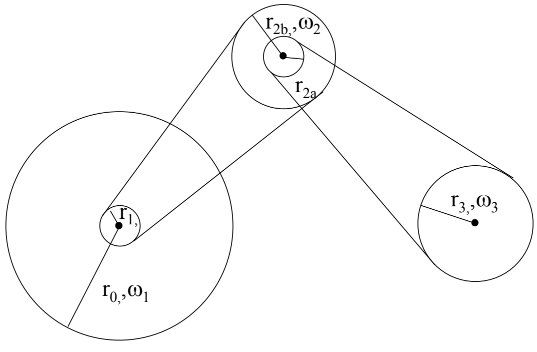
1.



$$v = 152 \quad miles/h = 244620 \quad m/h$$

$$Circonférence \;\; de \;\; la \;\; roue = 2\pi 0.235 \quad m = 1.476548m$$

$$\omega_1 = \frac{244620}{1.476548}\, tours/h = 165670.1369 \quad tours/h$$

$$\omega_1 = 2761.1689 \quad tours/\min$$

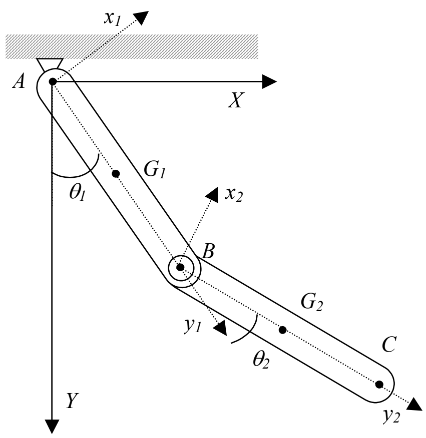
Tous les points de chaque chaîne ont la même vitesse

$$r_3\omega_3 = r_{2a}\omega_2 \quad \text{et} \quad r_{2b}\omega_2 = r_1\omega_1$$

$$\Rightarrow \omega_3 = \frac{r_1}{r_{2b}}\frac{r_{2a}}{r_3}\omega_1 = \frac{3}{10}\frac{3}{12}2761.1689 \quad tours/\min$$

$$\omega_3 = 207.08 \quad tours/\min$$

2.



$$\overline{\omega}_1 = -\dot{\theta}_1 \overline{1}_z \quad \text{et} \quad \overline{\omega}_2 = -\left(\dot{\theta}_1 + \dot{\theta}_2\right)\overline{1}_z$$

Cinématique du solide

$$\begin{cases} \overline{v}_{G_1} = \underbrace{\overline{v}_A}_{=0} + \overline{\omega}_1 \times \overline{AG_1} = \dfrac{L}{2}\dot{\theta}_1 \overline{1}_{x_1} \\ \overline{a}_{G_1} = \underbrace{\overline{a}_A}_{=0} + \overline{\varepsilon}_1 \times \overline{AG_1} + \overline{\omega}_1 \times \left(\overline{\omega}_1 \times \overline{AG_1}\right) = \dfrac{L}{2}\ddot{\theta}_1 \overline{1}_{x_1} - \dfrac{L}{2}\dot{\theta}_1^2 \overline{1}_{y_1} \end{cases}$$

$$\begin{cases} \overline{v}_{G_2} = \overline{v}_B + \overline{\omega}_2 \times \overline{DG_2} = L\dot{\theta}_1 \overline{1}_{x_1} + \dfrac{L}{2}\left(\dot{\theta}_1 + \dot{\theta}_2\right)\overline{1}_{x_2} \\ \quad = L\left(\dot{\theta}_1 + \cos\theta_2 \dfrac{\left(\dot{\theta}_1 + \dot{\theta}_2\right)}{2}\right)\overline{1}_{x_1} - L\left(\sin\theta_2 \dfrac{\left(\dot{\theta}_1 + \dot{\theta}_2\right)}{2}\right)\overline{1}_{y_1} \end{cases}$$

