MATH-H-405 - Decision engineering

Session 2 & 3: Multi-criteria decision aid

Exercice 1 - Treshold model
Show that the following preference model built with a treshold model:

$$\forall a, b \in A \begin{cases} aPb \Leftrightarrow g(a) > g(b) + q \\ aIb \Leftrightarrow |g(a) - g(b)| \leq q. \end{cases}$$

respects the following properties:

- aPb, bPc, $aId \Rightarrow dPc$
- aPb, bIc, $cPd \Rightarrow aPd$

Exercise 2 - The Datum point algorithm
Considering the following multi-criteria problem:

$$\max X_1 + 2X_2, \max X_1$$

under the following constraints:

$$\begin{aligned} X_1 &\leq 2 \\ X_1 + X_2 &\leq 3 \\ -X_1 + X_2 &\leq 1 \\ X_1, X_2 &\geq 0 \end{aligned}$$

Compute the efficient solutions set. Which are the solutions that are the closest to the Datum point considering the L_1 and L_2 distance.

Exercise 3 - Using the weighted sum
In a multi-criteria problem (where all the criteria are to be maximized), all the solutions of the Pareto frontier can be found by maximizing a weighted sum with well-chosen weights. Demonstrate or give a counterexample.

Exercise 4 - ELECTRE I
A company wants to rank 5 candidates A, B, C, D, E for a promotion (see table below). Five selection criteria have been chosen. Which are the results given by the ELECTRE I method, with a concordance index of 0.6 and a discordance index of 6?

	A	B	C	D	E	Weight
Reputation of the diploma	7	14	15	11	16	15
Skills	12	18	6	8	10	15
Personality	13	13	14	19	10	25
Spoken languages	18	16	19	13	19	25
Seniority	10	20	16	14	20	20

Exercice 5 - Additive model/representability
We consider the additive model: $U(a) = \sum_{j=1}^{n} U_j(g_j(a))$. On the basis of the table below, a decision maker gives you the following preferences:

$$a_9 P a_6 P a_8 P a_5 P a_3 I a_7 P a_2 I a_4 P a_1$$

	a_1	a_2	a_3	a_4	a_5	a_6	a_7	a_8	a_9
g_1	1	1	1	2	2	2	3	3	3
g_2	1	3	5	1	3	5	1	3	5

1. Verify the hypothesis of preferential independence
2. Compute the values of the marginal utilities so that we can represent the global preferences of the decision maker.

Reference: *"Multicriteria decision-aid"*, P. Vincke

Exercise 6 - Properties of the net flow of PROMETHEE
Demonstrate that $\sum_{i=1}^{N} \phi(a_i) = 0$.

Exercise 7 - Parallel between the voting theory and the multi-criteria ranking problems
Which Arrow's conditions are respected by the PROMETHEE II method?

Exercise 8 - Rank reversal
Demonstrate that a rank reversal between two actions a and b in the PROMETHEE II ranking can not occur when removing a third alternative action if:

$$|\phi(a) - \phi(b)| > \frac{2}{n-1}.$$

Exercise 9 - Preferential independence
As an employer, you want to hire workers on the basis of their age, their degree and their professional experience. Give a situation where the preferential independence hypothesis is not respected.

Exercise 10 - Consistent family of criteria - cohesion
For a family of criteria, give an example where the cohesion hypothesis is not respected.

Exercise 11 - Consistent family of criteria - exhaustivity
For a family of criteria, give an example where the exhaustivity hypothesis is not respected.

Exercise 12 - Dominance in PROMETHEE
Demonstrate: $aDb \Rightarrow \phi(a) > \phi(b)$

Exercise 13 - PROMETHEE II properties
Demonstrate that the deletion of a non discriminating criterion does not change the PROMETHEE II ranking.

Exercise 14 - Management of financial portfolios
Let us consider N financial assets and note X_i the proportion invested in the asset i with a mean return μ_i and a variance σ_i^2. Let σ_{ij} be the correlation between the asset i and the asset j. Let us define the vector of mean returns μ and the variance-covariance matrix Σ as follows:

$$\mu = \begin{pmatrix} \mu_1 \\ \mu_2 \\ \vdots \\ \mu_N \end{pmatrix} \text{ and } \Sigma = \begin{pmatrix} \sigma_1^2 & \sigma_{12} & \dots & \sigma_{1N} \\ \sigma_{21} & \sigma_2^2 & \dots & \sigma_{2N} \\ \vdots & \vdots & \dots & \vdots \\ \sigma_{N1} & \sigma_{N2} & \dots & \sigma_N^2 \end{pmatrix}.$$

A portfolio P is represented by the vector $X(X_1, X_2, \ldots, X_N)$ with $\sum_{i=1}^{N} X_i = 1$. Its mean return μ_p and its variance σ_P^2 are computed as follows:

$$\begin{aligned} \mu_p &= \sum_{i=1}^{N} X_i \mu_i = X\mu \\ \sigma_P^2 &= \sum_{i=1}^{N} \sum_{j=1, i \neq j}^{N} X_i X_j \sigma_{ij} + \sum_{i=1}^{N} X i^2 \sigma_i^2 = X \Sigma X'. \end{aligned}$$

Establish the expression of the efficient frontier.