}\right)\ddot{\phi}+mL\left(\frac{1}{2}\cos\phi-\frac{\sin(\beta-\phi)}{\sin\beta}\right)\dot{\phi}^2$$

$$=\underline{k\left(L-\left(L\cos\phi-\lambda\cos\beta\right)\right)+\left(f\cos\beta+\sin\beta\right)N_B}$$

$\Rightarrow N_B$ à remplacer dans l'équation de mouvement.

4.2. $\overline{OG}=\left(x_A-\frac{L}{2}\cos\phi\right)\overline{1}_x+\frac{L}{2}\sin\phi\overline{1}_y \Rightarrow \overline{v}_G=\left(\dot{x}_A+\frac{L}{2}\sin\phi\dot{\phi}\right)\overline{1}_x+\left(\frac{L}{2}\cos\phi\dot{\phi}\right)\overline{1}_y$

$$T=\left(\frac{1}{2}mv_{G_i}^2+\frac{1}{2}\overline{\omega}.\overline{\overline{I}}_{G_i}.\overline{\omega}\right)=\frac{1}{2}m\left(\dot{x}_A^2+\frac{L^2}{4}\dot{\phi}^2+L\dot{x}_A\sin\phi\dot{\phi}\right)+\frac{1}{2}m\frac{L^2}{12}\dot{\phi}^2$$

$$\Rightarrow T=\frac{1}{2}m\left(\dot{x}_A^2+L\dot{x}_A\sin\phi\dot{\phi}\right)+\frac{1}{2}m\frac{L^2}{3}\dot{\phi}^2$$

avec $x_A\sin\beta=L\sin(\beta-\phi)\Rightarrow x_A\sin\beta-L\sin(\beta-\phi)=0\Rightarrow \delta x_A\sin\beta+L\cos(\beta-\phi)\delta\phi=0$

$$\overline{OG}=\left(x_A-\frac{L}{2}\cos\phi\right)\overline{1}_x+\frac{L}{2}\sin\phi\,\overline{1}_y \Rightarrow \delta\overline{OG}\Big|_y=\frac{L}{2}\cos\phi\,\delta\phi \text{ et } \overline{F}_G=-mg\,\overline{1}_y$$

$$\overline{OA}=x_A\overline{1}_x \Rightarrow \delta\overline{OA}=\delta x_A\overline{1}_x \text{ et } \overline{F}_A=k\left(L-x_A\right)\overline{1}_x$$

$$\overline{OB}=x_A\overline{1}_x-L\cos\phi\,\overline{1}_x+L\sin\phi\,\overline{1}_y \Rightarrow \delta\overline{OB}=\left(\delta x_A+L\sin\phi\,\delta\phi\right)\overline{1}_x+L\cos\phi\,\delta\phi\,\overline{1}_y$$

$$\overline{F}_B=N_B\overline{1}_u-T_B\frac{\dot\phi}{|\dot\phi|}\overline{1}_v=\left(N_B\sin\beta+T_B\frac{\dot\phi}{|\dot\phi|}\cos\beta\right)\overline{1}_x+\left(N_B\cos\beta-T_B\frac{\dot\phi}{|\dot\phi|}\sin\beta\right)\overline{1}_y \text{ avec } T_B=fN_B\frac{\dot\phi}{|\dot\phi|}$$

$$Q_\phi\delta\phi=\sum\overline{F}_i.\delta\overline{OP}_i$$

$$\Rightarrow\begin{cases} Q_\phi=-mg\frac{L}{2}\cos\phi+L\sin\phi N_B\left(\sin\beta+f\frac{\dot\phi}{|\dot\phi|}\cos\beta\right)+L\cos\phi N_B\left(\cos\beta-f\frac{\dot\phi}{|\dot\phi|}\sin\beta\right)\\ Q_{x_A}=k\left(L-x_A\right)+N_B\left(\sin\beta+f\frac{\dot\phi}{|\dot\phi|}\cos\beta\right)\end{cases}$$

$$\Rightarrow\begin{cases} Q_\phi=-mg\frac{L}{2}\cos\phi+LN_B\left(\cos(\beta-\phi)-f\frac{\dot\phi}{|\dot\phi|}\sin(\beta-\phi)\right)\\ Q_{x_A}=k\left(L-x_A\right)+N_B\left(\sin\beta+f\frac{\dot\phi}{|\dot\phi|}\cos\beta\right)\end{cases}$$

$$\frac{d}{dt}\left(\frac{\partial T}{\partial\dot\phi}\right)-\left(\frac{\partial T}{\partial\phi}\right)=Q_\phi+\sum_i\lambda_i\frac{\partial\phi_i}{\partial q_j}$$

$$Q_\phi=-mg\frac{L}{2}\cos\phi+LN_B\left(\cos(\beta-\phi)-f\frac{\dot\phi}{|\dot\phi|}\sin(\beta-\phi)\right)$$

$$Q_{x_A}=k\left(L-x_A\right)+N_B\left(\sin\beta+f\frac{\dot\phi}{|\dot\phi|}\cos\beta\right)$$

$$\begin{cases} m\left(\frac{L}{2}\ddot x_A\sin\phi+\frac{L}{2}\dot x_A\cos\phi\dot\phi+\frac{L^2}{3}\ddot\phi-\frac{L}{2}\dot x_A\cos\phi\dot\phi\right)=m\left(\frac{L^2}{3}\ddot\phi+\frac{L}{2}\ddot x_A\sin\phi\right)=\\ \qquad -mg\frac{L}{2}\cos\phi+LN_B\left(\cos(\beta-\phi)-f\frac{\dot\phi}{|\dot\phi|}\sin(\beta-\phi)\right)+\lambda_1L\cos(\beta-\phi)\quad(1)\\ m\ddot x_A+\frac{1}{2}mL\sin\phi\ddot\phi+\frac{1}{2}mL\cos\phi\dot\phi^2=k\left(L-x_A\right)+N_B\left(\sin\beta+f\frac{\dot\phi}{|\dot\phi|}\cos\beta\right)+\lambda_1\sin\beta\quad(2)\\ x_A\sin\beta=L\sin(\beta-\phi);\ \dot x_A\sin\beta=-L\cos(\beta-\phi)\dot\phi;\ \ddot x_A\sin\beta=-L\sin(\beta-\phi)\dot\phi^2-L\cos(\beta-\phi)\ddot\phi\quad(3)\end{cases}$$

On remplace la coordonnée x_A ainsi que ses dérivées dans les équations (1) et (2) par les équations (3)
On isole le multiplicateur de Lagrange de l'équation (2) et on le réinjecte dans l'équation (1).
On retrouve la même équation de mouvement qu'au (4).1 après quelques remplacements.

$$mL^2\left(\frac{1}{3}+\frac{\cos^2(\beta-\phi)}{\sin^2\beta}-\sin\phi\frac{\cos(\beta-\phi)}{\sin\beta}\right)\ddot\phi+mL^2\left(\frac{\cos(\beta-\phi)\sin(\beta-\phi)}{\sin^2\beta}-\frac{\cos(\beta-2\phi)}{2\sin\beta}\right)\dot\phi^2$$

$$=-mg\frac{L}{2}\cos\phi-kL\frac{L\cos(\beta-\phi)}{\sin\beta}\left(\sin\beta-\sin(\beta-\phi)\right)-N_BLf\frac{\dot\phi}{|\dot\phi|}\frac{\cos\phi}{\sin\beta}$$

Corrigés disponible sur http://cfao.ulb.ac.be/cfao/teaching/meca2/tps.html