Introduction to Language Theory and Compilation: Exercises

Session 6: *First* sets, *Follow* sets and LL(1) parsing

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Prediction issues

- We've seen that parsers cannot easily make a choice when several conflicting possibilities arise.
  - Top-down parsing: choice between several Produce actions
  - Bottom-up parsing: choice between Shift and Reduce actions
- We had to rely on (horribly inefficient) backtracking techniques.
- We'll now attempt to make better use of the input to *predict* which action should be taken.
- The objective is to avoid backtracking altogether.

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First sets

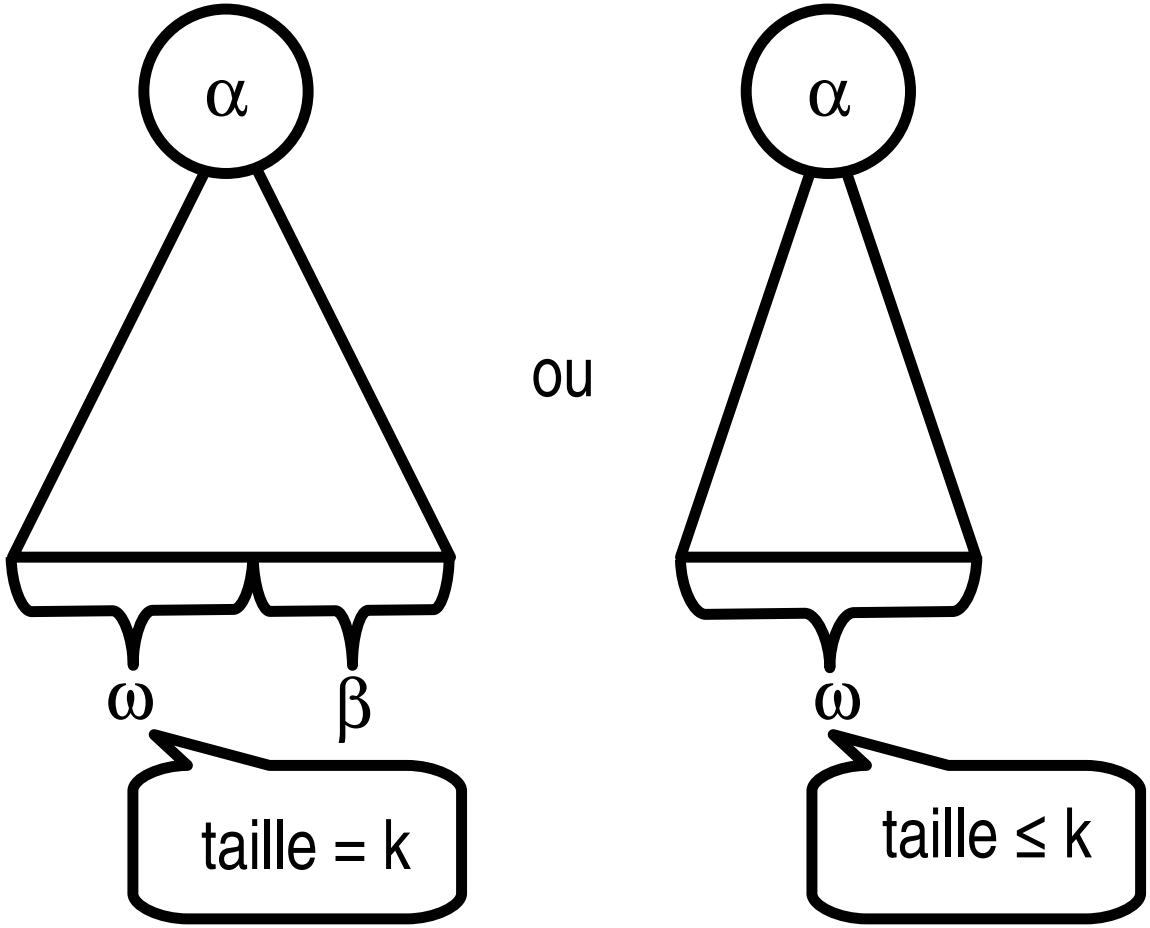
- Let $\alpha \in (V \cup T)^*$ be a sentential form. We define $\mathrm{First}^k(\alpha)$ as the set of strings of k first terminals that can be produced from α.
- Formally:

$$\forall \alpha \in (V \cup T)^* \ : \ \mathrm{First}^k(\alpha) =$$

$$\{\omega \in T^* | \alpha \stackrel{*}{\Rightarrow} \omega\beta$$

$$\wedge \ \big((|\omega| = k \wedge \beta \in T^*) \vee (|\omega| < k \wedge \beta = \varepsilon)\big)\}$$

