MATH-H-405 - Decision engineering

Session 4: Game theory

Exercice 1
Can the following games be solved by applying the dominance principle? If so, what are the equilibrium strategies?

1.

	C_1	C_2	C_3
R_1	$(1;9)$	$(2;9)$	$(0;9)$
R_2	$(3;1)$	$(5;2)$	$(2;1)$

2.

	C_1	C_2	C_3
R_1	$(1;3)$	$(8;2)$	$(1;4)$
R_2	$(3;2)$	$(3;1)$	$(2;7)$
R_3	$(1;5)$	$(9;2)$	$(1;4)$

3.

	C_1	C_2	C_3
R_1	$(1;3)$	$(2;4)$	$(1;0)$
R_2	$(3;3)$	$(5;2)$	$(0;1)$
R_3	$(2;5)$	$(2;0)$	$(1;8)$

4.

	C_1	C_2	C_3	C_4
R_1	$(1;1)$	$(1;1)$	$(1;4)$	$(2;2)$
R_2	$(3;0)$	$(5;2)$	$(2;3)$	$(3;3)$
R_3	$(1;1)$	$(2;2)$	$(1;4)$	$(8;2)$
R_4	$(0;6)$	$(2;4)$	$(1;7)$	$(2;8)$

Exercise 2
Find all Nash equilibria in the following games:

1.

	C_1	C_2
R_1	$(3;5)$	$(0;4)$
R_2	$(1;0)$	$(20;30)$

2.

	C_1	C_2
R_1	$(0;8)$	$(2;2)$
R_2	$(8;0)$	$(3;3)$

3.

	C_1	C_2	C_3
R_1	$(0;2)$	$(2;2)$	$(2;3)$
R_2	$(4;4)$	$(3;3)$	$(1;2)$
R_3	$(3;2)$	$(1;3)$	$(0;2)$

Exercise 3
Give an example of a matrix game having at least two Nash equilibria, one of which can be obtained by the method of elimination of dominated strategies and the other cannot be obtained by the method of elimination of dominated strategies.

Exercise 4
Verify that if a matrix game can be solved by using iterated elimination of strictly dominated strategies, the game has a unique Nash equilibrium that is precisely the strategy pair found trough eliminating strictly dominated strategies.

Exercise 5

1. Let G be a strategic form game whose strategy sets are open intervals and with twice differentiable payoff functions. Find a sufficient condition so that a strategy $(s_1^*, s_2^*, \ldots, s_n^*)$ is a Nash equilibrium.

2. Consider a two-person strategic game such that $S_1 = S_2 = \mathbb{R}$. The utility functions of the two players are : $u_1(x, y) = xy^2 - x^2$ and $u_2(x, y) = 8y - xy^2$. Find the Nash equilibrium of the game.

Exercise 6 - Second Price Auction
A seller has an expensive painting to sell at an auction that is valued for a certain price by n potential buyers. Each buyer k has their own valuation $v_k > 0$ of the painting. The buyers must simultaneously bid an amount; we denote the bid of the buyer i by $b_i \in (0, \infty)$. In a second price auction the highest bidder gets the painting and pays the second highest bid. If there is more than one buyer with the highest bid, the winner is decided by drawing among the highest bidders and she pays the highest bid. The remaining ones receive a payoff of zero.
We can formulate this auction as a strategic form game:

- n players (the n buyers)
- The strategy set of each player is $(0, \infty)$

1. Give the payoff of a player k.

2. Demonstrate that the strategy profile $(v_1, v_2, \ldots, v_n)$ is a Nash equilibrium for this game.

Exercise 7 - The Bertrand model
Consider a market with two firms that produce identical products. Let $q = A - p$ be the total quantity sold when the price is p. If both firms charge the same price, then each sells one-half of the total. If the firms charge different prices, then the firm with the lower price sells everything. The marginal cost of the firm i is c_i. The positive parameters c_1 and c_2 satisfy $c_1 \neq c_2$.

1. What is the strategic form game with the price p_i being the strategy of the firm i?

2. Show that the game does not have a Nash equilibrium.

References:

"Games and Decision Making", C.D. Aliprantis and S.K. Chakrabarti
"An Introduction to Decision Theory", M Peterson