$$\begin{cases} \overline{a}_{G_2} = \overline{a}_B + \overline{\varepsilon}_2 \times \overline{BG_2} + \overline{\omega}_2 \times \left(\overline{\omega}_2 \times \overline{BG_2}\right) \text{ avec } \overline{a}_B = L\ddot{\theta}_1 \overline{1}_{x_1} - L\dot{\theta}_1^2 \overline{1}_{y_1} \\ \overline{a}_{G_2} = L\ddot{\theta}_1 \overline{1}_{x_1} - L\dot{\theta}_1^2 \overline{1}_{y_1} + \dfrac{\ddot{\theta}_1 + \ddot{\theta}_2}{2} L\overline{1}_{x_2} - \dfrac{\left(\dot{\theta}_1 + \dot{\theta}_2\right)^2}{2} L\overline{1}_{y_2} \\ \quad = L\left(\ddot{\theta}_1 - \sin\theta_2 \dfrac{\left(\dot{\theta}_1 + \dot{\theta}_2\right)^2}{2} + \cos\theta_2 \dfrac{\ddot{\theta}_1 + \ddot{\theta}_2}{2}\right)\overline{1}_{x_1} - L\left(\dot{\theta}_1^2 + \cos\theta_2 \dfrac{\left(\dot{\theta}_1 + \dot{\theta}_2\right)^2}{2} + \sin\theta_2 \dfrac{\ddot{\theta}_1 + \ddot{\theta}_2}{2}\right)\overline{1}_{y_1} \end{cases}$$

dérivée dans un repère absolu

$$\begin{cases} \overline{AG_1} = \dfrac{L}{2}\left(\sin\theta_1 \overline{1}_X + \cos\theta_1 \overline{1}_Y\right) \quad \text{et} \quad \overline{v}_{G_1} = \left.\dfrac{d\overline{AG_1}}{dt}\right|_{OXY} = \dfrac{L}{2}\cos\theta_1\dot{\theta}_1 \overline{1}_X - \dfrac{L}{2}\sin\theta_1\dot{\theta}_1 \overline{1}_Y \\ \overline{a}_{G_1} = \left.\dfrac{d\overline{v}_{G_1}}{dt}\right|_{OXY} = \left(-\dfrac{L}{2}\sin\theta_1\dot{\theta}_1^2 + \dfrac{L}{2}\cos\theta_1\ddot{\theta}_1\right)\overline{1}_X - \left(\dfrac{L}{2}\cos\theta_1\dot{\theta}_1^2 + \dfrac{L}{2}\sin\theta_1\ddot{\theta}_1\right)\overline{1}_Y \end{cases}$$

$$\begin{cases} \overline{AG_2} = L\overline{1}_{x_1} + \dfrac{L}{2}\overline{1}_{x_2} = L\left(\cos\theta_1 + \cos\dfrac{\left(\theta_1 + \theta_2\right)}{2}\right)\overline{1}_X + L\left(\sin\theta_1 + \sin\dfrac{\left(\theta_1 + \theta_2\right)}{2}\right)\overline{1}_Y \\ \overline{v}_{G_2} = \left.\dfrac{d\overline{AG_2}}{dt}\right|_{OXY} = L\left(\cos\theta_1\dot{\theta}_1 + \cos\left(\theta_1 + \theta_2\right)\dfrac{\left(\dot{\theta}_1 + \dot{\theta}_2\right)}{2}\right)\overline{1}_X - L\left(\sin\theta_1\dot{\theta}_1 + \sin\left(\theta_1 + \theta_2\right)\dfrac{\left(\dot{\theta}_1 + \dot{\theta}_2\right)}{2}\right)\overline{1}_Y \\ \overline{a}_{G_2} = \left.\dfrac{d\overline{v}_{G_2}}{dt}\right|_{OXY} = L\left(-\sin\theta_1\dot{\theta}_1^2 + \cos\theta_1\ddot{\theta}_1 - \sin\left(\theta_1 + \theta_2\right)\dfrac{\left(\dot{\theta}_1 + \dot{\theta}_2\right)^2}{2} + \cos\left(\theta_1 + \theta_2\right)\dfrac{\left(\ddot{\theta}_1 + \ddot{\theta}_2\right)}{2}\right)\overline{1}_X \\ \qquad - L\left(\cos\theta_1\dot{\theta}_1^2 + \sin\theta_1\ddot{\theta}_1 + \cos\left(\theta_1 + \theta_2\right)\dfrac{\left(\dot{\theta}_1 + \dot{\theta}_2\right)^2}{2} + \sin\left(\theta_1 + \theta_2\right)\dfrac{\left(\ddot{\theta}_1 + \ddot{\theta}_2\right)}{2}\right)\overline{1}_Y \end{cases}$$

