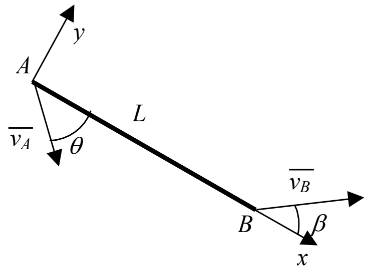
1.



$$\overline{v_B} = \overline{v_A} + \overline{\omega}_{AB} \times \overline{AB} \Rightarrow \begin{cases} v_B \cos\beta = v_A \cos\theta \\ v_B \sin\beta = -v_A \sin\theta + \omega_{AB} L \end{cases}$$

$$\omega_{AB} = \frac{1}{L}\left(v_B \sqrt{1-\left(\frac{v_A}{v_B}\right)^2 \cos^2\theta} + v_A \sin\theta \right)$$

2. $$\overline{AG} = \left[\frac{L}{2}\cos\theta + \frac{3L}{8}\cos(\theta+\alpha)\right]\overline{1}_x + \left[\frac{L}{2}\sin\theta + \frac{3L}{8}\sin(\theta+\alpha)\right]\overline{1}_y$$

Par dérivation des coordonnées dans le repère *Axy*

$$\overline{v}_G = \frac{d\overline{AG}}{dt} = \left[-\frac{L}{2}\sin\theta\dot{\theta} - \frac{3L}{8}\sin(\theta+\alpha)\dot{\theta}\right]\overline{1}_x + \left[\frac{L}{2}\cos\theta\dot{\theta} + \frac{3L}{8}\cos(\theta+\alpha)\dot{\theta}\right]\overline{1}_y$$

$$\overline{a}_G = \frac{d\overline{v}_G}{dt} = \left[-\frac{L}{2}\cos\theta\dot{\theta}^2 - \frac{3L}{8}\cos(\theta+\alpha)\dot{\theta}^2 - \frac{L}{2}\sin\theta\ddot{\theta} - \frac{3L}{8}\sin(\theta+\alpha)\ddot{\theta}\right]\overline{1}_x$$

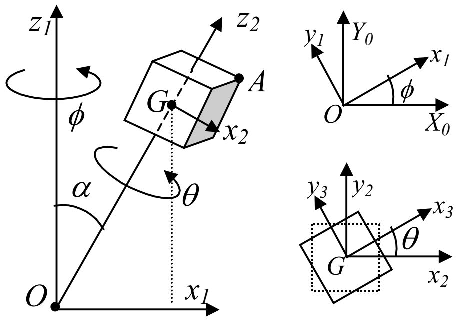
$$+\underline{\left[-\frac{L}{2}\sin\theta\dot{\theta}^2 - \frac{3L}{8}\sin(\theta+\alpha)\dot{\theta}^2 + \frac{L}{2}\cos\theta\ddot{\theta} + \frac{3L}{8}\cos(\theta+\alpha)\ddot{\theta}\right]}\overline{1}_y$$

Par la méthode des distribution des vitesse dans le repère *Axy* $(\overline{\omega}_{AG} = \dot{\theta}\overline{1}_z)$

$$\overline{v}_G = \overline{v}_A + \overline{\omega}_{AG} \times \overline{AG} = \dot{\theta}\overline{1}_z \times \left(\left[\frac{L}{2}\cos\theta + \frac{3L}{8}\cos(\theta+\alpha)\right]\overline{1}_x + \left[\frac{L}{2}\sin\theta + \frac{3L}{8}\sin(\theta+\alpha)\right]\overline{1}_y\right)$$

$$\overline{a}_G = \overline{a}_A + \overline{\omega}_{AG} \times \left(\overline{\omega}_{AG} \times \overline{AG}\right) + \overline{\varepsilon}_{AG} \times \overline{AG} \quad \text{avec} \quad \overline{\varepsilon}_{AG} = \ddot{\theta}\overline{1}_z$$

3



R_0 : Axe $OX_0Y_0Z_0$ fixe.
R_1 : Axe $Ox_1y_1z_1$ tourne autour de l'axe $Z_0 = z_1$ (ϕ)
R_2 : Axe $Gx_2y_2z_2$ incliné d'un angle α par rapport aux axes $Ox_1y_1z_1$, avec z_2 lié à la tige, tourne autour de l'axe z_1 (ϕ)
R_3 : Axe $Gx_3y_3z_3$ liés au cube et tourne autour de z_1 (ϕ) et z_2 (θ)
=> les vecteurs de Darboux :

$$\overline{\omega}_{R_1/R_0} = \dot{\phi}\overline{1}_{z_1} \; ; \; \overline{\omega}_{R_2/R_0} = \dot{\phi}\overline{1}_{z_1} \; ; \; \overline{\omega}_{R_3/R_0} = \dot{\phi}\overline{1}_{z_1} + \dot{\theta}\overline{1}_{z_2}$$

a. <u>Par la formule des distribution des vitesses exprimé dans le repère $Ox_2y_2z_2$</u>

