10/12/14

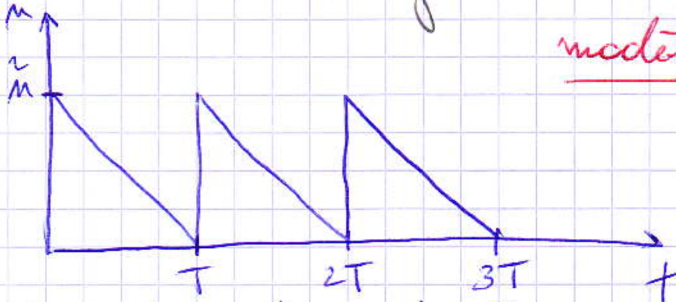
Inventory control

How much do you order? At what time?

modèle de Wilson

(1)

We know the demand in a deterministic way!



- demand : d [unit / time unit]
- replenishment order of n units
- setup cost : c_e
  - ↳ cost per replenishment order
- storage cost : c_s
  - ↳ cost per unit & per time unit

Trying to solve it:

~~cost $\Gamma(n) = c_e + c_s\, C_s\, n$~~

~~$C_s = \int c_s n \, dn = \int_0^T c_s (n - dt)\, dt$~~

"moyenne du" ↳ coût par unité de temps à diminuer

$$\Gamma(n) = \frac{c_e}{T} + c_s \cdot \frac{n}{2} = \frac{c_e}{n} d + c_s \cdot \frac{n}{2}$$

$T = \frac{n}{d}$

$\Rightarrow \frac{c_e}{\ell} d + c_s \cdot n^2$

$/2$ car n est le nbre de pièces à commander, pas le nbre de p. à un certain temps t ! On veut le coût par unité de temps. On prend la moyenne du nbre de pièces en stock pour avoir le coût.

Exam : dev. new model based on the ones viewed at the course.

demand : $d.T = n \Rightarrow T = \frac{n}{d}$

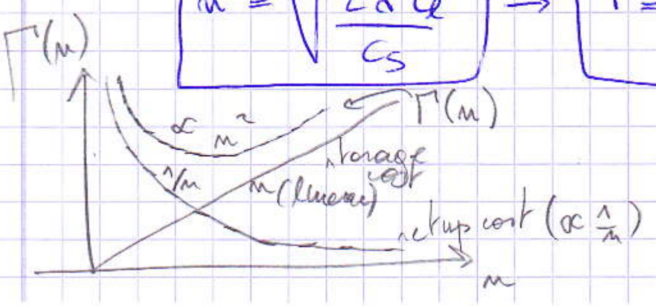
setup cost : $\frac{c_e}{T} = c_e \cdot \frac{d}{n}$

storage cost : $c_s \frac{n}{2}$

total cost per t.u. : $\Gamma(n) = c_e \frac{d}{n} + c_s \frac{n}{2}$

$$\frac{d\Gamma(n)}{dn} = -c_e \frac{d}{n^2} + \frac{c_s}{2} = 0$$

$$\boxed{\hat{n} = \sqrt{\frac{2 d c_e}{c_s}}} \rightarrow \boxed{\hat{T} = \frac{\hat{n}}{d} = \sqrt{\frac{2 c_e}{d c_s}}}$$



We ~~add~~ borrow money from the bank.
We add money in the model.
Money stock.

(2)
- demand : d unit / t.u
- replenishment order of n units
- setup cost : c_ℓ
  - ↳ cost per ...
- storage cost : $c_s = \alpha \left(c_a + \frac{c_\ell}{n} \right)$
  - ↳ cost per unit & per t.u.
- α : interest rate per t.u.
- c_a : purchase cost

$\Rightarrow$ demand : $dT = n \Rightarrow T = \frac{n}{d}$

setup cost : $\frac{c_\ell}{T} = c_\ell \frac{d}{n}$

storage cost : $c_s \frac{n}{2} \Rightarrow \alpha \left(c_a + \frac{c_\ell}{n} \right) \frac{n}{2} = \alpha c_a \frac{n}{2} + \alpha \frac{c_\ell}{2}$

total cost per t.u. : $\Gamma(n) = c_\ell \frac{d}{n} + \alpha c_a \frac{n}{2} + \alpha \frac{c_\ell}{2}$

$$\frac{d\Gamma(n)}{dn} = - c_\ell \frac{d}{n^2} + \alpha \frac{c_a}{2} = 0 \Rightarrow \boxed{\hat{n} = \sqrt{\frac{2 c_\ell d}{\alpha c_a}}} \quad \boxed{\hat{T} = \frac{\hat{n}}{d}}$$