Dérivée dans un repère relatif

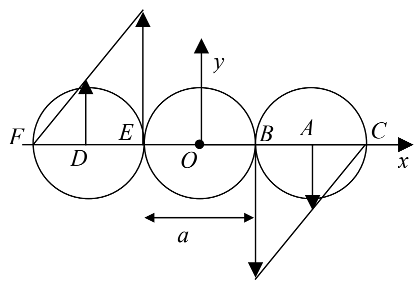
$$\left\{ \overline{AG_1} = \frac{L}{2}\bar{1}_{y_1} \quad \Rightarrow \quad \bar{v}_{G_1} = \underbrace{\left.\frac{d\,\overline{AG_1}}{dt}\right|_{Ox_1y_1}}_{=0} + \bar{\omega}_1 \times \overline{AG_1} = \frac{L}{2}\dot{\theta}_1\bar{1}_{x_1} \quad \text{et} \quad \bar{a}_{G_1} = \left.\frac{d\bar{v}_{G_1}}{dt}\right|_{Ox_1y_1} + \bar{\omega}_1 \times \bar{v}_{G_1} = \frac{L}{2}\ddot{\theta}_1\bar{1}_{x_1} - \frac{L}{2}\dot{\theta}_1^2\bar{1}_{y_1} \right.$$

$$\left\{ \begin{aligned} \overline{AG_2} &= L\bar{1}_{y_1} + \frac{L}{2}\bar{1}_{y_2} = \frac{L}{2}\sin\theta_2\,\bar{1}_{x_1} + \left(L + \frac{L}{2}\cos\theta_2\right)\bar{1}_{y_1} \\ \bar{v}_{G_2} &= \left.\frac{d\,\overline{AG_2}}{dt}\right|_{Ox_1y_1} + \bar{\omega}_1 \times \overline{AG_2} = L\left(\cos\theta_2\frac{\left(\dot{\theta}_1 + \dot{\theta}_2\right)}{2} + \dot{\theta}_1\right)\bar{1}_{x_1} - L\sin\theta_2\frac{\left(\dot{\theta}_2 + \dot{\theta}_1\right)}{2}\bar{1}_{y_1} \\ \bar{a}_{G_2} &= \left.\frac{d\bar{v}_{G_2}}{dt}\right|_{Ox_1y_1} + \bar{\omega}_1 \times \bar{v}_{G_2} \\ &= L\left(\ddot{\theta}_1 - \sin\theta_2\frac{\left(\dot{\theta}_1 + \dot{\theta}_2\right)^2}{2} + \cos\theta_2\frac{\ddot{\theta}_1 + \ddot{\theta}_2}{2}\right)\bar{1}_{x_1} - L\left(\dot{\theta}_1^2 + \cos\theta_2\frac{\left(\dot{\theta}_1 + \dot{\theta}_2\right)^2}{2} + \sin\theta_2\frac{\ddot{\theta}_1 + \ddot{\theta}_2}{2}\right)\bar{1}_{y_1} \end{aligned} \right.$$

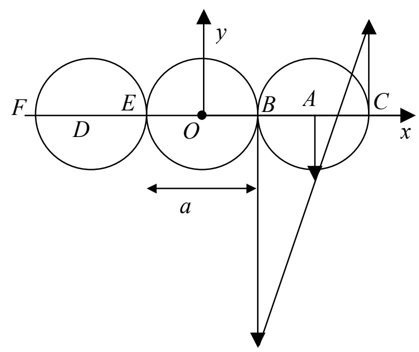
Vitesse d’entraînement et vitesse relative

$$\left\{ \begin{aligned} \bar{v}_{G_2} &= \left.\bar{v}_{G_2}\right|_{rel} + \left.\bar{v}_{G_2}\right|_{entr} = \underbrace{\frac{L}{2}\dot{\theta}_2\bar{1}_{y_2}}_{v_{rel}} + \underbrace{\left(\dot{\theta}_1\bar{1}_z\right) \times \left(\left(L + \frac{L}{2}\cos\theta_2\right)\bar{1}_{x_1} + \frac{L}{2}\sin\theta_2\,\bar{1}_{y_1}\right)}_{v_{entr}} \\ &= L\left(\dot{\theta}_1 + \cos\theta_2\frac{\dot{\theta}_1 + \dot{\theta}_2}{2}\right)\bar{1}_{x_1} - L\left(\sin\theta_2\frac{\dot{\theta}_1 + \dot{\theta}_2}{2}\right)\bar{1}_{y_1} \end{aligned} \right.$$

3.