$$\overline{\omega}_{Tige} = \dot{\phi}\overline{1}_{z_1} \quad \text{et} \quad \overline{\omega}_{Carré} = \dot{\phi}\overline{1}_{z_1} + \dot{\theta}\overline{1}_{z_2} = -\dot{\phi}\sin\alpha\,\overline{1}_{x_2} + \left(\dot{\theta} + \dot{\phi}\cos\alpha\right)\overline{1}_{z_2}$$

$$\overline{v}_G = \overline{v}_O + \overline{\omega}_{Tige} \times \overline{OG} = 0 + L\dot{\phi}\sin\alpha\,\overline{1}_y = L\dot{\phi}\sin\alpha\,\overline{1}_{y'}$$

$$\overline{GA} = \frac{d}{2}\left(\overline{1}_{x_3} + \overline{1}_{y_3} + \overline{1}_{z_3}\right) = \frac{d}{2}\left((\cos\theta - \sin\theta)\overline{1}_{x_2} + (\sin\theta + \cos\theta)\overline{1}_{y_2} + \overline{1}_{z_2}\right)$$

$$\overline{v}_A = \overline{v}_G + \overline{\omega}_{Carré} \times \overline{GA} = \begin{cases} -\frac{d}{2}\left(\left(\dot{\theta} + \dot{\phi}\cos\alpha\right)(\sin\theta + \cos\theta)\right)\overline{1}_{x_2} \\ +\left(\left(L + \frac{d}{2}\right)\dot{\phi}\sin\alpha + \frac{d}{2}\left(\dot{\theta} + \dot{\phi}\cos\alpha\right)(\cos\theta - \sin\theta)\right)\overline{1}_{y_2} \\ -\frac{d}{2}\dot{\phi}\sin\alpha(\sin\theta + \cos\theta)\overline{1}_{z_2} \end{cases}$$

b. <u>Par dérivation des coordonnées exprimées dans le repère $Ox_3y_3z_3$</u>

$$\overline{OA} = \frac{d}{2}\bar{1}_{x_3} + \frac{d}{2}\bar{1}_{y_3} + \left(\frac{d}{2} + L\right)\bar{1}_{z_3} \quad \text{et} \quad \bar{\omega}_{R_3/R_0} = -\dot{\phi}\sin\alpha\cos\theta\,\bar{1}_{x_3} + \dot{\phi}\sin\alpha\sin\theta\,\bar{1}_{y_3} + \left(\dot{\theta} + \dot{\phi}\cos\alpha\right)\bar{1}_{z_3}$$

$$\bar{v}_A = \frac{d\overline{OA}}{dt} = \underbrace{\left.\frac{d\overline{OA}}{dt}\right|_{x_3y_3z_3}}_{=0} + \bar{\omega}_{R_3/R_0} \times \overline{OA} = \begin{cases} \left(\left(\frac{d}{2} + L\right)\dot{\phi}\sin\alpha\sin\theta - \frac{d}{2}\left(\dot{\theta} + \dot{\phi}\cos\alpha\right)\right)\bar{1}_{x_3} \\ +\left(\left(\dot{\theta} + \dot{\phi}\cos\alpha\right)\frac{d}{2} + \left(\frac{d}{2} + L\right)\dot{\phi}\sin\alpha\cos\theta\right)\bar{1}_{y_3} \\ -\frac{d}{2}\dot{\phi}\sin\alpha\left(\cos\theta + \sin\theta\right)\bar{1}_{z_3} \end{cases}$$

b'. <u>Par dérivation des coordonnées exprimées dans le repère $Ox_2y_2z_2$</u>

$$\overline{OA} = \frac{d}{2}\left(\cos\theta - \sin\theta\right)\bar{1}_{x_2} + \frac{d}{2}\left(\sin\theta + \cos\theta\right)\bar{1}_{y_2} + \left(\frac{d}{2} + L\right)\bar{1}_{z_2} \quad \text{et} \quad \bar{\omega}_{R_2/R_0} = -\dot{\phi}\sin\alpha\,\bar{1}_{x_2} + \dot{\phi}\cos\alpha\,\bar{1}_{z_2}$$

$$\bar{v}_A = \frac{d\overline{OA}}{dt} = \left.\frac{d\overline{OA}}{dt}\right|_{R_2} + \underline{\bar{\omega}_{R_2/R_0}} \times \overline{OA} = \begin{cases} -\frac{d}{2}\left(\sin\theta + \cos\theta\right)\left(\dot{\theta} + \dot{\phi}\cos\alpha\right)\bar{1}_{x_2} \\ +\left(\left(\dot{\theta} + \dot{\phi}\cos\alpha\right)\frac{d}{2}\left(\cos\theta - \sin\theta\right) + \dot{\phi}\sin\alpha\left(\frac{d}{2} + L\right)\right)\bar{1}_{y_2} \\ -\left(\dot{\phi}\sin\alpha\frac{d}{2}\left(\sin\theta + \cos\theta\right)\right)\bar{1}_{z_2} \end{cases}$$

