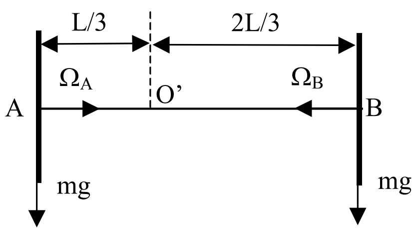
1.



$$\begin{cases}\overline{C}_{gyr} = \Gamma\left(\Omega_A - \Omega_B\right)\dot{\varphi}\ \overline{1}_x \\ \overline{m}_{e,O'} = \left(\dfrac{L}{3}mg - \dfrac{2L}{3}mg\right)\overline{1}_x = -\dfrac{L}{3}mg\,\overline{1}_x\end{cases}$$

$$\Rightarrow\ \Omega_A - \Omega_B = \frac{L}{3\Gamma\dot{\varphi}}mg = \frac{2L}{3R^2\dot{\varphi}}g$$

2.1

Energie cinétique : $T = \sum_{i=1}^{4}\left(\frac{1}{2}mv_{G_i}^2 + \frac{1}{2}\overline{\omega}.\overline{\overline{I}}_{G_i}.\overline{\omega}\right)$

$$T = \left(\frac{1}{2}2m\frac{(2l)^2}{12}\dot{\theta}^2\right)_1 + \left(\frac{m}{2}\left(l\dot{\theta}\right)^2\right)_2 + \left(\frac{1}{2}2m\frac{(2l)^2}{12}\dot{\theta}^2\right)_3 + \left(\frac{2m}{2}\frac{5}{4}l^2\dot{\theta}^2 - \frac{2m}{2}l^2\dot{\theta}^2\sin\frac{\theta}{2} + \frac{1}{2}2m\frac{(2l)^2}{12}\frac{\dot{\theta}^2}{4}\right)_4$$

$$= \left(\frac{5}{2} - \sin\frac{\theta}{2}\right)ml^2\dot{\theta}^2$$

avec
$$\begin{cases}\overline{JG}_4 = \left(l\cos\theta + l\cos\varphi\right)\overline{1}_x + \left(l\sin\theta - l\sin\varphi\right)\overline{1}_y \\ \overline{v}_{G_4} = \overline{v}_D + \left(-\dot{\varphi}\overline{1}_z\right)\times\overline{DG}_4 = l\dot{\theta}\left(-\sin\theta\,\overline{1}_x + \cos\theta\,\overline{1}_y\right) + \dfrac{\dot{\theta}}{2}l\left(\cos\dfrac{\theta}{2}\overline{1}_x + \sin\dfrac{\theta}{2}\overline{1}_y\right) = \dfrac{d\left(\overline{JG}_4\right)}{dt} \\ v_{G_4}^2 = l^2\dot{\theta}^2 + \dfrac{\dot{\theta}^2}{4}l^2 + l^2\dot{\theta}^2\left(-\sin\theta\cos\dfrac{\theta}{2} + \cos\theta\sin\dfrac{\theta}{2}\right) = \dfrac{5}{4}l^2\dot{\theta}^2 + l^2\dot{\theta}^2\sin\left(-\dfrac{\theta}{2}\right)\end{cases}$$

$$Q_\theta = \sum\overline{F}_h\frac{\partial\overline{\varphi}_h}{\partial q_\theta} = M - kl^2\sin\theta + mgl\cos\theta - 2mg\left(l\cos\theta + \frac{l}{2}\sin\frac{\theta}{2}\right) = M - kl^2\sin\theta - mgl\cos\theta - mgl\sin\frac{\theta}{2}$$

avec
$$\begin{cases}y_{G_2} = -l\sin\theta\ \Rightarrow\ \delta y_{G_2} = -l\cos\theta\,\delta\theta \\ y_{G_4} = l\sin\theta - l\sin\varphi = l\sin\theta - l\cos\dfrac{\theta}{2}\ \Rightarrow\ \delta y_{G_4} = \left(l\cos\theta + \dfrac{l}{2}\sin\dfrac{\theta}{2}\right)\delta\theta\end{cases}$$

$$\frac{d}{dt}\frac{\partial T}{\partial\dot{\theta}} - \frac{\partial T}{\partial\theta} = Q_\theta$$

$$\left(5 - 2\sin\frac{\theta}{2}\right)\ddot{\theta} - \frac{1}{2}\cos\frac{\theta}{2}\dot{\theta}^2 = \frac{M}{ml^2} - \frac{k}{m}\sin\theta - \frac{g}{l}\cos\theta - \frac{g}{l}\sin\frac{\theta}{2}$$

