

# Chapter 5

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## MOSFET differential amplifier



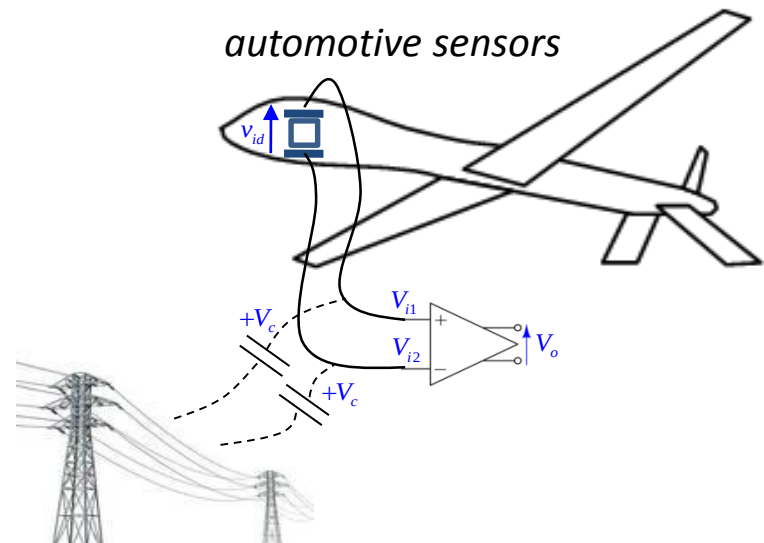
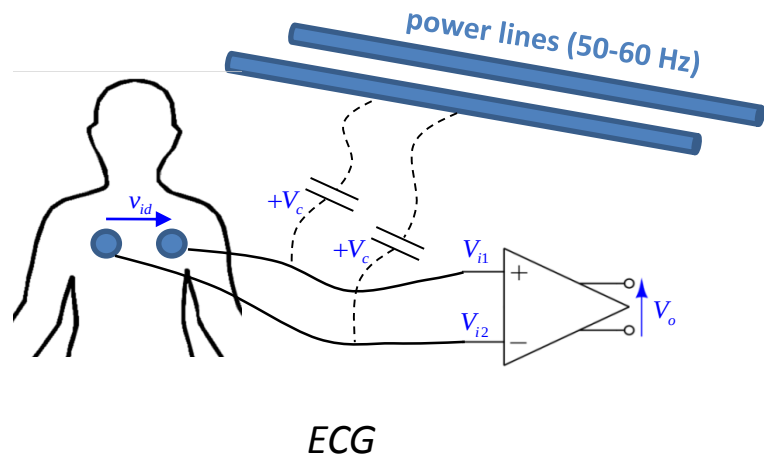
# Outline

- Introduction
- Ideal MOS differential pair
  - Common-mode voltage
  - Large-signal operation
  - Small-signal operation
- Nonidealities
  - Common-mode rejection ratio
  - Input offset voltage
- Differential amplifier with active load
- Frequency response of differential amplifier
- Multistage amplifiers

# Why a differential amplifier ?

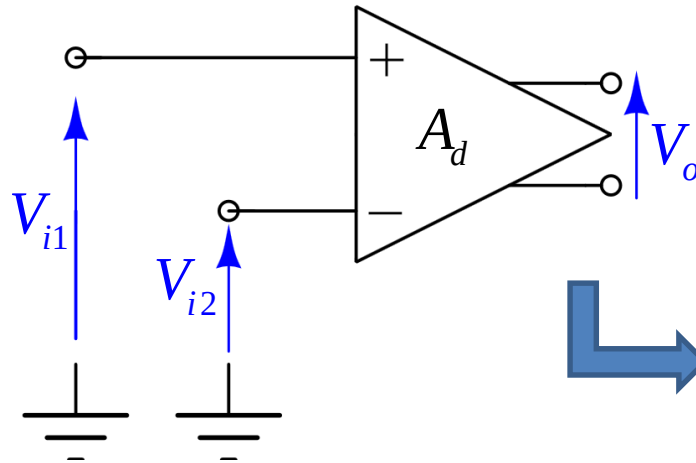
## ... *differential* voltage – polluted by common mode