c. $\bar{\omega}_{Cube} = -\dot{\phi}\sin\alpha\,\bar{1}_{x_2} + \left(\dot{\theta} + \dot{\phi}\cos\alpha\right)\bar{1}_{z_2}$

$$\bar{\varepsilon}_{Cube} = \left.\frac{d\bar{\omega}_{Cube}}{dt}\right|_{rel\text{ dans }R_2} + \bar{\omega}_{R_2/R_0} \times \bar{\omega}_{Cube} \quad \text{avec} \quad \bar{\omega}_{R_2/R_0} = -\dot{\phi}\sin\alpha\,\bar{1}_{x_2} + \dot{\phi}\cos\alpha\,\bar{1}_{z_2}$$

$$\bar{\varepsilon}_{Cube} = -\ddot{\phi}\sin\alpha\,\bar{1}_{x_2} + \left(\ddot{\theta} + \ddot{\phi}\cos\alpha\right)\bar{1}_{z_2} - \cos\alpha\sin\alpha\dot{\phi}^2\,\bar{1}_{y_2} + \left(\sin\alpha\dot{\phi}\dot{\theta} + \sin\alpha\cos\alpha\dot{\phi}^2\right)\bar{1}_{y_2}$$

$$\bar{\varepsilon}_{Cube} = -\ddot{\phi}\sin\alpha\,\bar{1}_{x_2} + \sin\alpha\dot{\phi}\dot{\theta}\,\bar{1}_{y_2} + \left(\ddot{\theta} + \ddot{\phi}\cos\alpha\right)\bar{1}_{z_2}$$

d. $\bar{\omega}_{Cube} = -\sin\alpha\cos\theta\dot{\phi}\,\bar{1}_{x_3} + \sin\alpha\sin\theta\dot{\phi}\,\bar{1}_{y_3} + \left(\dot{\theta} + \cos\alpha\dot{\phi}\right)\bar{1}_{z_3}$ avec $\bar{\omega}_{R_3/R_0} = \bar{\omega}_{Cube}$

$$\bar{\varepsilon}_{Cube} = \left.\frac{d\bar{\omega}_{Cube}}{dt}\right|_{rel\text{ dans }R_3} + \underbrace{\bar{\omega}_{R_3/R_0} \times \bar{\omega}_{Cube}}_{=0}$$

$$\bar{\varepsilon}_{Cube} = -\sin\alpha\cos\theta\ddot{\phi}\,\bar{1}_{x_3} + \dot{\phi}\sin\alpha\sin\theta\dot{\theta}\,\bar{1}_{x_3} + \sin\alpha\sin\theta\ddot{\phi}\,\bar{1}_{y_3} + \sin\alpha\cos\theta\dot{\theta}\dot{\phi}\,\bar{1}_{y_3} + \left(\ddot{\theta} + \ddot{\phi}\cos\alpha\right)\bar{1}_{z_3}$$

$$\bar{\varepsilon}_{Cube} = \left(\sin\alpha\sin\theta\dot{\phi}\dot{\theta} - \sin\alpha\cos\theta\ddot{\phi}\right)\bar{1}_{x_3} + \left(\sin\alpha\cos\theta\dot{\phi}\dot{\theta} + \sin\alpha\sin\theta\ddot{\phi}\right)\bar{1}_{y_3} + \left(\ddot{\theta} + \ddot{\phi}\cos\alpha\right)\bar{1}_{z_3}$$

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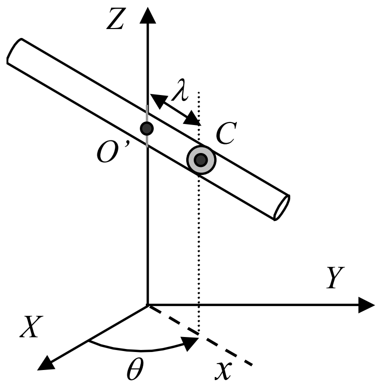
4. $\begin{cases} \bar{1}_x = \cos\theta\,\bar{1}_X + \sin\theta\,\bar{1}_Y \\ \bar{1}_y = -\sin\theta\,\bar{1}_X + \cos\theta\,\bar{1}_Y \\ \bar{1}_z = \bar{1}_z \end{cases}$ et $\bar{\omega} = \omega\,\bar{1}_z$



$$\bar{v}_C = \bar{v}_{C-rel} + \bar{v}_{C-entr} \quad \text{avec} \quad \begin{cases} \bar{v}_{C-rel} = \left.\frac{d\overline{O'C}}{dt}\right|_{rel} = \dot{\lambda}\,\bar{1}_x \\ \bar{v}_{C-entr} = \bar{v}_{O'} + \bar{\omega} \times \underline{\overline{O'C}} = 0 + \lambda\omega\,\bar{1}_y \end{cases}$$