(3) Take into account the fact that when we purchase a large amount of units, the cost decreases.
Degressive cost

$$c_a = \begin{cases} c_1 & \text{if } 0 \leq n < a \\ c_2 & \text{if } n \geq a \end{cases} \quad \text{"tarif dégressif"}$$

$$\Rightarrow c_s = \begin{cases} \alpha \left(c_1 + \frac{c_\ell}{n} \right) & 0 \leq n < a \\ \alpha \left(c_2 + \frac{c_\ell}{n} \right) & a \leq n \end{cases}$$

$\Gamma(n) = c_\ell \frac{d}{n} + c_s \frac{n}{2}$

$\Gamma_1(n) = c_\ell \frac{d}{n} + \alpha \left(c_1 + \frac{c_\ell}{n} \right) \frac{n}{2}$

$\Rightarrow \hat{n}_1 = \sqrt{\frac{2 c_\ell d}{2 c_1}} \quad \hat{n}_1 < \hat{n}_2$

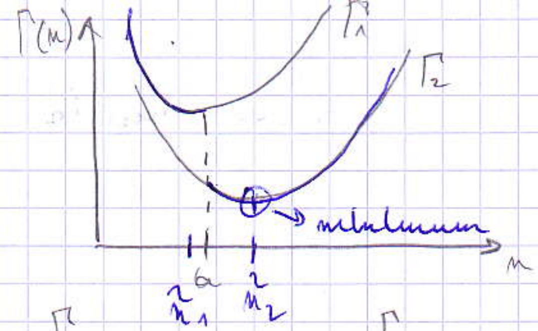
$\Gamma_2(n) = c_\ell \frac{d}{n} + \alpha \left(c_2 + \frac{c_\ell}{n} \right) \frac{n}{2}$

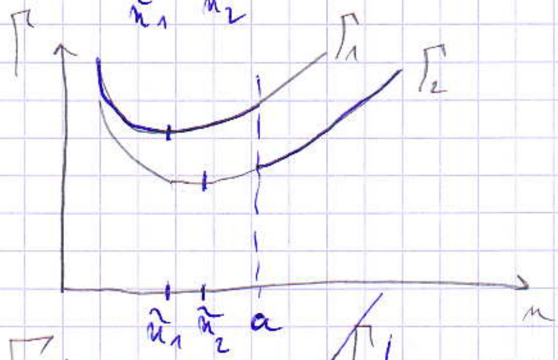
$\Rightarrow \hat{n}_2 = \sqrt{\frac{2 c_\ell d}{\alpha c_2}}$

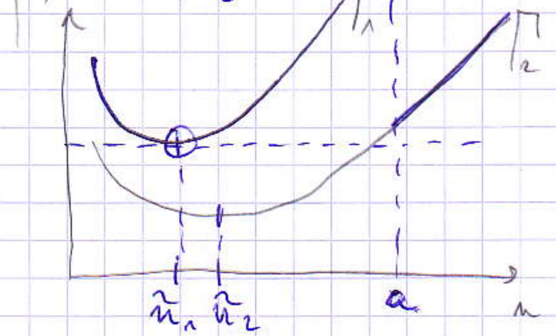
3 cases:

- $a < \hat{n}_2 \Rightarrow \hat{n} = \hat{n}_2$
- $\begin{cases} a > \hat{n}_2 \\ \Gamma_2(a) < \Gamma_1(\hat{n}_1) \end{cases} \Rightarrow \hat{n} = a$

  minimize cost fct $\Gamma(n)$
- $\begin{cases} a > \hat{n}_2 \\ \Gamma_2(a) > \Gamma_1(\hat{n}_1) \end{cases} \Rightarrow \hat{n} = \hat{n}_1$







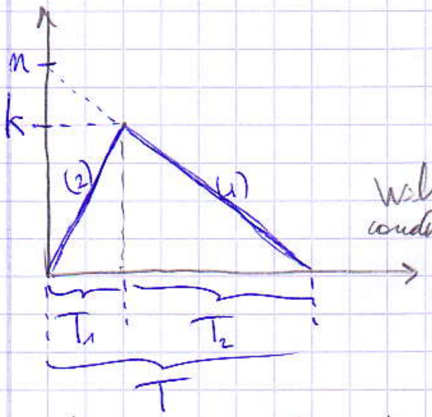
(1) Before "a", we follow the curve Γ_1. When we reach a, we switch to Γ_2.