- Many sensors provide a differential voltage
  - ⇒ When transmitting this differential voltage over cables, they become susceptible to common-mode voltage (power lines, RF interference, magnetic fields, ...)
  - ⇒ Common-mode voltage also appears if ground of sensor is different from ground of circuit
  - ⇒ Differential voltage can be small w.r.t. common-mode voltage



# Ideal differential amplifier

only amplifies the *difference* between input voltages



$$\begin{cases} V_{i1} = V_c + \frac{V_{id}}{2} \\ V_{i2} = V_c - \frac{V_{id}}{2} \end{cases} \Leftrightarrow \begin{cases} V_c = \frac{V_{i1} + V_{i2}}{2} \\ V_{id} = V_{i1} - V_{i2} \end{cases}$$

$$V_o = A_c \cdot \left( \frac{V_{i1} + V_{i2}}{2} \right) + A_d \cdot (V_{i1} - V_{i2})$$
$$= A_c V_c + A_d v_{id}$$

- Ideal differential amplifier

⇒ High differential gain  $A_d$

⇒ Small common gain  $A_c$

⇒ High input resistance

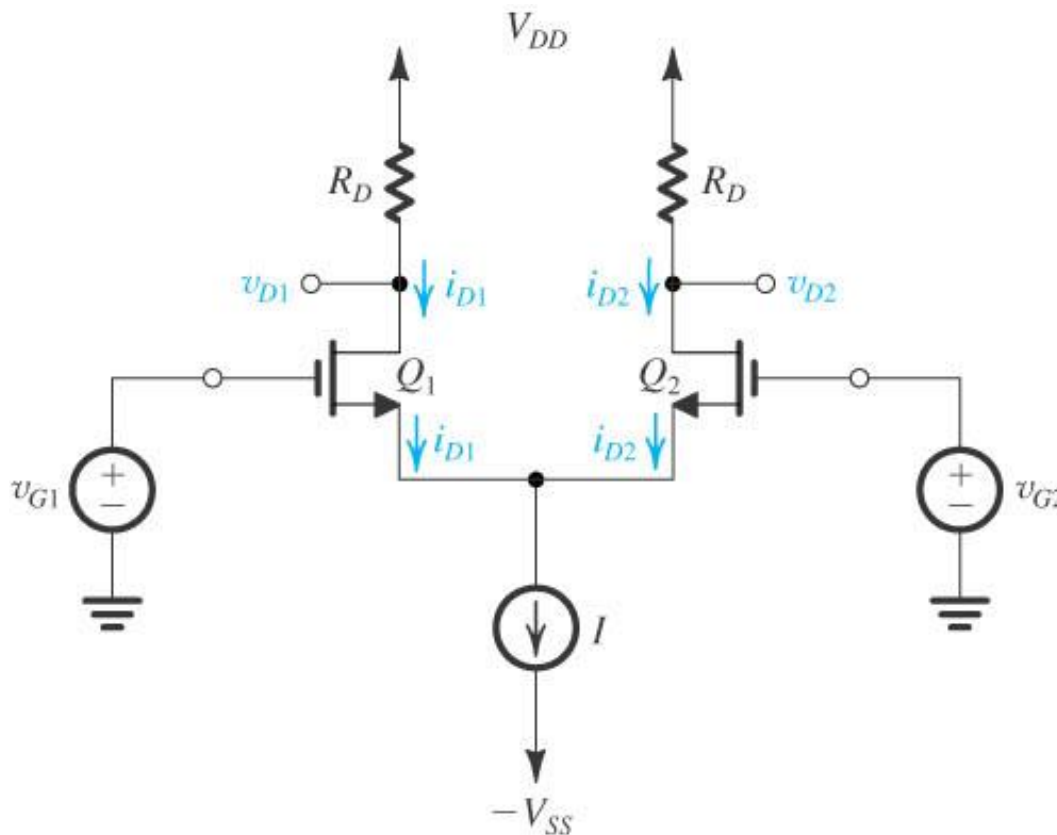
- How to build this in MOSFET technology ?

$$V_o = A_d \cdot (V_{i1} - V_{i2})$$

# Ideal MOS differential pair

## 2 matched transistors, biased by current source

- 2 inputs are the two gates of the transistors  
⇒ High input impedance !!!