$$\bar{a}_C = \bar{a}_{C-rel} + \bar{a}_{C-entr} + \bar{a}_{C-cor}$$

$$\text{avec} \begin{cases} \bar{a}_{C-rel} = \ddot{\lambda}\,\bar{1}_x \\ \bar{a}_{C-cor} = 2\bar{\omega} \times \bar{v}_{C-rel} = 2\omega\,\bar{1}_z \times \dot{\lambda}\,\bar{1}_x = 2\omega\dot{\lambda}\,\bar{1}_y \\ \bar{a}_{C-entr} = \bar{a}_{O'} + \bar{\omega}_T \times \left(\bar{\omega}_T \times \underline{\overline{O'C}}\right) + \bar{\varepsilon}_T \times \underline{\overline{O'C}} = 0 - \lambda\omega^2\,\bar{1}_x + 0 \end{cases}$$

$$=> \bar{a}_C = \ddot{\lambda}\,\bar{1}_x - \lambda\omega^2\,\bar{1}_x + 2\omega\dot{\lambda}\,\bar{1}_y$$

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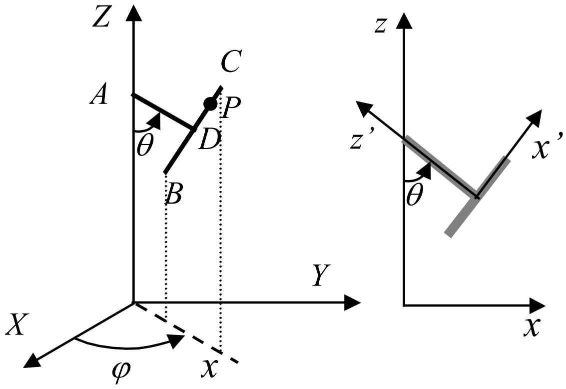
5. $\begin{cases} \overline{1}_{x'} = \cos\theta\,\overline{1}_x + \sin\theta\,\overline{1}_z \\ \overline{1}_{y'} = \overline{1}_y \\ \overline{1}_{z'} = -\sin\theta\,\overline{1}_x + \cos\theta\,\overline{1}_z \end{cases}$ et $\begin{cases} \overline{1}_x = \cos\theta\,\overline{1}_{x'} - \sin\theta\,\overline{1}_{z'} \\ \overline{1}_y = \overline{1}_{y'} \\ \overline{1}_z = \sin\theta\,\overline{1}_{x'} + \cos\theta\,\overline{1}_{z'} \end{cases}$



Cinématique (formule de distribution des vitesses) avec les vecteurs exprimés dans le repère *x'y'z'* :

a. $\overline{\omega}_T = \sin\theta\dot{\phi}\,\overline{1}_{x'} - \dot{\theta}\,\overline{1}_{y'} + \cos\theta\dot{\phi}\,\overline{1}_{z'}$ et $\overline{\omega}_{x'y'z'/XYZ} = \overline{\omega}_T = \sin\theta\dot{\phi}\,\overline{1}_{x'} - \dot{\theta}\,\overline{1}_{y'} + \cos\theta\dot{\phi}\,\overline{1}_{z'}$

$$=> \overline{\varepsilon}_T = \left.\frac{d\overline{\omega}_T}{dt}\right|_{rel} + \overline{\omega}_{x'y'z'/XYZ} \times \overline{\omega}_T = \left(\cos\theta\dot{\theta}\dot{\phi} + \sin\theta\ddot{\phi}\right)\overline{1}_{x'} - \ddot{\theta}\,\overline{1}_{y'} + \left(\cos\theta\ddot{\phi} - \sin\theta\dot{\theta}\dot{\phi}\right)\overline{1}_{z'}$$

$\overline{AD} = -L\,\overline{1}_{z'}$

$\overline{v}_D = \overline{v}_A + \overline{\omega}_T \times \overline{AD} = 0 + L\dot{\theta}\,\overline{1}_{x'} + L\sin\theta\dot{\phi}\,\overline{1}_{y'}$

$$\overline{a}_D = \overline{a}_A + \overline{\omega}_T \times \left(\overline{\omega}_T \times \overline{AD}\right) + \overline{\varepsilon}_T \times \overline{AD} = L\left(\ddot{\theta} - \sin\theta\cos\theta\dot{\phi}^2\right)\overline{1}_{x'} + L\left(2\cos\theta\dot{\theta}\dot{\phi} + \sin\theta\ddot{\phi}\right)\overline{1}_{y'} + \left(L\sin^2\theta\dot{\phi}^2 + L\dot{\theta}^2\right)\overline{1}_{z'}$$

b. $\overline{AP} = \lambda\,\overline{1}_{x'} - L\,\overline{1}_{z'}$

$\overline{v}_{P_{entr}} = \overline{v}_D + \overline{\omega}_T \times \overline{DP} = L\dot{\theta}\,\overline{1}_{x'} + \left(L\sin\theta\dot{\phi} + \lambda\cos\theta\dot{\phi}\right)\overline{1}_{y'} + \lambda\dot{\theta}\,\overline{1}_{z'}$ et $\overline{v}_{P_{rel}} = \dot{\lambda}\,\overline{1}_{x'}$

