Math-H-405 - Decision engineering

Session 5: Decision making under risk and uncertainty

Exercise 1-Problem Statement
T. Bone Pucket, a corporate raider, has acquired a textile company and is contemplating the future of one of its major plants, located in South Carolina. Three alternatives decisions are being considered:

1. Expand the plant and produce lightweight, durable materials for possible sales to military, a market with little foreign competition;
2. Maintain the status quo at the plant, continuing production of textile goods that are subject to heavy foreign competition; or
3. Sell the plant now.

If one of the first two alternatives is chosen, the plant will still be sold at the end of a year. The amount of profit that could be earned by selling the plant in a year depends on foreign market conditions, including the status of trade embargo bull in Congress. The following payoff table describes this decision situation:

Decision	**Good Foreign Competitive Conditions**	**Poor Foreign Competitive Conditions**
Expand	\$800,000	\$500,000
Maintain status quo	\$1,300,000	\$-150,000
Sell now	\$320,000	\$320,000

1. Determine the best decision by using the following decision criteria:
   (a) Maximax
   (b) Maximin
   (c) Minimax regret
   (d) Hurwicz ($\alpha = 0.7$)
   (e) Equal likelihood
2. Assume that it is possible to estimate a probability of 0.70 that good foreign competitive conditions will exist and probability of .30 that poor conditions will exist. Determine the best decision by using expected value and expected opportunity loss.
3. Compute the expected value of perfect information
4. Develop a decision tree, with expected values at the probability nodes.

5. T. Bone Puckett has hired a consulting firm to provide a report future political and market situations. The report will be positive (P) or negative (N), indicating either a good (g) or poor (p) future foreign competitive situation. The conditional probability of each report outcome, given each state of nature, is :

$$P(P|g) = 0.7$$
$$P(N|g) = 0.3$$
$$P(P|p) = 0.2$$
$$P(N|p) = 0.8$$

   Determine the posterior probabilities by using Bayes's rule.

6. Perform a decision tree analysis by using the posterior probability obtained in 5.

Reference: *"Introduction to Management Science"*, B.W. Taylor III, P. Hall

Exercise 2
We want to optimize the process of the urgency service of a hospital. Currently, the patients are received by a doctor that examines them and then sends them to the most-suited service for their pathology. To simplify the problem, we will consider two types of patients:

1. The patients suffering from mild pathologies. They represent 70% of the patients. In this case, the diagnosis lasts 6 minutes ;

2. The patients suffering from severe pathologies. In this case, the diagnosis lasts 14 minutes.

a. What is the mean time for a diagnosis?

b. In order to optimize the mean time of a diagnosis, we consider the following system: the arriving patient has to answer 5 standard questions by yes or no. If the patient suffers from a severe pathology, we are sure that at least one question will be answered by yes. However, due to anxiety, 20% of the patients that only suffer from a mild pathology will also answer at least one question by yes. Answering the questionnaire takes 2 minutes. We consider that the given answers will allow the doctor to reduce the diagnosis time by 50% in the case of mild pathology. What is the mean admission time?

c. The patients who answered by at least one yes receive another questionnaire with 5 standard questions (more specific). The answer are still limited to yes or no and this test takes 2 minutes. Those new answers will allow the doctor to reduce the diagnosis of severe pathology to 6 minutes (again, a patient suffering from a severe pathology will answer by yes at least once). On the other hand, only 10% of the patients suffering from a mild pathology will give one yes at this second questionnaire. What is the mean admission time?

Exercise 3
Suppose that you prefer gamble 1 to gamble 2, and gamble 4 to gamble 3. Show that your preference are incompatible with the principle of maximizing the expected utility, no matter what your utility of money is.

	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$
Gamble 1	\$50	\$50	\$50
Gamble 2	\$100	\$50	\$0
Gamble 3	\$50	\$0	\$50
Gamble 4	\$100	\$0	\$0

Exercise 4
Construct a decision matrix that corresponds to the following regret table.

a_1	5	45	0
a_2	0	30	20
a_3	10	0	25

Exercise 5
According to the minimax regret principle, regret is defined as the difference between the best outcome obtainable under a given state and the outcome obtained under that state with the chosen act. Now consider an alternative version of this principle, according to which regret is defined (relative to each act) as the difference between the best outcome and the worst outcome obtainable with the act in question. This modification of the minimax regret rule would give us a highly implausible decision rule. Why?

Reference: *"An Introduction to DECISION Theory"*, M. Peterson

Exercise 6 (by Pr. D. Bouyssou)
You are an online book reseller who uses a systematic mailing system to 10 000 persons who have subscribed to your service.

An editor propose you an exceptional offer: he gives you the possibility to acquire a batch of a new book for a reduced cost so that you can sell it for a reduced price at the same time of the release in bookshops and so benefit from the promotional launch campaign. The contract specifies the acquisition of an indivisible batch of 10 000 books for a global price of 100 000 €. Each book would be sold fo 20 € knowing that you have to pay 2 € as additional cost per unit solf for the handling and the shipment. Usually, the online selling of a book is done a few months after its release in bookstores so you can predict with a relative accuracy that your customers will be "smart enough to not miss such an amazing offer". In the current case, you are doubtful.

In order to simplify your analysis, you consider at first two possibilities that you think equiprobable:

- A large success where you estimate that 80% of you customers will buy the book.
- A limited success where you estimate that only 20% of your customers will buy the book.

1. Establish the decision tree associated to this problem and examine what is your optimal strategy (we will suppose that you respect the axioms of von Neumann-Morgenstern and that your utility function is linear).
2. How much will you be ready to pay to obtain a perfect information related to this problem?
3. How much will you be ready to pay for a quick survey that will allow you to know (with certainty) the decision to purchase the book or not of 2 of your 10 000 customers chosen randomly? How do you interpret the value you found?