(2) $\hat{n}_2$ is the optimum from Γ_2, but is not reached (we are still on Γ_1).

$\hat{n}_1$ is the optimum for Γ_1, but $\Gamma_2(a) < \Gamma_1(\hat{n}_1)$, we thus choose "a" over $\hat{n}_1$.

(3) Although we switch to Γ_2 when we reach "a", $\Gamma_2(a)$ is the minimum of the "blue" part of Γ_2, which is already larger than the minimum of Γ_1, which is $\hat{n}_1$.

(4)



When conditions H

- Replenishment rate: v units p.t.u.
- Demand: d units per t.u.
- Setup cost per t.u.: $c_e \frac{d}{n}$
- Storage cost per t.u.: $c_s \frac{k}{2} = c_s\left(\frac{v-d}{v} n\right)\frac{1}{2}$

We do not actually order k. We order n, which is a virtual point.

$(1): y = \cancel{k}\, n - dt$

$(2): y = (v-d)t$

$\Leftrightarrow \begin{cases} y = \frac{v-d}{v} \cdot n \\ t = \frac{n}{v} \end{cases}$

$\hookrightarrow (1) \cap (2) \rightarrow y = k$

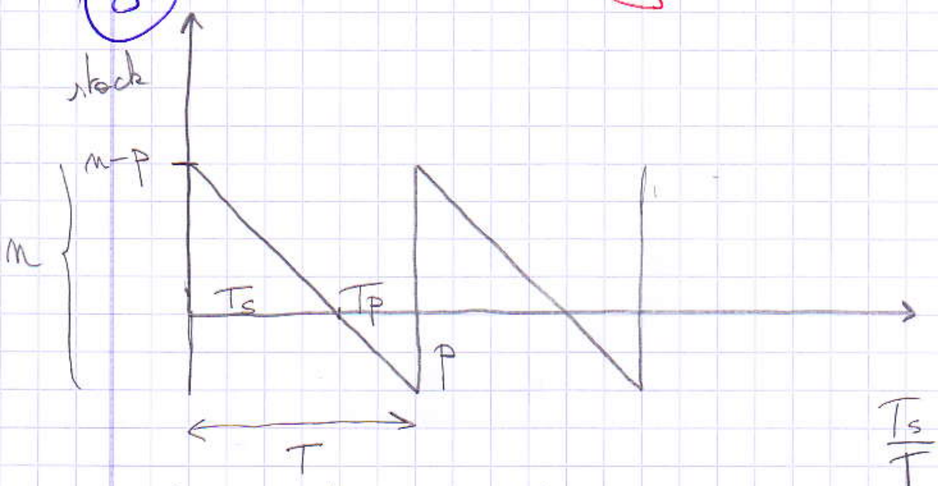
• Total cost per t.u. : $\Gamma(n) = c_\ell \frac{d}{n} + c_s \left(\frac{v-d}{v} n \right) \frac{1}{2}$

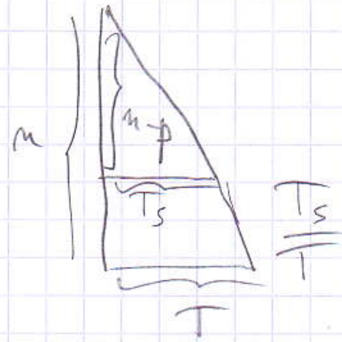
$$\Rightarrow \frac{\partial \Gamma(n)}{\partial n} = -c_\ell \frac{d}{n^2} + \frac{c_s}{2} \left(\frac{v-d}{v} \right) = 0$$

$$\Rightarrow \hat{n} = \sqrt{\frac{2 d c_\ell v}{c_s (v-d)}}$$

$$\Rightarrow \text{if } v \to \infty : \hat{n} = \sqrt{\frac{2 d c_\ell}{c_s}}$$