$=> \overline{v}_P = \left(L\dot{\theta} + \dot{\lambda}\right)\overline{1}_{x'} + \left(L\sin\theta\dot{\phi} + \lambda\cos\theta\dot{\phi}\right)\overline{1}_{y'} + \lambda\dot{\theta}\,\overline{1}_{z'}$

Cinématique (formule de distribution des vitesses) avec les vecteurs exprimés dans le repère *xyz* :

a. $\overline{\omega}_T = -\dot{\theta}\,\overline{1}_y + \dot{\phi}\,\overline{1}_z$; $\overline{\omega}_{xyz/XYZ} = \dot{\phi}\,\overline{1}_z$ $=>$ $\overline{\varepsilon}_T = \left.\frac{d\overline{\omega}_2}{dt}\right|_{rel} + \overline{\omega}_{xyz/XYZ} \times \overline{\omega}_T = \dot{\theta}\dot{\phi}\,\overline{1}_x - \ddot{\theta}\,\overline{1}_y + \ddot{\phi}\,\overline{1}_z$;

$\overline{AD} = L\sin\theta\,\overline{1}_x - L\cos\theta\,\overline{1}_z$

$\overline{v}_D = \overline{v}_A + \overline{\omega}_T \times \overline{AD} = L\cos\theta\dot{\theta}\,\overline{1}_x + L\sin\theta\dot{\phi}\,\overline{1}_y + L\sin\theta\dot{\theta}\,\overline{1}_z$

$$\overline{a}_D = \overline{a}_A + \overline{\omega}_T \times \left(\overline{\omega}_T \times \overline{AD}\right) + \overline{\varepsilon}_T \times \overline{AD} = \left|\begin{array}{l} \left(L\cos\theta\ddot{\theta} - L\sin\theta\dot{\phi}^2 - L\sin\theta\dot{\theta}^2\right)\overline{1}_x \\ +\left(2L\cos\theta\dot{\theta}\dot{\phi} + L\sin\theta\ddot{\phi}\right)\overline{1}_y \\ +\left(L\cos\theta\dot{\theta}^2 + L\sin\theta\ddot{\theta}\right)\overline{1}_z \end{array}\right.$$

b. $\overline{AP} = \left(L\sin\theta + \lambda\cos\theta\right)\overline{1}_x + \left(-L\cos\theta + \lambda\sin\theta\right)\overline{1}_z$

$\overline{v}_{P_{entr}} = \overline{v}_A + \overline{\omega}_{xyz/XYZ} \times \overline{AP} = \left(L\sin\theta + \lambda\cos\theta\right)\dot{\phi}\,\overline{1}_y$

$$\overline{v}_{P_{rel}} = \left.\frac{d\overline{AP}}{dt}\right|_{rel-xyz} = \left(L\cos\theta\dot{\theta} + \dot{\lambda}\cos\theta - \lambda\sin\theta\dot{\theta}\right)\overline{1}_x + \left(L\sin\theta\dot{\theta} + \dot{\lambda}\sin\theta + \lambda\cos\theta\dot{\theta}\right)\overline{1}_z$$

$$=> \overline{v}_P = \left(L\cos\theta\dot{\theta} - \lambda\sin\theta\dot{\theta} + \dot{\lambda}\cos\theta\right)\overline{1}_x + \left(L\sin\theta\dot{\phi} + \lambda\cos\theta\dot{\phi}\right)\overline{1}_y + \left(L\sin\theta\dot{\theta} + \lambda\cos\theta\dot{\theta} + \dot{\lambda}\sin\theta\right)\overline{1}_z$$

Par dérivation des vecteurs dans le repère *x'y'z'* :

a. $\overline{\omega}_{x'y'z'/XYZ} = \sin\theta\dot{\phi}\,\overline{1}_{x'} - \dot{\theta}\,\overline{1}_{y'} + \cos\theta\dot{\phi}\,\overline{1}_{z'}$

$$\overline{AD} = -L\,\overline{1}_{z'} => \text{A fixe : } \overline{v}_D = \underbrace{\left.\frac{d\overline{AD}}{dt}\right|_{rel}}_{\overline{v}_{D_{rel}}} + \underbrace{\overline{\omega}_{x'y'z'/XYZ} \times \overline{AD}}_{\overline{v}_{D_{entr}}} = 0 + L\sin\theta\dot{\phi}\,\overline{1}_{y'} + L\dot{\theta}\,\overline{1}_{x'}$$