2.2.a $\frac{d}{dt}\overline{M}_D = \sum\overline{m}_{e,D} + 2m\overline{v}_G\times\overline{v}_D$ pour la tige DE seule

$$\overline{M}_D = \overline{\overline{I}}_D.\overline{\omega} + 2m\overline{DG}\times\overline{v}_D = \left(\frac{2}{3}ml^2\dot{\theta} - 2ml^2\sin\frac{\theta}{2}\dot{\theta}\right)\overline{1}_z$$

$$\overline{m}_{e,D} = -2mgl\sin\frac{\theta}{2}\overline{1}_z + 2l\sin\frac{\theta}{2}R_K\,\overline{1}_z$$

$$\Rightarrow\ ml^2\ddot{\theta}\left(\frac{2}{3} - 2\sin\frac{\theta}{2}\right) - ml^2\dot{\theta}^2\cos\frac{\theta}{2} = -2mgl\sin\frac{\theta}{2} + 2l\sin\frac{\theta}{2}R_K + ml^2\dot{\theta}^2\cos\frac{\theta}{2}$$

$$\Rightarrow R_K = \frac{ml}{2\sin\frac{\theta}{2}}\left(\left(\frac{2}{3} - 2\sin\frac{\theta}{2}\right)\ddot{\theta} - 2\cos\frac{\theta}{2}\dot{\theta}^2\right) + mg$$

2.2.b $\frac{d}{dt}\overline{M}_B = \sum\overline{m}_{e,D}$ pour la tige AB seule

$$\overline{M}_B = \overline{\overline{I}}_B.\overline{\omega} + 2m\overline{BG}\times\overline{v}_B = \frac{2m(2l)^2}{3}\dot{\theta}\ \overline{1}_z + -2ml^2\dot{\theta}\left(\cos^2\theta + \sin^2\theta\right)\overline{1}_z = \frac{2ml^2}{3}\dot{\theta}\overline{1}_z$$

$$\Rightarrow \frac{2ml^2}{3}\ddot{\theta} = M - 2mgl\cos\theta + Y_H l\cos\theta - X_H l\sin\theta$$

$$\frac{d}{dt}\overline{M}_C = \sum \overline{m}_{e,C} + 3m\overline{v}_{G_{ABC}} \times \overline{v}_C$$ pour la tige ABC (G_{ABC}=centre de masse su système)

$$\overline{M}_C = \left(\overline{M}_B + \overline{CB} \times \overline{R}\right)_{AB} + \left(\overline{\overline{I}}_C.\overline{\omega} + m\overline{CG}_{BC} \times \overline{v}_C\right)_{BC} = \left[\left(\frac{2ml^2}{3}\dot{\theta}\right)_{AB} + \left(m\frac{l^2\dot{\theta}}{2}\cos\theta\right)_{BC}\right]\overline{1}_z = ml^2\dot{\theta}\left(\frac{2}{3}+\frac{1}{2}\cos\theta\right)\overline{1}_z$$

$$ml^2\left(\ddot{\theta}\left(\frac{2}{3}+\frac{1}{2}\cos\theta\right)-\frac{1}{2}\sin\theta\dot{\theta}^2\right) = M + 2mg\left(l - l\cos\theta\right) - Y_H\left(l - l\cos\theta\right) - X_H l\sin\theta + \frac{l}{2}mg$$

$$ml^2\left(\ddot{\theta}\left(\frac{2}{3}+\frac{1}{2}\cos\theta\right)-\frac{1}{2}\sin\theta\dot{\theta}^2\right) = M - 2mgl\cos\theta - X_H l\sin\theta + l\cos\theta Y_H - Y_H l + \frac{5}{2}lmg$$