### Assumptions:

- Transistors identical
- Drain resistors identical
- Perfect current source

### Common mode and differential mode:

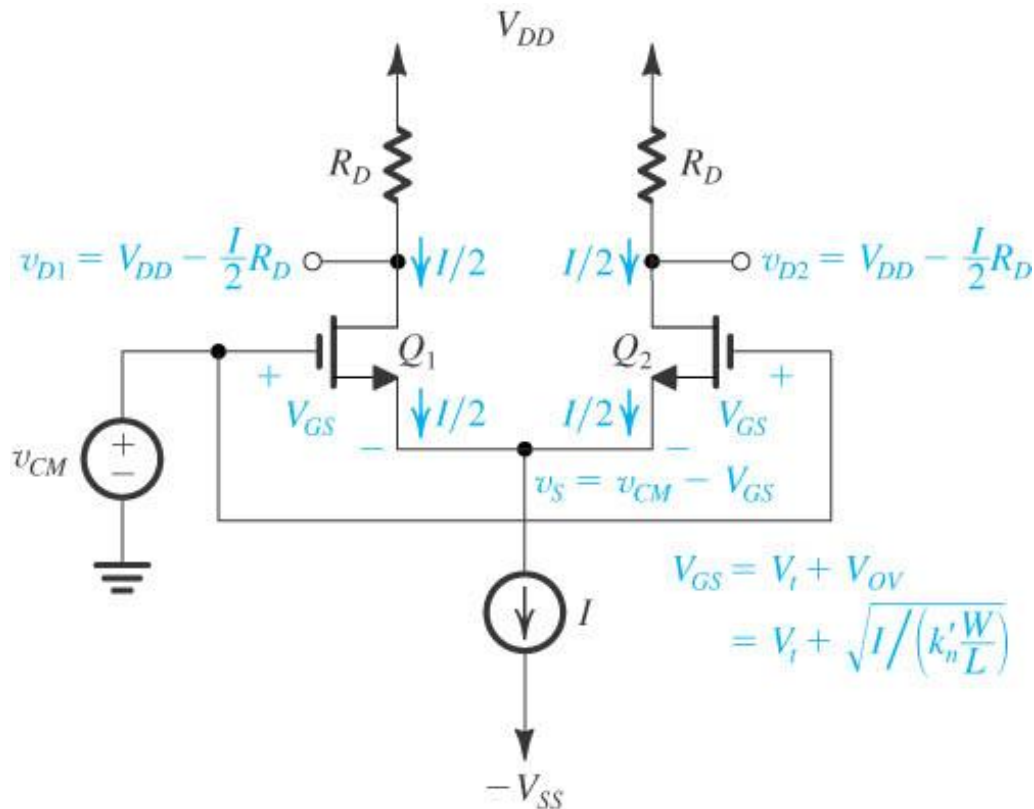
$$\begin{cases} v_{CM} = \frac{v_{G1} + v_{G2}}{2} \\ v_{id} = v_{G1} - v_{G2} \end{cases}$$

$$\Leftrightarrow \begin{cases} v_{G1} = v_{CM} + \frac{v_{id}}{2} \\ v_{G2} = v_{CM} - \frac{v_{id}}{2} \end{cases}$$