$$\overline{a}_D = \left.\frac{d\,\overline{v}_D}{dt}\right|_{rel} + \overline{\omega}_{x'y'z'/XYZ} \times \overline{v}_D = L\left(\ddot{\theta} - \sin\theta\cos\theta\dot{\phi}^2\right)\overline{1}_{x'} + L\left(2\cos\theta\dot{\theta}\dot{\phi} + \sin\theta\ddot{\phi}\right)\overline{1}_{y'} + L\left(\sin^2\theta\dot{\phi}^2 + \dot{\theta}^2\right)\overline{1}_{z'}$$

$$\overline{AP} = \lambda \overline{1}_{x'} - L\overline{1}_{z'} \Rightarrow \bar{v}_P = \left.\frac{d\overline{AP}}{dt}\right|_{rel} + \bar{\omega}_{x'y'z'/XYZ} \times \overline{AP} = \left(L\dot{\theta} + \dot{\lambda}\right)\overline{1}_{x'} + \left(\lambda\cos\theta\dot{\phi} + L\sin\theta\dot{\phi}\right)\overline{1}_{y'} + \lambda\dot{\theta}\overline{1}_{z'}$$

$$\bar{a}_P = \left.\frac{d\bar{v}_P}{dt}\right|_{rel} + \bar{\omega}_{x'y'z'/XYZ} \times \bar{v}_P =$$

$$\begin{cases} \left(L\ddot{\theta} + \ddot{\lambda} - \left(\lambda\cos^2\theta + L\cos\theta\sin\theta\right)\dot{\phi}^2 - \lambda\dot{\theta}^2\right)\overline{1}_{x'} \\ +\left(2\dot{\lambda}\cos\theta\dot{\phi} + \left(-\lambda sin\theta + 2L\cos\theta\right)\dot{\theta}\dot{\phi} + \left(\lambda\cos\theta + L\sin\theta\right)\ddot{\phi} - \lambda\sin\theta\dot{\theta}\dot{\phi}\right)\overline{1}_{y'} \\ +\left(\lambda\ddot{\theta} + 2\dot{\lambda}\dot{\theta} + L\dot{\theta}^2 + \left(\lambda\sin\theta\cos\theta + L\sin^2\theta\right)\dot{\phi}^2\right)\overline{1}_{z'} \end{cases}$$

Par dérivation dans les axes *xyz*: $\bar{\omega}_{xyz/XYZ} = \dot{\phi}\overline{1}_z$

$$\overline{AD} = L\sin\theta\overline{1}_x - L\cos\theta\overline{1}_z \Rightarrow \bar{v}_D = \left.\frac{d\overline{AD}}{dt}\right|_{rel} + \bar{\omega}_{xyz/XYZ} \times \overline{AD} = L\cos\dot{\theta}\overline{1}_x + L\sin\theta\dot{\phi}\overline{1}_y + L\sin\theta\dot{\theta}\overline{1}_z$$

Par dérivation des vecteurs dans le repère *xyz* :

$$\bar{\omega}_{xyz/XYZ} = \dot{\phi}\overline{1}_z$$

$$\overline{AD} = L\sin\theta\overline{1}_x - L\cos\theta\overline{1}_z \Rightarrow \bar{v}_D = \left.\frac{d\overline{AD}}{dt}\right|_{rel} + \bar{\omega}_{xyz/XYZ} \times \overline{AD} = L\cos\theta\dot{\theta}\overline{1}_x + L\sin\theta\dot{\phi}\overline{1}_y + L\sin\theta\dot{\theta}\overline{1}_z$$

$$\bar{a}_D = \left.\frac{d\,\bar{v}_D}{dt}\right|_{rel} + \bar{\omega}_{xyz/XYZ} \times \bar{v}_D = \begin{cases} \left(L\cos\theta\ddot{\theta} - L\sin\theta\dot{\theta}^2 + L\sin\theta\dot{\phi}^2\right)\overline{1}_x \\ +\left(2L\cos\theta\dot{\theta}\dot{\phi} + L\sin\theta\ddot{\phi}\right)\overline{1}_y \\ +\left(L\cos\theta\dot{\phi}^2 + L\sin\theta\ddot{\theta}\right)\overline{1}_z \end{cases}$$

b. $\overline{AP} = \left(\lambda\cos\theta + L\sin\theta\right)\overline{1}_x + \left(\lambda\sin\theta - L\cos\theta\right)\overline{1}_z$

$$\bar{v}_P = \left.\frac{d\overline{AP}}{dt}\right|_{rel} + \bar{\omega}_{xyz/XYZ} \times \overline{AP} = \begin{cases} \left(-\lambda\sin\theta\dot{\theta} + \dot{\lambda}\cos\theta + L\cos\theta\dot{\theta}\right)\overline{1}_x \\ +\left(\lambda\cos\theta\dot{\phi} + L\sin\theta\dot{\phi}\right)\overline{1}_y \\ +\left(\lambda\cos\theta\dot{\theta} + \dot{\lambda}\sin\theta + L\sin\theta\dot{\theta}\right)\overline{1}_z \end{cases}$$

$$\bar{a}_P = \left.\frac{d\,\bar{v}_P}{dt}\right|_{rel} + \bar{\omega}_{xyz/XYZ} \times \bar{v}_P$$

