INFO-F-404 : Operating Systems II

1 Exercises

We will use following notations :

- O_i - Release time of Task τ_i
- C_i - Worst case execution time of Task τ_i
- D_i - Deadline of Task τ_i
- T_i - Period of Task τ_i

Exercise 1a : Rate Monotonic

Task index	Release time	WCET	Deadline	Period
Task τ_1	0	10	50	50
Task τ_2	0	20	80	80
Task τ_3	0	10	100	100
Task τ_4	0	50	200	200

Table 1: System of 4 periodic, synchronous tasks with implicit deadline.

a) Verify that this system could be scheduled using RM algorithm.

Answer : We can choose between two tests, let's start with the most simple (utilization of the processor):

$$U_{tot}(\tau) = \sum_{i=1}^{n} U_i = \sum_{i=1}^{n} \frac{C_i}{T_i}$$

$$U_{tot}(\tau) = \frac{10}{50} + \frac{20}{80} + \frac{10}{100} + \frac{50}{200} = 0.2 + 0.25 + 0.1 + 0.25 = 0.8$$

$$U_{tot} \leq ln(2), (ln(2) \approx 0.69)$$

We have $69\% < 80\% \leq 100\%$, so we can not claim anything about feasibility of this system. Note that we can also use $U_{tot} \leq n(2^{1/n} - 1)$, in this case we have $n = 4$, so $n(2^{1/n} - 1) = 0.7568$, but we still have $80\% > 75.68\%$.

We have to use the second method (based on the worst response time of the first job):

1. First of all, sort tasks by priority (assigned by RM, $p_i = \frac{1}{T_i}$) in decreasing order: $\tau_1 > \tau_2 > \tau_3 > \tau_4$.
2. After that we have to find the worst response time of the first job of each task. We can do so by finding the fixed point:
   - $W_0 = C_i$
   - $W_{k+1} = C_i + \sum_{j=1}^{i-1} \left\lceil \frac{W_k}{T_j} \right\rceil \cdot C_j$

   till $W_{k+1} = W_k$ or $W_k > D_i$ (system is not feasible).

   For our system, we have:

   (a) τ_1:
      i. $W_0 = 10$
      ii. $W_1 = 10 + 0 = 10$
      iii. Fixed point found, $W_1 \leq D_1 \rightarrow$ OK

   (b) τ_2:
      i. $W_0 = 20$
      ii. $W_1 = 20 + \lceil \frac{20}{50} \rceil \cdot 10 = 20 + 10 = 30$
      iii. $W_2 = 20 + \lceil \frac{30}{50} \rceil \cdot 10 = 20 + 10 = 30$
      iv. Fixed point found, $W_2 \leq D_2 \rightarrow$ OK

   (c) τ_3:
      i. $W_0 = 10$
      ii. $W_1 = 10 + \lceil \frac{10}{50} \rceil \cdot 10 + \lceil \frac{10}{80} \rceil \cdot 20 = 10 + 10 + 20 = 40$
      iii. $W_2 = 10 + \lceil \frac{40}{50} \rceil \cdot 10 + \lceil \frac{40}{80} \rceil \cdot 20 = 10 + 10 + 20 = 40$
      iv. Fixed point found, $W_2 \leq D_3 \rightarrow$ OK

   (d) τ_4:
      i. $W_0 = 50$
      ii. $W_1 = 50 + \lceil \frac{50}{50} \rceil \cdot 10 + \lceil \frac{50}{80} \rceil \cdot 20 + \lceil \frac{50}{100} \rceil \cdot 10 = 50 + 10 + 20 + 10 = 90$
      iii. $W_2 = 50 + \lceil \frac{90}{50} \rceil \cdot 10 + \lceil \frac{90}{80} \rceil \cdot 20 + \lceil \frac{90}{100} \rceil \cdot 10 = 50 + 20 + 40 + 10 = 120$
      iv. $W_3 = 50 + \lceil \frac{120}{50} \rceil \cdot 10 + \lceil \frac{120}{80} \rceil \cdot 20 + \lceil \frac{120}{100} \rceil \cdot 10 = 50 + 30 + 40 + 20 = 140$
      v. $W_4 = 50 + \lceil \frac{140}{50} \rceil \cdot 10 + \lceil \frac{140}{80} \rceil \cdot 20 + \lceil \frac{140}{100} \rceil \cdot 10 = 50 + 30 + 40 + 20 = 140$
      vi. Fixed point found, $W_4 \leq D_4 \rightarrow$ OK

b) Plot the scheduling of these 4 tasks using RM. Each job takes its worst case execution time (WCET) to end.