$$M - 2mgl\cos\theta + Y_H l\cos\theta - \frac{2ml^2}{3}\ddot{\theta} = X_H l\sin\theta$$

$$\Rightarrow Y_H = \frac{5}{2}mg + \frac{1}{2}ml\left(\sin\theta\dot{\theta}^2 - \cos\theta\ddot{\theta}\right)$$

$$\Rightarrow X_H = \frac{1}{l\sin\theta}\left[M - 2mgl\cos\theta + \left(\frac{5}{2}mg + \frac{1}{2}ml\left(\sin\theta\dot{\theta}^2 - \cos\theta\ddot{\theta}\right)\right)l\cos\theta - \frac{2ml^2}{3}\ddot{\theta}\right]$$

3.1 Energie cinétique : $T=\sum_{i=1}^{3}\left(\frac{1}{2}mv_{G_i}^2+\frac{1}{2}\overline{\omega}.\overline{\overline{I}}_{G_i}.\overline{\omega}\right)$

$$T=\left(\frac{m}{2}l^2\dot{\theta}^2+\frac{1}{2}m\frac{(2l)^2}{12}\dot{\theta}^2\right)_1+\left(\frac{m}{2}\left(l^2\dot{\theta}^2+8l^2\cos^2\theta\dot{\theta}^2\right)+\frac{1}{2}m\frac{(2l)^2}{12}\dot{\theta}^2\right)_2+\left(\frac{M}{2}16l^2\cos^2\theta\dot{\theta}^2\right)_3$$

$$=\left(\frac{4}{3}+4\cos^2\theta\right)ml^2\dot{\theta}^2+6M\cos^2\theta l^2\dot{\theta}^2=\left(\frac{4}{3}m+2\left(2m+3M\right)\cos^2\theta\right)l^2\dot{\theta}^2$$

$$\frac{d}{dt}\frac{\partial T}{\partial\dot{\theta}}-\frac{\partial T}{\partial\theta}=Q_\theta=\sum\overline{F}_h\frac{\partial\overline{\varphi}_h}{\partial q_\theta}=-3l\cos\theta mg-l\cos\theta mg-4l\cos\theta Mg+2l\sin\theta F$$

$$4\left(\frac{2}{3}m+\left(2m+3M\right)\cos^2\theta\right)l^2\ddot{\theta}+4\left(\left(2m+3M\right)\cos\theta\sin\theta\right)l^2\dot{\theta}^2=-4l\left(m+M\right)g\cos\theta+2lF\sin\theta$$

$\overline{v}_D=4l\cos\theta\dot{\theta}$ et $V=-\int Q_\theta d\theta=4lmg\sin\theta+4lMg\sin\theta+2l\cos\theta F$

$$T+V=Cte\ \Rightarrow 2\left(\frac{2}{3}m+\left(2m+3M\right)\cos^2\theta\right)l^2\dot{\theta}^2+4l\left(m+M\right)g\sin\theta+2l\cos\theta F=2lF$$

$$v_D{}^2=16l^2\cos^2\theta\dot{\theta}^2=\frac{16\cos^2\theta}{\left(\frac{2}{3}m+2\left(2m+3M\right)\cos^2\theta\right)}\left(\left(1-\cos\theta\right)lF-2l\left(m+M\right)g\sin\theta\right)$$

3.2 $Q_\theta=-4lmg\cos\theta-4l\cos\theta Mg+4l\sin\theta\cos\theta F$

$T+V=Cte$ avec $V=-\int Q_\theta d\theta=4lmg\sin\theta+4lMg\sin\theta+l\cos\left(2\theta\right)F$

$$\frac{7}{3}ml^2\dot{\theta}^2+2\left(m+3M\right)l^2\cos^2\theta\dot{\theta}^2+4l\left(m+M\right)g\sin\theta+l\cos\left(2\theta\right)F=2lF$$

$$v_D{}^2=16l^2\cos^2\theta\dot{\theta}^2=\frac{16\cos^2\theta}{\left(\frac{2}{3}m+2\left(2m+3M\right)\cos^2\theta\right)}\left(\left(1-\frac{1}{2}\cos\left(2\theta\right)\right)lF-2l\left(m+M\right)g\sin\theta\right)$$

