MATH-H-405 – Decision Engineering Prof. Yves DE SMET Academic year 2012-2013 – 1st session

Last name	
First name	
Faculty & Section	

Q1 (/10)	Q2 $(/10)$	Q3 $(/10)$	Q4 (/10)	Total (/40)	Total $(/20)$

Please give precise and concise answers and use the notations seen in the course.

1. Voting theory

Arrow's theorem is based on 5 specific conditions. If we consider BORDA's rule, which of them are respected and which ones are not ? Justify each of your answers by providing a formal proof or a counter-example.

2. Multicriteria decision aid

Let $A = \{a_1, a_2, ..., a_n\}$ be a set of alternatives and $F = \{f_1, f_2, ..., f_q\}$ be a set of criteria (w.l.g. criteria are assumed to be maximized). We assume that the Decision Maker has provided a set of weights w_j characterizing the relative importance of the different criteria. Of course, we have $w_j \ge 0$ and $\sum_{j=1}^{q} w_j = 1$. We consider two ranking methods:

- the WEIGHTED SUM;
- PROMETHEE II.

The same weights are assumed to be used in both methods. For PROMETHEE II, we assume that: the indifference threshold is set to zero and the preference threshold of criterion j is

$$p_j = \max_{k=1,..,n} f_j(a_k) - \min_{l=1,..,n} f_j(a_l)$$

With these settings, show that both rankings are the same.

3. Game theory

We consider the extension of the *Median voter theorem* to three candidates. Without loss of generality, we will assume that $s_1 \leq s_2 \leq s_3$ where s_i is the position of candidate *i*. We ask you to:

- (a) Formalize the utilities of the three candidates;
- (b) Show that no Nash equilibrium can be found in this case.

4. Decision under uncertainty (inspired by [1])

The Athletic Department of Leland University is considering whether to hold an extensive campaign next year to raise funds for a new athletic field. The response to the campaign depends heavily upon the success of the football team this fall. In the past, the football team has had winning seasons 60 percent of the time. If the football team has a winning season (W) this fall, then many of the alumnae and alumni will contribute and the campaign will raise \$3 million. If the team has a losing season (L), few will contribute and the campaign will lose \$2 million. If no campaign is undertaken, no costs are incurred. On September 1, just before the football season begins, the Athletic Department needs to make its decision about whether to hold the campaign next year.

- (a) Should the campaign be undertaken? Justify.
- (b) What is the Expected Value of Perfect Information?
- (c) A famous football guru, William Walsh, has offered his services to help evaluate whether the team will have a winning season. For \$100,000, he will carefully evaluate the team throughout spring practice and then throughout preseason workouts. William then will provide his prediction on September 1 regarding what kind of season, W or L, the team will have. In similar situations in the past, his predictions for a winning season have been correct 75 percent of the time. For a losing season, his predictions have been correct 80 percent of the time.

Should the campaign be undertaken? Justify.

^[1] F.S. Hillier and G.J. Lieberman, Introduction to Operations Research (McGraw-Hill, 2005)