Answer : see Figure 1. All deadlines were met between 0 and 200, thus we know that this system is feasible (thanks to the feasibility interval $[0, \max\{D_i | i = 1, \ldots, n\})$)

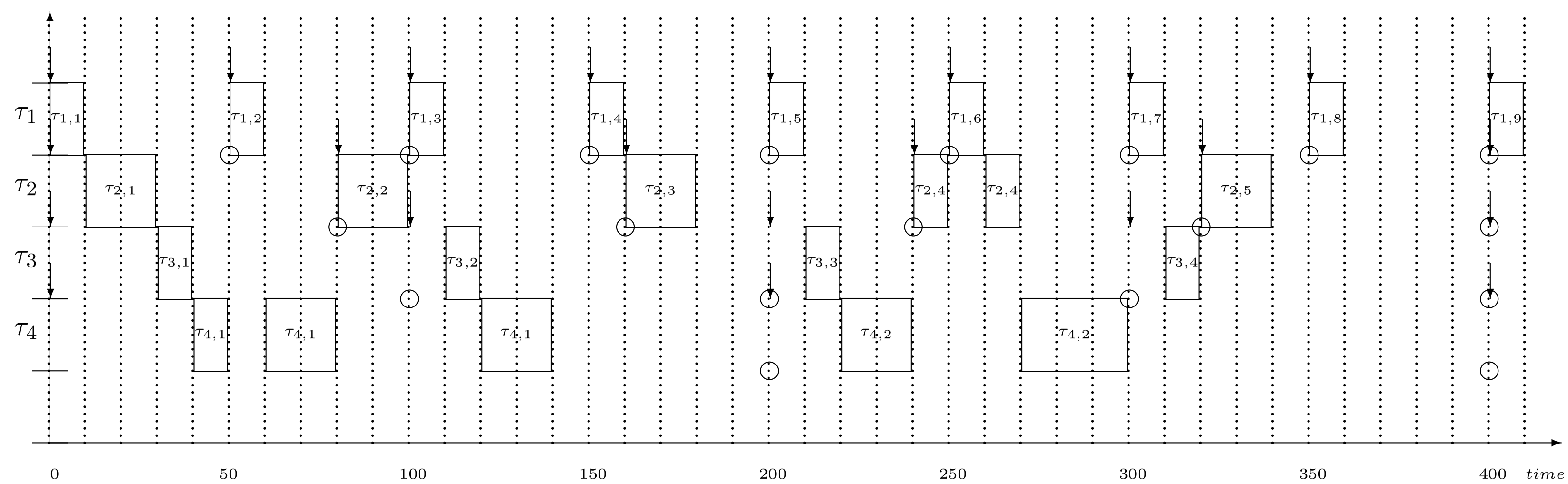

Figure 1: *RM Scheduling.*

c) Try to find a periodic system such as its utilization (U_{total}) is in range $[0.7; 1]$ and which could not be scheduled using RM algorithm.

Answer : see Table 2.

Task index	**Release time**	**WCET**	**Deadline**	**Period**
Tâche τ_1	0	20	40	40
Tâche τ_2	0	30	60	60

Table 2: RM can not schedule this system without missing a deadline.

Exercise 2 : Deadline Monotonic

Task index	Release time	WCET	Deadline	Period
Task τ_1	0	10	50	50
Task τ_2	0	20	40	80
Task τ_3	0	10	30	100
Task τ_4	0	50	150	200

Table 3: System of 4 periodic, synchronous tasks with constrained deadline.

a) Verify that this system could be scheduled using DM algorithm.