First sets – intuitive illustration



First sets – construction algorithm

begin

  foreach $a \in T$ **do** $\mathrm{First}^k(a) \leftarrow \{a\}$

  foreach $A \in V$ **do** $\mathrm{First}^k(A) \leftarrow \emptyset$

  repeat

    foreach $A \in V$ **do**

      $\mathrm{First}^k(A) \leftarrow \mathrm{First}^k(A) \cup \{x \in T^* \mid A \rightarrow Y_1 Y_2 \ldots Y_n \wedge$
      $x \in \mathrm{First}^k(Y_1) \oplus^k \mathrm{First}^k(Y_2) \oplus^k \cdots \oplus^k \mathrm{First}^k(Y_n)\}$

  until *stability*

Follow sets

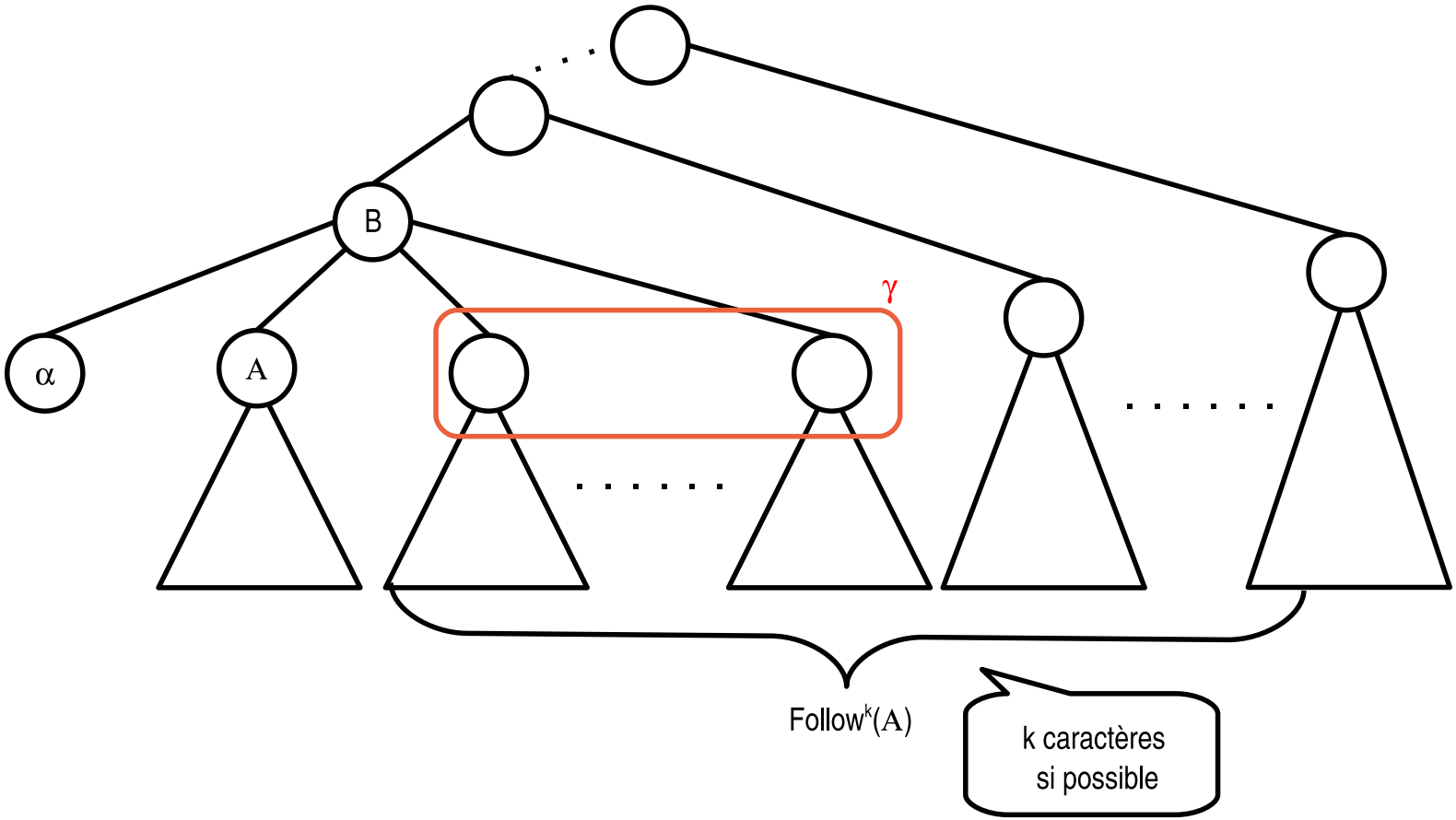
- Let A be a variable. We define $\mathrm{Follow}^k(A)$ as the set of k first terminals of the strings that can follow the productions of A.
- Computation:

$$\forall A \in V \setminus \{S\} : \mathrm{Follow}^k(A) =$$

$$\bigcup_{B \to \alpha A \gamma} \{\mathrm{First}^k(\gamma) \oplus^k \mathrm{Follow}^k(B)\}$$

$$\bigcup (\mathit{if}\ S \overset{+}{\Rightarrow} \alpha A\ \mathit{then}\ \{\varepsilon\}\ \mathit{else}\ \emptyset)$$

Follow sets – intuitive illustration



Follow sets – construction algorithm

begin

  foreach $A \in V$ **do** $\mathrm{Follow}^k(A) \leftarrow \emptyset$;

  ;

  repeat

    if $B \rightarrow \alpha A \beta \in P$ **then**

      $\mathrm{Follow}^k(A) \leftarrow \mathrm{Follow}^k(A) \cup \{\mathrm{First}^k(\beta) \oplus^k \mathrm{Follow}^k(B)\}$;

  until *stability*;

Exercise 1

With regards to the grammar given on paper:

1. Give the $\mathrm{First}^1(A)$ and the $\mathrm{Follow}^1(A)$ sets for each $A \in V$.
2. Give the $\mathrm{First}^2(\text{<expression>})$ and the $\mathrm{Follow}^2(\text{<expression>})$ sets.

Solution for exercise 1.1

Symbol	**First**^{1}()	**Follow**^{1}()
S	`begin`	
program	`begin`	`$`
statement list	`ID read write`	`end`
statement tail	`ID read write` ε	`end`
statement	`ID read write`	`ID read write end`
id list	`ID`	`)`
id tail	`,` ε	`)`
expr list	`( ID INTLIT`	`)`
expr tail	`,` ε	`)`
expression	`( ID INTLIT`	`; , )`
primary tail	`+ -` ε	`; , )`
primary	`( ID INTLIT`	`+ - ; , )`
add op	`+ -`	`( ID INTLIT`

Solution for exercise 1.2

- $\text{First}^2(< \textit{expression} >) = \{$ `((`, `(ID`, `(INTLIT`, `ID+`, `ID-`, `INTLIT+`, `INTLIT-`, `ID`, `INTLIT` $\}$

- $\text{Follow}^2(< \textit{expression} >) = \{$ `,(`, `,ID`, `,INTLIT`, `);`, `)+`, `)-`, `),`, `))`, `;read`, `;write`, `;ID`, `;end`$\}$

LL(1) grammars

- A grammar is said to be LL(1) if it can be recognized by a top-down parser with one lookahead symbol.
- Formally:

A grammar G is LL(1) if and only if:
For each production $A \to \alpha_1$ and $A \to \alpha_2$ $(\alpha_1 \neq \alpha_2)$ we have:
$$\mathrm{First}^1(\alpha_1 \mathrm{Follow}^1(A)) \cap \mathrm{First}^1(\alpha_2 \mathrm{Follow}^1(A)) = \emptyset$$

Action table

- We can easily construct an *action table* for an LL(1) grammar.
- The table tells us what action should be taken depending on the current symbol on top of the parser's stack and the current lookahead symbol.