# Ideal MOS differential pair

## Common-mode input voltage

- Identical voltage at both inputs



Symmetry arguments:

$$i_{D1} = i_{D2} = I / 2$$

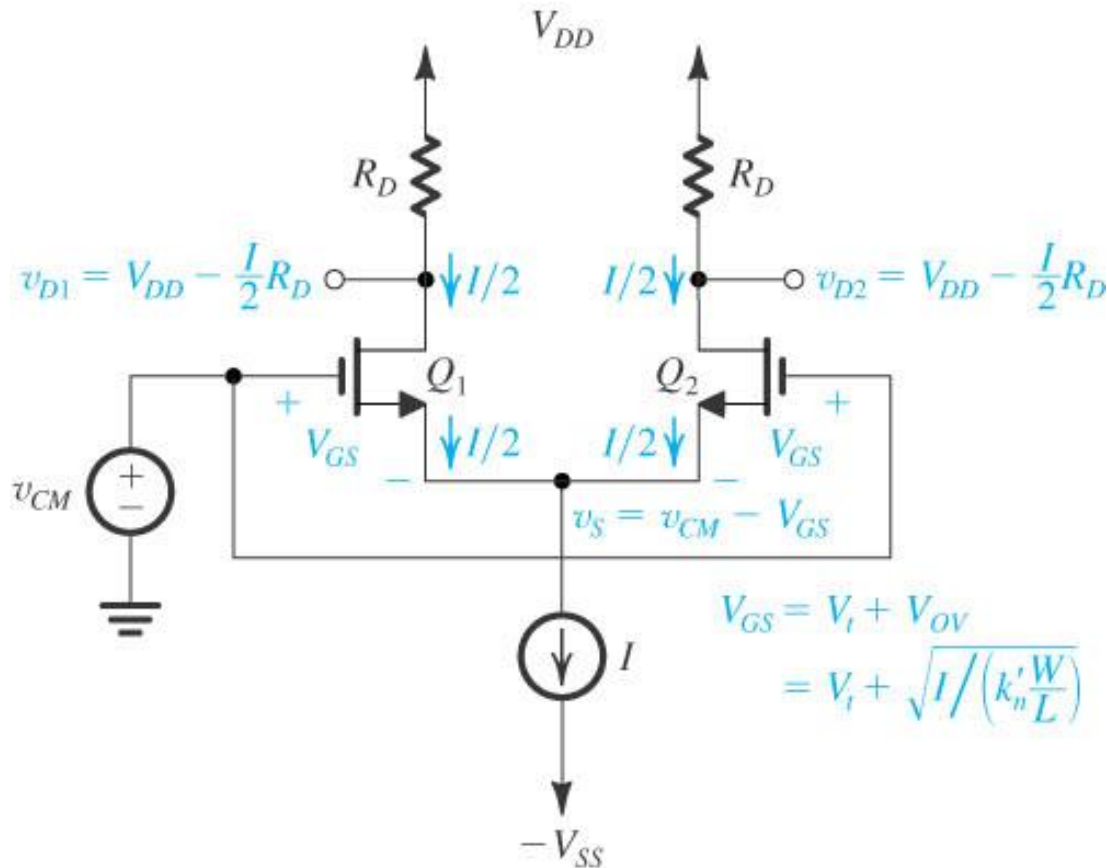
$$\Rightarrow v_{D1} = V_{DD} - R_D I / 2 = v_{D2}$$

$$\Rightarrow v_o = v_{D2} - v_{D1} = 0$$

$\Rightarrow$  Common gain is zero

# Ideal MOS differential pair

## Max. and min. common-mode input voltage



$Q_1$  and  $Q_2$  in saturation:

$$v_{DS} > (v_{GS} - V_t)$$

$$\Rightarrow V_{DD} - R_D i_D - v_S > v_{CM} - v_S - V_t$$

$$\Rightarrow v_{CM} < V_{DD} - \frac{R_D I}{2} + V_t$$

Current source must operate properly:

$\Rightarrow$  Minimum voltage  $V_{CS}$  over current source

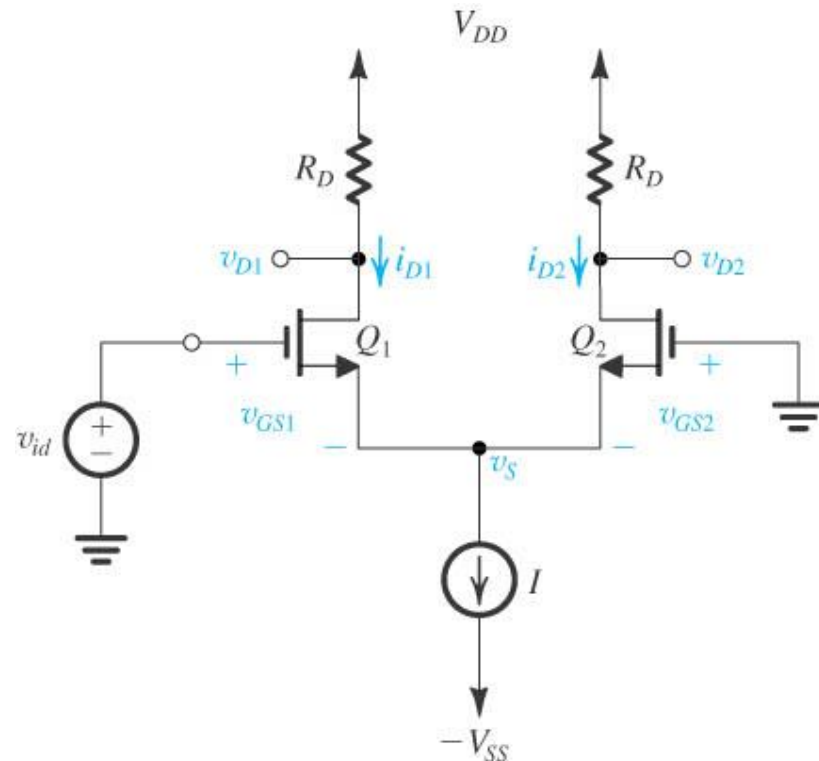
$$v_S - (-V_{SS}) > V_{CS}$$

$$\Rightarrow v_{CM} - V_{GS} + V_{SS} > V_{CS}$$

$$\Rightarrow v_{CM} > -V_{SS} + V_{CS} + V_{GS}$$

# Ideal MOS differential pair

## Differential input voltage: large-signal behavior



$$\begin{cases} v_{id} = v_{GS1} + v_S \\ 0 = v_{GS2} + v_S \end{cases} \Rightarrow v_{id} = (v_{GS1} - v_{GS2})$$