(a) $\bar{\omega}_S = 0$



(b) $\bar{\omega}_S \neq 0$

(a) Le bras DOA tourne à la vitesse angulaire $\omega = 90$ tours/min

$$\bar{\omega}_{S_{DOA}} = -\frac{90.2.\pi}{60}\text{ rad/s }\bar{1}_z = -3\pi\text{ rad/s }\bar{1}_z$$

$$\left. \begin{aligned} \bar{v}_{A\in S_{DOA}} &= \bar{v}_O + \bar{\omega}_{S_{DOA}} \times \overline{OA} = -\omega_{S_{DOA}}a\,\bar{1}_y \\ \bar{v}_{A\in S_{BAC}} &= \underbrace{\bar{v}_C}_{=0} + \bar{\omega}_{S_{BAC}} \times \overline{CA} = -\omega_{S_{BAC}}\frac{a}{2}\bar{1}_y \end{aligned} \right\} \Rightarrow \omega_{S_{BAC}} = 2\omega_{S_{DOA}}$$

$$\left. \begin{aligned} \bar{v}_{B\in S_{BAC}} &= \underbrace{\bar{v}_C}_{=0} + \bar{\omega}_{S_{BAC}} \times \overline{CB} = -\omega_{S_{BAC}}a\,\bar{1}_y \\ \bar{v}_{B\in S_{BOE}} &= \underbrace{\bar{v}_O}_{=0} + \bar{\omega}_{S_{BOE}} \times \overline{OB} = -\omega_{S_{BOE}}\frac{a}{2}\bar{1}_y \end{aligned} \right\} \Rightarrow 2\omega_{S_{BAC}} = \omega_{S_{BOE}}$$

$$\Rightarrow \left\{ \begin{aligned} \omega_{S_{BOE}} &= 4\omega_{S_{DOA}} \\ \bar{\omega}_{S_{BOE}} &= -12\pi\text{ rad/s }\bar{1}_z = -37{,}7\text{ rad/s }\bar{1}_z \end{aligned} \right.$$

(b) Le bras *DOA* tourne à la vitesse angulaire ω_{DOA} = 90 tours/min et le solide *S* à la vitesse ω_S = 80 tours/min

$$\overline{\omega}_{S_{DOA}} = -3\pi \text{ rad/s } \overline{1}_z \quad \text{et} \quad \overline{\omega}_S = \frac{80.2.\pi}{60} \text{ rad/s } \overline{1}_z = \frac{8\pi}{3} \text{ rad/s } \overline{1}_z \quad \text{et} \quad \overline{\omega}_{S_{BOE}} = -\omega_{S_{BOE}}\overline{1}_z$$

$$\left.\begin{array}{l}\left.\begin{array}{l}\overline{v}_{A\in S_{DOA}} = \overline{v}_O + \overline{\omega}_{S_{DOA}} \times \overline{OA} = -\omega_{S_{DOA}} a\overline{1}_y \\ \overline{v}_{A\in S_{BAC}} = \overline{v}_C + \overline{\omega}_{S_{BAC}} \times \overline{CA} = \left(\omega_S \frac{3a}{2} - \omega_{S_{BAC}}\frac{a}{2}\right)\overline{1}_y\end{array}\right\} \Rightarrow 3\omega_S + 2\omega_{S_{DOA}} = \omega_{S_{BAC}} \\ \left.\begin{array}{l}\overline{v}_{B\in S_{BAC}} = \overline{v}_A + \overline{\omega}_{S_{BAC}} \times \overline{AB} = \left(-\omega_{S_{DOA}} a - \omega_{S_{BAC}}\frac{a}{2}\right)\overline{1}_y \\ \overline{v}_{B\in S_{BOE}} = \underbrace{\overline{v}_O}_{=0} + \overline{\omega}_{S_{BOE}} \times \overline{OB} = -\omega_{S_{BOE}}\frac{a}{2}\overline{1}_y\end{array}\right\} \Rightarrow -2\omega_{S_{DOA}} - \omega_{S_{BAC}} = -\omega_{S_{BOE}}\end{array}\right\} \Rightarrow \begin{cases}\omega_{S_{BOE}} = 4\omega_{S_{DOA}} + 3\omega_S \\ \overline{\omega}_{S_{BOE}} = -600 \text{ t/min } \overline{1}_z \\ \overline{\omega}_{S_{BOE}} = -20\pi \text{ rad/s } \overline{1}_z\end{cases}$$