3.3 $\frac{v_D{}^2(1)}{v_D{}^2(2)}=\frac{\left(1-\cos\theta\right)lF-2lmg\sin\theta}{\left(1-\frac{1}{2}\cos\left(2\theta\right)\right)lF-2lmg\sin\theta}=\frac{2\left(1-\cos\theta\right)}{2-\cos\left(2\theta\right)}$ si les barres sont de masses négligeables.

3.1 Energie cinétique : $T=\sum_{i=1}^{2}\left(\frac{1}{2}mv_{G_i}^2+\frac{1}{2}\overline{\omega}.\overline{\overline{I}}_{G_i}.\overline{\omega}\right)$

$$T=\left(\frac{m}{2}l^2\dot{\theta}^2+\frac{1}{2}m\frac{l^2}{3}\dot{\theta}^2\right)_1+\left(\frac{m}{2}\left(l^2\dot{\theta}^2+8l^2\cos^2\theta\dot{\theta}^2\right)+\frac{1}{2}m\frac{l^2}{3}\dot{\theta}^2\right)_2$$

$$=4\left(\frac{1}{3}+\cos^2\theta\right)ml^2\dot{\theta}^2$$

$$\frac{d}{dt}\frac{\partial T}{\partial\dot{\theta}}-\frac{\partial T}{\partial\theta}=Q_\theta=\sum\overline{F}_h\frac{\partial\overline{\varphi}_h}{\partial q_\theta}=-3l\cos\theta mg-l\cos\theta mg-4l\cos\theta Mg+2l\sin\theta F$$

$$8\left(\frac{1}{3}+\cos^2\theta\right)ml^2\ddot{\theta}-16\cos\theta\sin\theta ml^2\dot{\theta}+8\cos\theta\sin\theta ml^2\dot{\theta}^2=-4l\left(m+M\right)g\cos\theta+2lF\sin\theta$$

$\overline{v}_D=4l\cos\theta\dot{\theta}$ et $V=-\int Q_\theta d\theta=4lmg\sin\theta+4lMg\sin\theta+2l\cos\theta F$

$$T+V=Cte\ \Rightarrow 4\left(\frac{1}{3}+\cos^2\theta\right)ml^2\dot{\theta}^2+4l\left(m+M\right)g\sin\theta+2l\cos\theta F=2lF$$

$$v_D{}^2=16l^2\cos^2\theta\dot{\theta}^2=\frac{4\cos^2\theta}{\left(\frac{1}{3}+\cos^2\theta\right)}\left(\left(1-\cos\theta\right)2lF-4l\left(m+M\right)g\sin\theta\right)$$

3.2 $Q_\theta = -4lmg\cos\theta - 4l\cos\theta Mg + 4l\sin\theta\cos\theta F$

$T + V = Cte$ avec $V = -\int Q_\theta d\theta = 4lmg\sin\theta + 4lMg\sin\theta + l\cos(2\theta)F$

=> $4\left(\frac{1}{3} + \cos^2\theta\right)ml^2\dot{\theta}^2 + 4l(m+M)g\sin\theta + l\cos(2\theta)F = 2lF$

$$v_D{}^2 = 16l^2\cos^2\theta\dot{\theta}^2 = \frac{4\cos^2\theta}{\left(\frac{1}{3}+\cos^2\theta\right)}\left(\left(1-\frac{1}{2}\cos 2\theta\right)2lF - 4l(m+M)g\sin\theta\right)$$

3.3 $\frac{v_D{}^2(1)}{v_D{}^2(2)} = \frac{(1-\cos\theta)lF - 2lmg\sin\theta}{\left(1-\frac{1}{2}\cos(2\theta)\right)lF - 2lmg\sin\theta} = \frac{2(1-\cos\theta)}{2-\cos(2\theta)}$ si les barres sont de masses négligeables.