5 Wilson + shortage





$$\frac{T_s}{T} = \frac{n-p}{n}$$

setup cost : $c_\ell \frac{d}{n}$

storage cost : $c_s \cdot \frac{n-p}{2} \frac{n-p}{n} = c_s \frac{(n-p)^2}{2n}$

shortage cost : $c_p \frac{p}{2} \frac{p}{n} = c_p \frac{p^2}{2n}$

Thalès $= \frac{T_p}{T}$

$$\Gamma(n,p) = c_p \frac{d}{n} + c_s \frac{(n-p)^2}{2n} + c_p \frac{p^2}{2n}$$

$c_p \to \infty$ $\Rightarrow$ same results as 1st model (Wilson)

$$\frac{\partial \Gamma(n,p)}{\partial n} = 0 \Rightarrow \hat{n} = \sqrt{\frac{2 d c_\ell}{c_s}} \sqrt{\frac{c_s + c_p}{c_p}} \Rightarrow \hat{T}_s = \frac{\hat{n} - \hat{p}}{d} \quad (*)$$

$$\frac{\partial \Gamma(n,p)}{\partial p} = 0 \Rightarrow \hat{p} = \sqrt{\frac{2 d c_\ell c_s}{c_s c_p + c_p^2}} = \frac{c_s}{c_s + c_p} \hat{n} \Rightarrow \hat{T}_p = \frac{\hat{p}}{d}$$

If the shortage cost is ∞, we're back on the basic Wilson model :

(*) $\frac{T_s}{T} = \frac{n-p}{n}$, or $T = \frac{n}{d} \Rightarrow T_s = \frac{n-p}{d} \Rightarrow \hat{T}_s = \frac{\hat{n} - \hat{p}}{d}$

"shortage cost & loss on unsold goods"

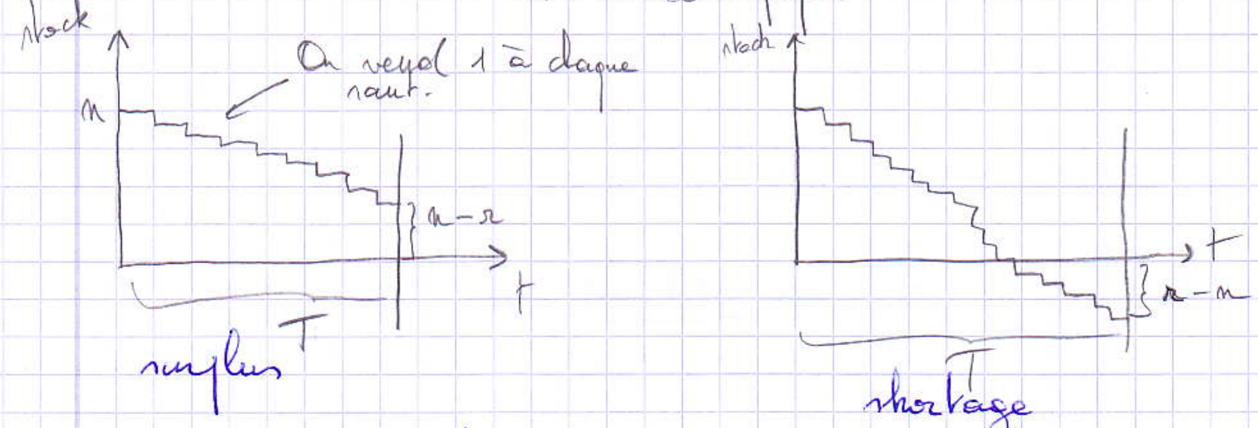
Don't know the demand (only have stats).