$$= \begin{cases} \left(\ddot{\lambda}\cos\theta - 2\dot{\lambda}\sin\theta\dot{\theta} + \left(L\cos\theta - \lambda\sin\theta\right)\ddot{\theta} - \left(\lambda\cos\theta + L\sin\theta\right)\dot{\theta}^2 - \left(\lambda\cos\theta + L\sin\theta\right)\dot{\phi}^2\right)\overline{1}_x \\ +\left(\left(\lambda\cos\theta + L\sin\theta\right)\ddot{\phi} + 2\dot{\lambda}\cos\theta\dot{\phi} + \left(2L\cos\theta - 2\lambda\sin\theta\right)\dot{\theta}\dot{\phi}\right)\overline{1}_y \\ +\left(2\dot{\lambda}\cos\theta\dot{\theta} + \ddot{\lambda}\sin\theta + \left(L\cos\theta - \lambda\sin\theta\right)\dot{\theta}^2 + \left(\lambda\cos\theta + L\sin\theta\right)\ddot{\theta}\right)\overline{1}_z \end{cases}$$

Utilisation des formules du mouvement relatif dans le repère *x'y'z'* :

D = origine du repère mobile ($\bar{\omega}_{x'y'z'/XYZ} = \sin\theta\dot{\phi}\overline{1}_{x'} - \dot{\theta}\overline{1}_{y'} + \cos\theta\dot{\phi}\overline{1}_{z'} = \bar{\omega}_T$)

$$\Rightarrow \bar{\varepsilon}_T = \left(\cos\theta\dot{\theta}\dot{\phi} + \sin\theta\ddot{\phi}\right)\overline{1}_{x'} - \ddot{\theta}\overline{1}_{y'} + \left(\cos\theta\ddot{\phi} - \sin\theta\dot{\theta}\dot{\phi}\right)\overline{1}_{z'}$$

b.

$$\bar{v}_P = \bar{v}_{P-rel} + \bar{v}_{P-entr} \quad \text{avec} \quad \begin{cases} \bar{v}_{P-rel} = \left.\frac{d\overline{DP}}{dt}\right|_{rel-x'y'z'} = \dot{\lambda}\overline{1}_{x'} \\ \bar{v}_{P-entr} = \bar{v}_D + \bar{\omega}_T \times \overline{DP} = L\dot{\theta}\overline{1}_{x'} + \left(L\sin\theta\dot{\phi} + \lambda\cos\theta\dot{\phi}\right)\overline{1}_{y'} + \lambda\dot{\theta}\overline{1}_{z'} \end{cases}$$

$$\bar{a}_P = \bar{a}_{P-rel} + \bar{a}_{P-entr} + \bar{a}_{P-cor} \quad \text{avec}$$

$$\bar{a}_{P-rel} = \ddot{\lambda}\overline{1}_{x'}$$

$$\bar{a}_{P-cor} = 2\bar{\omega}_{x'y'z'/XYZ} \times \bar{v}_{rel} = 2\left(\sin\theta\dot{\phi}\overline{1}_{x'} - \dot{\theta}\overline{1}_{y'} + \cos\theta\dot{\phi}\overline{1}_{z'}\right) \times \dot{\lambda}\overline{1}_{x'} = +2\dot{\lambda}\cos\theta\dot{\phi}\overline{1}_{y'} + 2\dot{\theta}\dot{\lambda}\overline{1}_{z'}$$

$$\bar{a}_{P-entr} = \bar{a}_D + \bar{\omega}_T \times \left(\bar{\omega}_T \times \overline{DP}\right) + \bar{\varepsilon}_T \times \overline{DP} \quad \text{avec } \overline{DP} = \lambda \bar{1}_{x'}$$

$$\bar{a}_{P-entr} = \begin{cases} L\left(\ddot{\theta} - \sin\theta\cos\theta\dot{\phi}^2\right)\bar{1}_{x'} + L\left(2\cos\theta\dot{\theta}\dot{\phi} + \sin\theta\ddot{\phi}\right)\bar{1}_{y'} + \left(L\sin^2\theta\dot{\phi}^2 + L\dot{\theta}^2\right)\bar{1}_{z'} \\ +\left(-\lambda\sin\theta\dot{\theta}\dot{\phi}\bar{1}_{y'} - \lambda\dot{\theta}^2\bar{1}_{x'}\right) + \left(\lambda\cos\theta\sin\theta\dot{\phi}^2\bar{1}_{z'} - \lambda\cos^2\theta\dot{\phi}^2\bar{1}_{x'}\right) \\ +\lambda\ddot{\theta}\bar{1}_{z'} + \lambda\left(\cos\theta\ddot{\phi} - \sin\theta\dot{\theta}\dot{\phi}\right)\bar{1}_{y'} \end{cases}$$