Answer : Since we have a system with constrained deadline, we can not use the method based on the utilization factor. We have to use the second method:

1. Sort tasks by priority (assigned by DM, $p_i = \frac{1}{D_i}$) in decreasing order $\tau_3 > \tau_2 > \tau_1 > \tau_4$.
2. Find the worst response time of the first job of each task :
   (a) τ_3:
      i. $W_0 = 10$
      ii. $W_1 = 10 + 0 = 10$
      iii. Fixed point found, $W_1 \leq D_3 \rightarrow$ OK
   (b) τ_2:
      i. $W_0 = 20$
      ii. $W_1 = 20 + \lceil \frac{20}{100} \rceil \cdot 10 = 20 + 10 = 30$
      iii. $W_2 = 20 + \lceil \frac{30}{100} \rceil \cdot 10 = 20 + 10 = 30$
      iv. Fixed point found, $W_2 \leq D_2 \rightarrow$ OK
   (c) τ_1:
      i. $W_0 = 10$
      ii. $W_1 = 10 + \lceil \frac{10}{100} \rceil \cdot 10 + \lceil \frac{10}{80} \rceil \cdot 20 = 10 + 10 + 20 = 40$
      iii. $W_2 = 10 + \lceil \frac{40}{100} \rceil \cdot 10 + \lceil \frac{40}{80} \rceil \cdot 20 = 10 + 10 + 20 = 40$
      iv. Fixed point found, $W_2 \leq D_1 \rightarrow$ OK
   (d) τ_4:
      i. $W_0 = 50$
      ii. $W_1 = 50 + \lceil \frac{50}{100} \rceil \cdot 10 + \lceil \frac{50}{80} \rceil \cdot 20 + \lceil \frac{50}{50} \rceil \cdot 10 = 50 + 10 + 20 + 10 = 90$
      iii. $W_2 = 50 + \lceil \frac{90}{100} \rceil \cdot 10 + \lceil \frac{90}{80} \rceil \cdot 20 + \lceil \frac{90}{50} \rceil \cdot 10 = 50 + 20 + 40 + 10 = 120$
      iv. $W_3 = 50 + \lceil \frac{120}{100} \rceil \cdot 10 + \lceil \frac{120}{80} \rceil \cdot 20 + \lceil \frac{120}{50} \rceil \cdot 10 = 50 + 30 + 40 + 20 = 140$
      v. $W_3 = 50 + \lceil \frac{140}{100} \rceil \cdot 10 + \lceil \frac{140}{80} \rceil \cdot 20 + \lceil \frac{140}{50} \rceil \cdot 10 = 50 + 30 + 40 + 20 = 140$
      vi. Fixed point found, $W_4 \leq D_4 \rightarrow$ OK

b) Plot the scheduling of these 4 tasks using DM. Each job takes its worst case execution time (WCET) to end.

Answer : see Figure 2. Once again, all deadlines were met between 0 and 150, thus we know that this system is feasible (thanks to the feasibility interval $[0, \max\{D_i | i = 1, \ldots, n\})$).