(1) Cost for shortage & loose money if unsold products.

ex: Newspaper (don't know the demand for each day) random demand (discrete)

$$\xi : \{r, p_r\} \quad , p_r = \mathbb{P}[\xi = r] , \sum_{r=0}^{\infty} p_r = 1$$

↳ P to sell r papers



c_1 : cost (per unit) of unsold goods

c_2 : cost (p.u.) of shortage "

$$\gamma(n, r) = \begin{cases} c_1 (n - r) & 0 \leq r \leq n \\ c_2 (r - n) & r \geq n \end{cases}$$

$$\Gamma(n) = E[\gamma(n, r)] = \sum_{r=0}^{\infty} \gamma(n, r) \cdot p_r$$

$$= \sum_{r=0}^{n} c_1 (n - r) \cdot p_r + \sum_{r=n+1}^{\infty} c_2 (r - n) \cdot p_r$$

↳ $r = n \Rightarrow c_2 = 0$

↳ = to $\sum_{r=n}^{\infty} \dots$

Pas besoin de changer les indices car il y a des valeurs nulles de toutes façons! Sert à rien.

$$\Gamma(n+1) = \sum_{r=0}^{n} c_1 (n + 1 - r) \cdot p_r + \sum_{r=n+1}^{\infty} c_2 (r - n - 1) \cdot p_r$$

$\Delta \Gamma(n) < 0$ cost ↘

> 0 " ↗

$$\Delta \Gamma(n) = \Gamma(n+1) - \Gamma(n) = c_1 \sum_{r=0}^{n} p_r - c_2 \sum_{r=n+1}^{\infty} p_r$$

$$= (c_1 + c_2) \sum_{r=0}^{n} p_r - c_2$$

$$= (c_1 + c_2) \mathbb{P}[\xi \leq n] - c_2$$

↳ P cumulative

$$\sum_{r=0}^{n} p_r + \sum_{r=n+1}^{\infty} p_r = 1 \iff \sum_{r=n+1}^{\infty} p_r = 1 - \sum_{r=0}^{n} p_r$$

$\Rightarrow$ n optimal if $\Delta\Gamma(n-1) < 0$ & $\Delta\Gamma(n) > 0$

$$\begin{cases} (c_1+c_2)\,\mathbb{P}[\xi \leq n-1] - c_2 < 0 \\ (c_1+c_2)\,\mathbb{P}[\xi \leq n] - c_2 > 0 \end{cases}$$

$$\Rightarrow \mathbb{P}[\xi \leq n-1] < \frac{c_2}{c_1+c_2} < \mathbb{P}[\xi \leq n]$$

ex. :

n	p_r	$\mathbb{P}[\xi \leq n]$
0	0,90	0,90
1	0,05	0,95
2	0,02	0,97
3	0,01	0,98
4	0,01	0,99
5	0,01	1,00

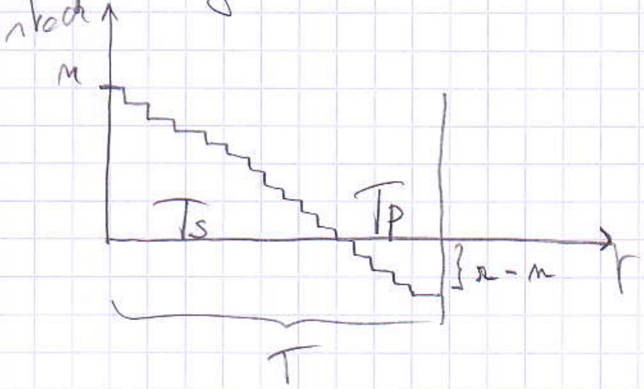
$c_1 = 5000$
$c_2 = 100\,000$

$\frac{c_2}{c_1+c_2} = 0,952$

$\boxed{\hat{n} = 2}$

(2) shortage cost & storage cost

Approximation of the demand by constant linear rate.



$$\frac{T_s}{T} = \frac{n}{r}$$

$$\frac{T_p}{T} = \frac{r-n}{r}$$

c_s : cost (per unit) of ~~shortage~~ storage

c_p : cost p.u. of shortage

$0 \leq r \leq n$

$r > n$

$r > n$

$$\gamma(n, r) = \begin{cases} \text{average storage cost (surplus)} : c_s\left(n - \frac{r}{2}\right) \\ \text{" " " (shortage)} : c_s\left(\frac{n}{2} \cdot \frac{n}{r}\right) = c_s \frac{n^2}{2r} \\ \text{" shortage "} : c_p\left(\frac{r-n}{2} \cdot \frac{r-n}{r}\right) = c_p \frac{(r-n)^2}{2r} \end{cases}$$