$$\bar{a}_P = \begin{cases} \left(\ddot{\lambda} + L\ddot{\theta} - \lambda\dot{\theta}^2 - \left(L\sin\theta\cos\theta + \lambda\cos^2\theta\right)\dot{\phi}^2\right)\bar{1}_{x'} \\ +\left(2\dot{\lambda}\cos\theta\dot{\phi} + \left(2L\cos\theta - \lambda\sin\theta\right)\dot{\theta}\dot{\phi} + \left(L\sin\theta + \lambda\cos\theta\right)\ddot{\phi} - \lambda\sin\theta\dot{\theta}\dot{\phi}\right)\bar{1}_{y'} \\ +\left(2\dot{\theta}\dot{\lambda} + \left(\lambda\cos\theta\sin\theta + L\sin^2\theta\right)\dot{\phi}^2 + L\dot{\theta}^2 + \lambda\ddot{\theta}\right)\bar{1}_{z'} \end{cases}$$

Par l'utilisation des formules du mouvement relatif dans le repère *xyz* :

A = origine du repère mobile xyz $\quad (\bar{\omega}_1 = \bar{\omega}_{xyz/XYZ} = \dot{\phi}\bar{1}_z)$

$$\overline{AP} = \left(L\sin\theta + \lambda\cos\theta\right)\bar{1}_x + \left(\lambda\sin\theta - L\cos\theta\right)\bar{1}_z$$

$$\bar{v}_P = \bar{v}_{P-rel} + \bar{v}_{P-entr} \quad \text{avec} \begin{cases} \bar{v}_{P-rel} = \left.\dfrac{d\overline{AP}}{dt}\right|_{rel} = \left(L\cos\theta\dot{\theta} + \dot{\lambda}\cos\theta - \lambda\sin\theta\dot{\theta}\right)\bar{1}_x + \left(\dot{\lambda}\sin\theta + \lambda\cos\theta\dot{\theta} + L\sin\theta\dot{\theta}\right)\bar{1}_z \\ \bar{v}_{P-entr} = \bar{v}_A + \bar{\omega}_1 \times \overline{AP} = \left(L\sin\theta + \lambda\cos\theta\right)\dot{\phi}\bar{1}_y \end{cases}$$

$\bar{a}_P = \bar{a}_{P-rel} + \bar{a}_{P-entr} + \bar{a}_{P-cor}$ avec

$$\begin{cases} \bar{a}_{P-rel} = \begin{cases} \left(+\ddot{\lambda}\cos\theta - 2\dot{\lambda}\sin\theta\dot{\theta} - \left(L\sin\theta + \lambda\cos\theta\right)\dot{\theta}^2 + \left(L\cos\theta - \lambda\sin\theta\right)\ddot{\theta}\right)\bar{1}_x \\ +\left(\ddot{\lambda}\sin\theta + 2\dot{\lambda}\cos\theta\dot{\theta} + \left(L\cos\theta - \lambda\sin\theta\right)\dot{\theta}^2 + \left(\lambda\cos\theta + L\sin\theta\right)\ddot{\theta}\right)\bar{1}_z \end{cases} \\ \bar{a}_{P-cor} = 2\bar{\omega}_{xyz/XYZ} \times \bar{v}_{P-rel} = 2\left(\left(L\cos\theta - \lambda\sin\theta\right)\dot{\theta}\dot{\phi} + \dot{\lambda}\cos\theta\dot{\phi}\right)\bar{1}_y \\ \bar{a}_{P-entr} = \bar{a}_A + \bar{\omega}_1 \times \left(\bar{\omega}_1 \times \overline{AP}\right) + \bar{\varepsilon}_1 \times \overline{AP} = -\left(L\sin\theta + \lambda\cos\theta\right)\dot{\phi}^2\bar{1}_x + \left(L\sin\theta + \lambda\cos\theta\right)\ddot{\phi}\bar{1}_y \end{cases}$$

$$\bar{a}_P = \begin{cases} \left(+\ddot{\lambda}\cos\theta - 2\dot{\lambda}\sin\theta\dot{\theta} - \left(L\sin\theta + \lambda\cos\theta\right)\dot{\theta}^2 - \left(L\sin\theta + \lambda\cos\theta\right)\dot{\phi}^2 + \left(L\cos\theta - \lambda\sin\theta\right)\ddot{\theta}\right)\bar{1}_x \\ \left(+2\dot{\lambda}\cos\theta\dot{\phi} + \left(2L\cos\theta - 2\lambda\sin\theta\right)\dot{\theta}\dot{\phi} + \left(L\sin\theta + \lambda\cos\theta\right)\ddot{\phi}\right)\bar{1}_y \\ +\left(\ddot{\lambda}\sin\theta + 2\dot{\lambda}\cos\theta\dot{\theta} + \left(L\cos\theta - \lambda\sin\theta\right)\dot{\theta}^2 + \left(\lambda\cos\theta + L\sin\theta\right)\ddot{\theta}\right)\bar{1}_z \end{cases}$$

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http://cfao.ulb.ac.be/cfao/ >Teaching>mécaII>Tps.