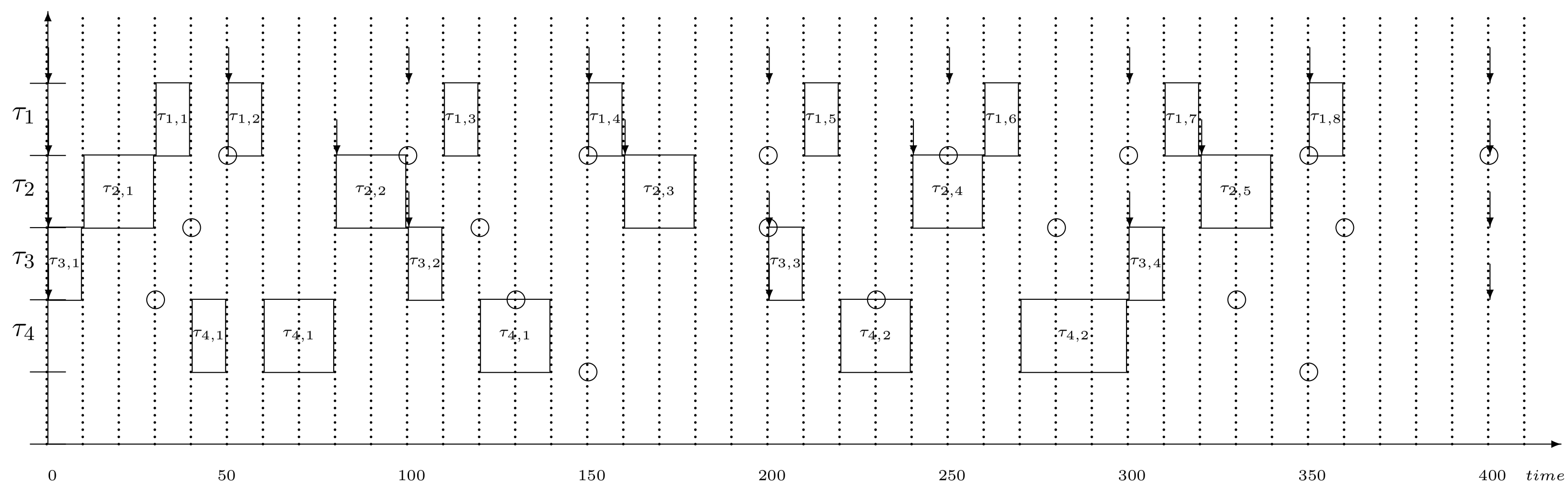

Figure 2: *DM Scheduling.*

Exercise 3 : Systems with arbitrary deadline

Task index	Release time	WCET	Deadline	Period
Taske τ_1	0	10	50	50
Taske τ_2	0	20	40	80
Taske τ_3	0	10	150	100
Taske τ_4	0	50	220	200

Table 4: System of 3 periodic, synchronous tasks with arbitrary deadline.

a) Find the feasibility interval for this system.

Answer : Since we deal with a system with arbitrary deadline we can not use test methods based on the utilization nor the worst response time. It means that we have to simulate the execution of this system to find out if the is a way to assign fix priorities to tasks in order to schedule the system. Thus we have to find the feasibility interval.

For systems with arbitrary deadline the feasibility interval is : $[0, \lambda_n)$, where λ is the fixed point of the following equation:

$$\lambda_n = \sum_{i=1}^{n} \left\lceil \frac{\lambda_n}{T_i} \right\rceil \cdot C_i$$

We can calculate λ using the following iterative method:

- $W_0 = \sum_{i=1}^{n} C_i$
- $W_{k+1} = \sum_{i=1}^{n} \left\lceil \frac{W_k}{T_i} \right\rceil \cdot C_i$

So for our system described by Table 4 we have:

1. $W_0 = 10 + 20 + 10 + 50 = 90$
2. $W_1 = \lceil \frac{90}{50} \rceil \cdot 10 + \lceil \frac{90}{80} \rceil \cdot 20 + \lceil \frac{90}{100} \rceil \cdot 10 + \lceil \frac{90}{200} \rceil \cdot 50 = 20 + 40 + 10 + 50 = 120$
3. $W_2 = \lceil \frac{120}{50} \rceil \cdot 10 + \lceil \frac{120}{80} \rceil \cdot 20 + \lceil \frac{120}{100} \rceil \cdot 10 + \lceil \frac{120}{200} \rceil \cdot 50 = 30 + 40 + 20 + 50 = 140$
4. $W_3 = \lceil \frac{140}{50} \rceil \cdot 10 + \lceil \frac{140}{80} \rceil \cdot 20 + \lceil \frac{140}{100} \rceil \cdot 10 + \lceil \frac{140}{200} \rceil \cdot 50 = 30 + 40 + 20 + 50 = 140$
5. Fixed-point is found, the feasibility interval is: $[0, W_3) = [0, 140)$