4.1 $\overline{\omega} = \dot{\varphi}\overline{1}_z + \dot{\theta}\overline{1}_y = -\dot{\theta}\sin\varphi\overline{1}_X + \dot{\theta}\cos\varphi\overline{1}_Y + \dot{\varphi}\overline{1}_Z$

4.2

$$\overline{v}_H = \frac{d\overline{OH}}{dt} = \frac{d\left(L\sin\theta\overline{1}_Z\right)}{dt} = L\cos\theta\dot{\theta}\overline{1}_Z$$

$$\overline{v}_A = \frac{d\overline{OA}}{dt} = \frac{d}{dt}\left(L\cos\theta\overline{1}_x - \frac{L}{2}\overline{1}_y\right) = -L\sin\theta\dot{\theta}\overline{1}_x + \dot{\varphi}\overline{1}_Z \times \overline{OA} = \left(-L\sin\theta\dot{\theta} + \frac{L}{2}\dot{\varphi}\right)\overline{1}_x + L\dot{\varphi}\cos\theta\overline{1}_y$$

$$\overline{v}_B = \frac{d}{dt}\left(L\cos\theta\overline{1}_x + \frac{L}{2}\overline{1}_y\right) = \left(-L\sin\theta\dot{\theta} - \frac{L}{2}\dot{\varphi}\right)\overline{1}_x + L\dot{\varphi}\cos\theta\overline{1}_y$$

$$\overline{v}_E = -L\sin\theta\dot{\theta}\overline{1}_x + L\dot{\varphi}\cos\theta\overline{1}_y$$

4.3 invariant $\overline{v}.\overline{\omega} = L\cos\theta\dot{\theta}\dot{\varphi}$

1. $\dot{\theta} = \dot{\varphi} = 0$ Solide immobile

2. $\dot{\theta} = 0; \dot{\varphi} \neq 0$ Rotation continue uniforme $\overline{\omega} = \dot{\varphi}\overline{1}_Z$

3. $\dot{\theta} \neq 0; \dot{\varphi} = 0$ Rotation instantanée CIR=(X=Lcosθ;Y=Lsinθ)

3. $\dot{\theta} \neq 0; \dot{\varphi} \neq 0$ Mvt hélicoidal instantané $\overline{\omega} = \dot{\varphi}\overline{1}_Z + \dot{\theta}\overline{1}_y$

Axe hélicoidal : droite de point P tq $\overline{v}_P // \overline{\omega}$

$$\overline{v}_P = \left(\frac{\overline{\omega}.\overline{v}_A}{\overline{\omega}.\overline{\omega}}\right).\overline{\omega} = \frac{L\cos\theta\dot{\theta}\dot{\varphi}}{\dot{\theta}^2 + \dot{\varphi}^2}\left(\dot{\theta}\overline{1}_y + \dot{\varphi}\overline{1}_z\right)$$

$$\overline{HP} = \frac{\overline{\omega} \times \overline{v}_A}{\overline{\omega}.\overline{\omega}} + \lambda\overline{\omega} = \frac{L\cos\theta\dot{\theta}^2}{\dot{\theta}^2 + \dot{\varphi}^2}\overline{1}_x + \lambda\left(\dot{\theta}\overline{1}_y + \dot{\varphi}\overline{1}_z\right)$$

5.1 $\overline{\omega} = q\overline{1}_Z + p\overline{1}_x - \dot{\theta}\overline{1}_y = \left(p + q\sin\theta\right)\overline{1}_x - \dot{\theta}\overline{1}_y + q\cos\theta\overline{1}_z$

$$\overline{\alpha} = \left.\frac{d\overline{\omega}}{dt}\right|_{abs} = \left(\dot{p} + \dot{q}\sin\theta + q\cos\theta\dot{\theta}\right)\overline{1}_x - \ddot{\theta}\overline{1}_y + \left(\dot{q}\cos\theta - q\sin\theta\dot{\theta}\right)\overline{1}_z + \overline{\Omega} \times \overline{\omega}$$

$$= \left(\dot{p} + \dot{q}\sin\theta + q\cos\theta\dot{\theta}\right)\overline{1}_x - \left(\ddot{\theta} - pq\cos\theta\right)\overline{1}_y + \left(\dot{q}\cos\theta - q\sin\theta\dot{\theta} + p\dot{\theta}\right)\overline{1}_z$$

avec $\overline{\Omega} = q\sin\theta\overline{1}_x - \dot{\theta}\overline{1}_y + q\cos\theta\overline{1}_z$