$$
\begin{array}{lcll}
S & \rightarrow & aS & (1) \\
 & \rightarrow & b & (2)
\end{array}
$$

	a	b	c	$\cdots$
S	P1	P2	$\times$	$\cdots$
a	M	$\times$	$\times$	$\cdots$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	

Action table construction algorithm

begin

- $M \leftarrow \times$;
- **foreach** $A \rightarrow \alpha$ **do**
  - **foreach** $a \in First^1(\alpha)$ **do**
    - $M[A, a] \leftarrow M[A, a] \cup \text{Produce}(A \rightarrow \alpha)$;
  - **if** $\varepsilon \in First^1(\alpha)$ **then**
    - **foreach** $a \in Follow^1(A)$ **do**
      - $M[A, a] \leftarrow M[A, a] \cup \text{Produce}(A \rightarrow \alpha)$;
- **foreach** $a \in T$ **do** $M[a, a] \leftarrow \text{Match}$;
- ;
- $M[\$, \varepsilon] \leftarrow \text{Accept}$;

Exercise 2

Which of these grammars are LL(1)?

- 1

$$\begin{aligned} S &\rightarrow ABBA \\ A &\rightarrow a \mid \varepsilon \\ B &\rightarrow b \mid \varepsilon \end{aligned}$$

- 2

$$\begin{aligned} S &\rightarrow aSe \mid B \\ B &\rightarrow bBe \mid C \\ C &\rightarrow cCe \mid d \end{aligned}$$

- 3

$$\begin{aligned} S &\rightarrow ABc \\ A &\rightarrow a \mid \varepsilon \\ B &\rightarrow b \mid \varepsilon \end{aligned}$$

- 4

$$\begin{aligned} S &\rightarrow Ab \\ A &\rightarrow a \mid B \mid \varepsilon \\ B &\rightarrow b \mid \varepsilon \end{aligned}$$

Solution for exercise 2

1. not LL(1) because $a \in \text{Follow}^1(A)$, $a \in \text{First}^1(A)$ and we have the rule $A \to \varepsilon$. Thus,
   $M[A, a] = \{\text{Produce } 2, \text{Produce } 3\}$.
2. LL(1).
3. LL(1).
4. not LL(1) because $b \in \text{First}^1(A)$, $b \in \text{Follow}^1(B)$ and we have the rule $B \to \varepsilon$. Thus,
   $M[A, b] = \{\text{Produce } 3, \text{Produce } 4\}$.

Exercise 3

Give the action table for the following grammar:

(1)	<S>	→	<expr> $
(2)	<expr>	→	– <expr>
(3)	<expr>	→	(<expr>)
(4)	<expr>	→	<var> <expr-tail>
(5)	<expr-tail>	→	– <expr>
(6)	<expr-tail>	→	ε
(7)	<var>	→	ID <var-tail>
(8)	<var-tail>	→	(<expr>)
(9)	<var-tail>	→	ε

Solution for exercise 3

	-	(	)	ID	$
<S>	1	1	×	1	×
<Expr>	2	3	×	4	×
<ExprTail>	5	×	6	×	6
<Var>	×	×	×	7	×
<VarTail>	9	8	9	×	9

Recursive descent parser

How can we implement a top-down parser?

- We create one function per variable.
- Each function calls a `next_token()` function to read the next token on the input.
- We use the latter as *lookahead symbol* and proceed to pick the right action.
  - Call(s) to `match(token)`
  - Call(s) to a function corresponding to another variable (i.e. Produce)
- We may also use a `syntax_error()` function to explicitly signal that parsing failed.

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Recursive descent parser – example

<statement> → `id` := <expr> ; | `read` (<id list>) ; | `write` (<expr list>) ;

```
void statement()
{
   token tok = next_token();
   switch(tok)
   {
   case ID:
       match(ID); match(ASSIGNOP); expression(); match(SEMICOLON);
       break;
   case READ:
       match(READ); match(LPAREN); id_list(); match(RPAREN);
       match(SEMICOLON); break;
   case WRITE:
       match(WRITE); match(LPAREN); expr_list(); match(RPAREN);
       match(SEMICOLON); break;
   default:
       syntax_error(tok); break;
   }
}
```

Exercise 4

Using the grammar given on paper, program a recursive descent parser for rules (14) through (21).