Exercise sheet n° 3 :

3-1. *Data compression.* Determine whether the following codes are nonsingular, uniquely decodable, instantaneous :

(a) The code $\{0, 0\}$.

(b) The code $\{0, 010, 01, 10\}$.

(c) The code $\{10, 00, 11, 110\}$.

(d) The code $\{0, 10, 110, 111\}$.

3-2. We consider a source $\{x_i, p_i\}$ with $i = 1, \cdots, m$. The symbols x_i (emitted with probabilities p_i) are encoded in sequences using an alphabet of cardinality D, such that the decoding is instantaneous. For $m = 6$ and lengths of the codewords $\{l_i\} = \{1, 1, 1, 2, 2, 3\}$, find a lower bound for D. Is this code optimal ?

3-3. *Huffman code.* A source emits a random variable X which can take four values with probabilities $(\frac{1}{10}, \frac{2}{10}, \frac{3}{10}, \frac{4}{10})$.

(a) Construct a binary Huffman code for X.

(b) Construct a ternary Huffman code for X.

(c) Construct a binary Shannon code for X and compare its expected length with the code of (a).

3-4. *Huffman code.* A source emits a random variable X which can take four values with probabilities $(\frac{1}{3}, \frac{1}{3}, \frac{1}{4}, \frac{1}{12})$.

(a) Construct a binary Huffman code for X.

(b) Construct a binary Shannon code for X and compare it with the code of (a).

3-5. We have a non-balanced coin with probability p to obtain "head" ("1") and probability $1 - p$ to obtain "tail" ("0"). Alice flips this coin as many times as needed to obtain "head" for the first time, and would like to communicate to Bob the number of flips k that were needed. A naive method is to send to Bob the sequence of the outcomes of the coin flips encoded in a chain of bits of length k, like $000 \cdots 01$ (where 0 stands for "tail" and 1 for "head").

(a) What is the expected length of this naive code ? Compare it with the entropy of the random variable k. In which case the naive code is optimal ? Use

$$\sum_{i=0}^{\infty} a^i = \frac{1}{1-a} \qquad \sum_{i=0}^{\infty} i\, a^i = \frac{a}{(1-a)^2}$$

(b) Alice decides now to encode her random variable k with a Shannon code, with the aim to approach $H(k)$. Compare the expected lengths of the naiv code and the Shannon code in the limit $p \to 0$.

Web site :

http ://quic.ulb.ac.be/teaching/