$$v_o = v_{D2} - v_{D1} = (V_{DD} - R_D i_{D2}) - (V_{DD} - R_D i_{D1})$$

$$\Rightarrow i_{D1} - i_{D2} = \frac{v_o}{R_D}$$

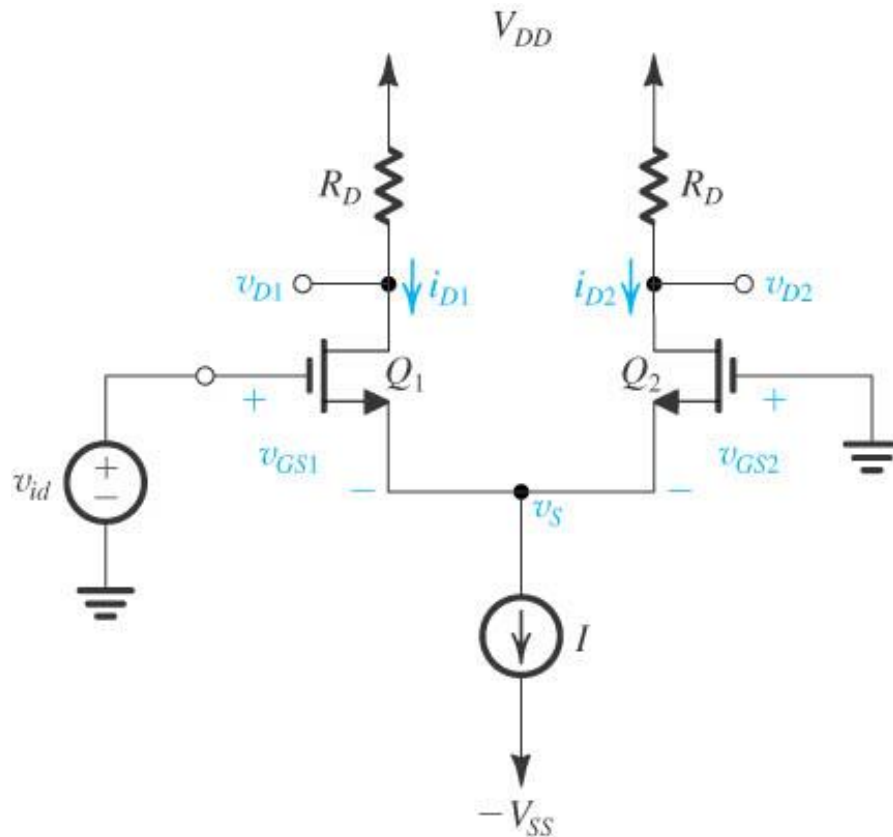
$$i_{D1} + i_{D2} = I$$

$$\Rightarrow \begin{cases} i_{d1} = \frac{I_0}{2} + \frac{v_o}{2R_d} \\ i_{d2} = \frac{I_0}{2} - \frac{v_o}{2R_d} \end{cases}$$



# Ideal MOS differential pair

## Differential input voltage: large-signal behavior



$$\begin{cases} i_{D1} = \frac{1}{2} k'_n \frac{W}{L} (v_{GS1} - V_t)^2 \\ i_{D2} = \frac{1}{2} k'_n \frac{W}{L} (v_{GS2} - V_t)^2 \end{cases}$$

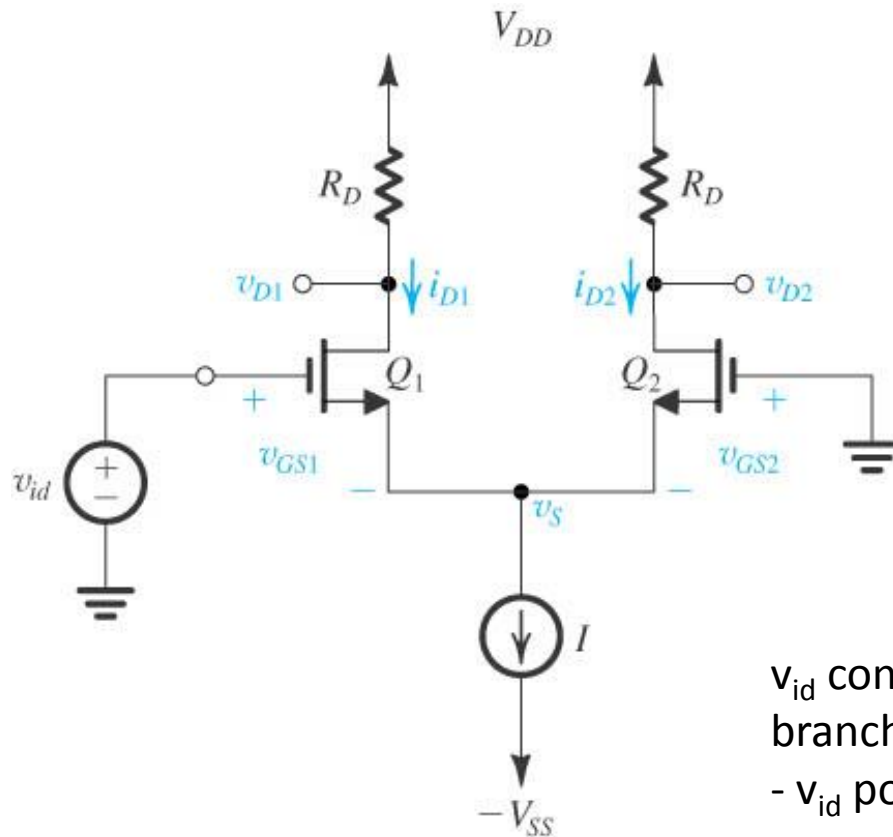
$$\Rightarrow \underbrace{v_{GS1} - v_{GS2}}_{=v_{id}} = \sqrt{\frac{2}{k'_n} \frac{L}{W}} (\sqrt{i_{D1}} - \sqrt{i_{D2}})$$

$$\begin{cases} i_{D1} = \frac{I}{2} + \frac{v_o}{2R_D} \\ i_{D2} = \frac{I}{2} - \frac{v_o}{2R_D} \end{cases}$$

$$v_o = R_D v_{id} \sqrt{k'_n \frac{W}{L} I} \sqrt{1 - \frac{1}{2} k'_n \frac{W}{L} \frac{v_{id}^2}{2I}}$$