$\overline{v}_A = \overline{v}_D + \overline{\omega} \times \overline{DA}$ où $\overline{\omega} \times \overline{DA} = R\dot{\theta}\overline{1}_x + \left(p + q\sin\theta\right)R\overline{1}_y$ où *D* est le centre du disque ($\overline{AD} // \overline{1}_z$)

$$\overline{v}_D = \overline{v}_O + \overline{\Omega} \times \overline{OD} = -bq\overline{1}_{x_0} + aq\cos\theta\overline{1}_y + a\dot{\theta}\overline{1}_z = -bq\cos\theta\overline{1}_x + aq\cos\theta\overline{1}_y + \left(a\dot{\theta} + bq\sin\theta\right)\overline{1}_z$$

$$\overline{v}_A = \left(R\dot{\theta} - bq\cos\theta\right)\overline{1}_x + \left(aq\cos\theta + \left(p + q\sin\theta\right)R\right)\overline{1}_y + \left(a\dot{\theta} + bq\sin\theta\right)\overline{1}_z$$

$$\overline{a}_A = \left.\frac{d\overline{v}_A}{dt}\right|_{rel} + \overline{\Omega}\times\overline{v}_A =$$

$$\left(R\ddot{\theta}+bq\sin\theta\dot{\theta}-b\dot{q}\cos\theta\right)\overline{1}_x+\left(a\dot{q}\cos\theta-aq\sin\theta\dot{\theta}+\left(\dot{p}+q\cos\theta\dot{\theta}+\dot{q}\sin\theta\right)R\right)\overline{1}_y+\left(a\ddot{\theta}+bq\cos\theta\dot{\theta}+b\dot{q}\sin\theta\right)\overline{1}_z$$

$$+\begin{vmatrix} \overline{1}_x & \overline{1}_y & \overline{1}_z \\ q\sin\theta & -\dot{\theta} & q\cos\theta \\ \left(R\dot{\theta}-bq\cos\theta\right) & \left(aq\cos\theta+\left(p+q\sin\theta\right)R\right) & \left(a\dot{\theta}+bq\sin\theta\right) \end{vmatrix}$$

$$=\left(R\ddot{\theta}+bq\sin\theta\dot{\theta}-b\dot{q}\cos\theta-\dot{\theta}\left(a\dot{\theta}+bq\sin\theta\right)-q\cos\theta\left(aq\cos\theta+\left(p+q\sin\theta\right)R\right)\right)\overline{1}_x$$

$$+\left(a\dot{q}\cos\theta-aq\sin\theta\dot{\theta}+\left(\dot{p}+q\cos\theta\dot{\theta}+\dot{q}\sin\theta\right)R+q\cos\theta\left(R\dot{\theta}-bq\cos\theta\right)-q\sin\theta q\sin\theta\right)\overline{1}_y$$

$$+\left(a\ddot{\theta}+bq\cos\theta\dot{\theta}+b\dot{q}\sin\theta+q\sin\theta\left(aq\cos\theta+\left(p+q\sin\theta\right)R\right)+\dot{\theta}\left(R\dot{\theta}-bq\cos\theta\right)\right)\overline{1}_z$$

$$=\left(R\ddot{\theta}-a\dot{\theta}^2-b\dot{q}\cos\theta-aq^2\cos^2\theta-R\cos\theta qp-Rq^2\cos\theta\sin\theta\right)\overline{1}_x$$

$$+\left(a\dot{q}\cos\theta-a\sin\theta\dot{\theta}q+\dot{p}R+R\cos\theta\dot{\theta}q+R\sin\theta\dot{q}+R\cos\theta\dot{\theta}q-bq^2\cos^2\theta-q^2\sin^2\theta\right)\overline{1}_y$$

$$+\left(a\ddot{\theta}+R\dot{\theta}^2+b\dot{q}\sin\theta+aq^2\sin\theta\cos\theta+R\sin\theta qp+Rq^2\sin^2\theta\right)\overline{1}_z$$
