TP 2 – Linear Time Temporal Logic

LTL Syntax Formulas of LTL are built over a set of atomic propositions $\mathcal{P}$ and satisfy the following syntax :

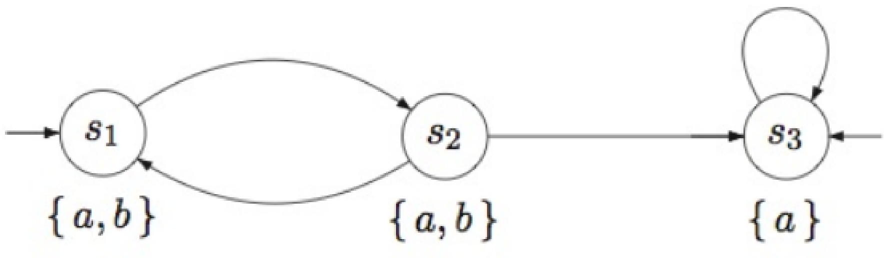
$$\phi \ ::= \ \top \mid \bot \mid p \mid \neg\phi \mid \phi \vee \phi \mid \phi \wedge \phi \mid \bigcirc\phi \mid \phi \ U \ \phi \mid \phi \ \tilde{U} \ \phi$$

where $p \in \mathcal{P}$.

A state s of a Kripke structure satisfies an LTL formula ϕ if all the paths from s satisfy ϕ.

A Kripke structure $\mathcal{K}$ satisfies an LTL formula ϕ if all its initial states satisfy ϕ.

Exercice 1 For the following Kripke structure,



and the following statements, replace ? by either $\models$ or $\not\models$:

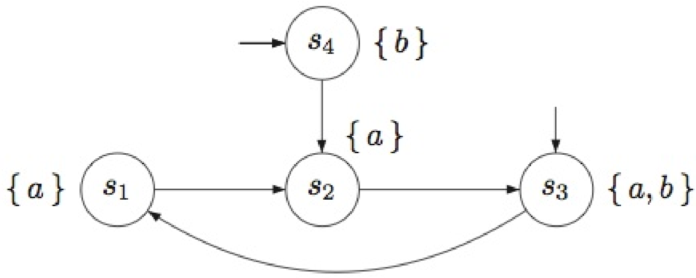
1. $\mathcal{K}$? $\Box a$
2. s_1 ? $\bigcirc (a \wedge b)$
3. s_2 ? $\bigcirc (a \wedge b)$
4. s_3 ? $\bigcirc (a \wedge b)$
5. $\mathcal{K}$? $\bigcirc (a \wedge b)$
6. $\mathcal{K}$? $\Box(\neg b \rightarrow \Box(a \wedge \neg b))$
7. $\mathcal{K}$? $b \ U \ (a \wedge \neg b)$

Exercice 2 For each of the following formulas

1. $\bigcirc a$
2. $\bigcirc \bigcirc \bigcirc a$

3. $\Box b$
4. $\Box\Diamond a$
5. $\Box(a\ U\ b)$
6. $\Diamond(a\ U\ b)$

and the following Kripke structure :

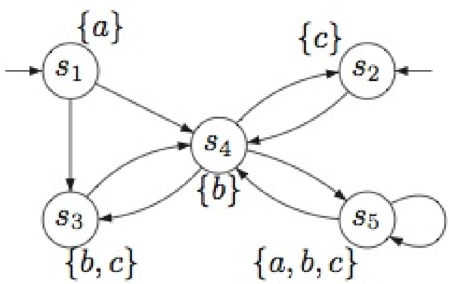


give the set of states for which the formulas are satisfied.

Exercice 3 For each of the following formulas

1. $\Diamond\Box c$
2. $\Box\Diamond c$
3. $\bigcirc(\neg c) \rightarrow \bigcirc\bigcirc c$
4. $\Box a$
5. $a\ U\ (b \vee c)$
6. $(\bigcirc\bigcirc b)\ U\ (b \vee c)$

tell whether they are satisfied by the following Kripke structure :



Exercice 4 Let $\mathcal{P} = \{p\}$. Express in LTL that p is true at most once.

Exercice 5 Prove the following equivalences :

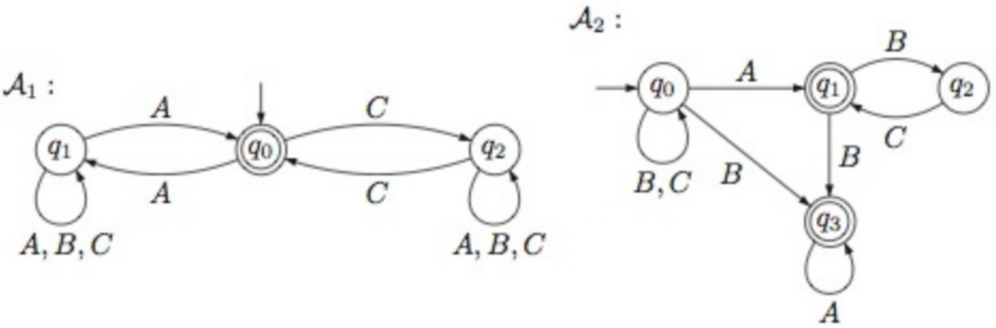
1. $\neg\bigcirc\phi \equiv \bigcirc\neg\phi$
2. $\neg\Diamond\phi \equiv \Box\neg\phi$
3. $\neg\Box\phi \equiv \Diamond\neg\phi$

4. $\Diamond\Diamond\phi \equiv \Diamond\phi$
5. $\Box\Box\phi \equiv \Box\phi$
6. $\Diamond\Box\Diamond\phi \equiv \Box\Diamond\phi$
7. $\Diamond\phi \equiv \phi \vee \bigcirc\Diamond\phi$
8. $\Box\phi \equiv \phi \wedge \bigcirc\Box\phi$
9. $\phi\ U\ \psi \equiv \psi \vee (\phi \wedge \bigcirc(\phi\ U\ \psi))$

Exercice 6 Consider a printer system with two users A and B. We suppose that there is only one printer. For all users $\alpha \in \{A, B\}$, we have the following command :
- req_α : α wants to use the printer (he sends a request),
- use_α : α uses the printer,
- rel_α : α releases the printer.

Specify in LTL the following properties :
1. mutual exclusion : at most one user uses the printer at a time
2. every user uses the printer during a finite amount of time (consecutively)
3. No starving : every user that wants to use the printer eventually uses it
4. alternance : if the two users want to use the printer infinitely often, then they must alternate in using it

Exercice 7 Give regular expressions equivalent to the following Büchi automata :



Exercice 8 Give a Büchi automaton whose language cannot be defined by a deterministic Büchi automaton.

Exercice 9 Construct Büchi automata equivalent to the following LTL formulas :
1. $\Box(a \vee \neg \bigcirc b)$
2. $\Diamond a \vee \Box \diamond (a \rightarrow b)$
3. $\bigcirc \bigcirc (a \vee \Diamond\Box b)$

Exercice 10 Construct a Büchi automaton which is not equivalent to any LTL formula.

Exercice 11 Apply the LTL model-checking algorithm to check that $\mathcal{K} \models \Box a$, where $\mathcal{K}$ is the structure defined in Exercice 1.