$$\Gamma(n) = E\left[\gamma(n, r)\right] = c_s \sum_{r=0}^{n} \left(n - \frac{r}{2}\right) p_r + c_s \sum_{r=n+1}^{\infty} \frac{n^2}{2r} p_r + c_p \sum_{r=n+1}^{\infty} \frac{(r-n)^2}{2r} p_r$$

$$\Gamma(n+1) = c_s \sum_{r=0}^{n+1} \left(n+1 - \frac{r}{2}\right) p_r + c_s \sum_{r=n+2}^{\infty} \frac{(n+1)^2}{2r} p_r + c_p \sum_{r=n+2}^{\infty} \frac{(r-n-1)^2}{2r} p_r$$

$$\Delta\Gamma(n) = \Gamma(n+1) - \Gamma(n) = (c_s + c_p) \underbrace{\left(\mathbb{P}[\xi \leq n] + \left(n + \frac{1}{2}\right) \sum_{r=n+1}^{\infty} \frac{p_r}{r}\right)}_{L(n)} - c_p$$

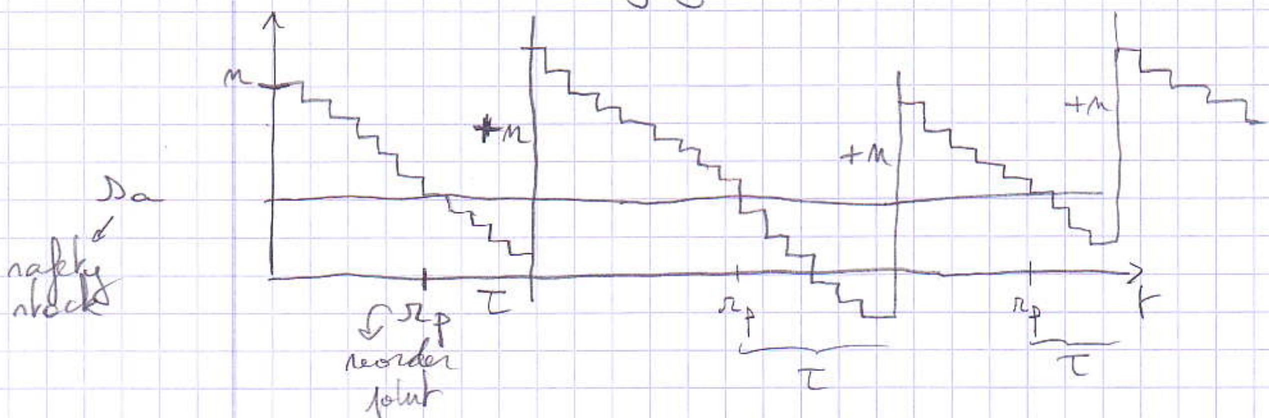
$$\Delta\Gamma(n-1) < 0 : (c_s + c_p) L(n-1) - c_p < 0$$
$$\Delta\Gamma(n) > 0 : (c_s + c_p) L(n) - c_p > 0$$

$$L(n-1) < \frac{c_p}{c_s + c_p} < L(n)$$

3 "Heuristique à 2 magasins":

Heuristic for continuous review (two bins) with delivery delay → On fait commande qui est livrée avec délai.

↳ 2 stocks : On vend de la salade au comptoir (stock 1). Quand on n'a plus de salade au c., on va chercher dans le frigo (stock 2).



Not always possible to get an exact solution

→ approx.

c_s : cost (p.u.) of storage per t.u.

c_p : cost (per u.) of shortage per t.u.

c_e : setup cost (per order, n units)

opt. n, s_a

Approx. ⇓ heuristic

1) Average demand is constant
2) stock will evolve linearly
3) first compute n, then we consider a value of s_a for n fixed.

$\overline{D}$: demand

θ : period (long)
pas forcément T
$\theta \geq T$

$d = \frac{\overline{D}}{\theta} = cte$

deterministic n°5 (see yesterday) $\Rightarrow \hat{n} = \sqrt{\frac{2 c_e d}{c_s}} \sqrt{\frac{c_s + c_p}{c_p}}$

$s_a \Rightarrow L(s_a - 1) < \frac{c_p}{c_s + c_p} < L(s_a)$