# Ideal MOS differential pair

## Differential input voltage: large-signal behavior



$$v_o = R_D v_{id} \sqrt{k'_n \frac{W}{L} I} \sqrt{1 - \frac{1}{2} k'_n \frac{W}{L} \frac{v_{id}^2}{2I}}$$

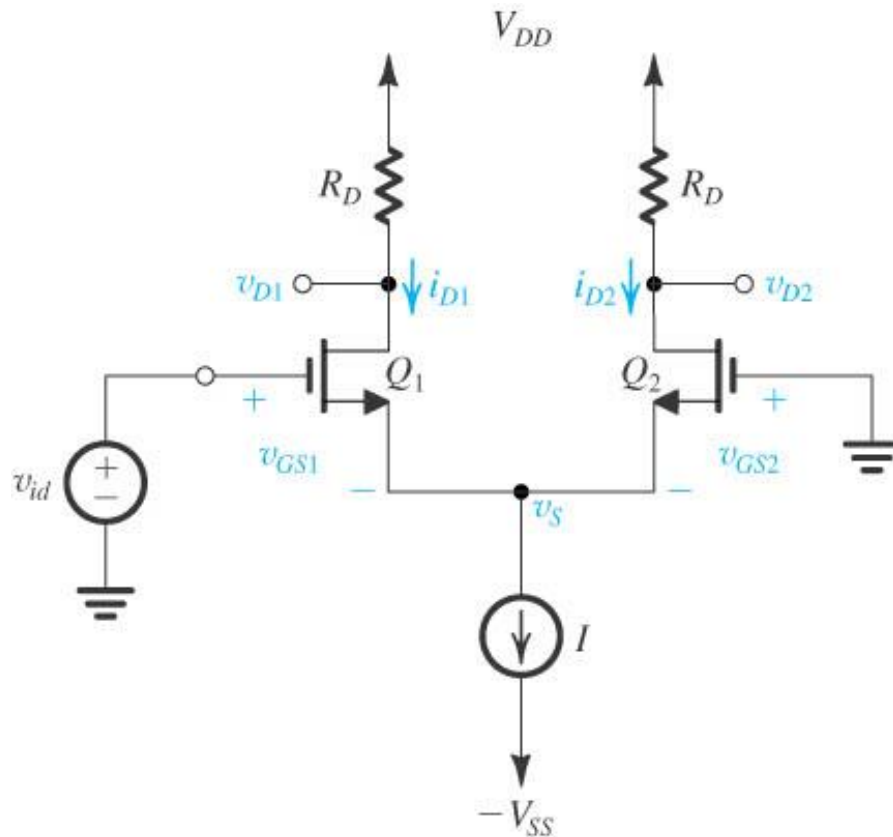
$$\Rightarrow \begin{cases} i_{D1} = \frac{I}{2} + \sqrt{k'_n \frac{W}{L} I} \left( \frac{v_{id}}{2} \right) \sqrt{1 - \frac{(v_{id}/2)^2}{1/k'_n \frac{W}{L}}} \\ i_{D2} = \frac{I}{2} - \sqrt{k'_n \frac{W}{L} I} \left( \frac{v_{id}}{2} \right) \sqrt{1 - \frac{(v_{id}/2)^2}{1/k'_n \frac{W}{L}}} \end{cases}$$

$v_{id}$  controls how much current flows through each branch of the circuit

- $v_{id}$  positive: more current through left branch
- $v_{id}$  negative: more current through right branch

# Ideal MOS differential pair

## Differential input voltage: large-signal behavior



Note that when  $v_{id} = 0$  :

$$i_{D1} = i_{D2} = I / 2$$

$$v_{GS1} = v_{GS2} = V_{GS}$$

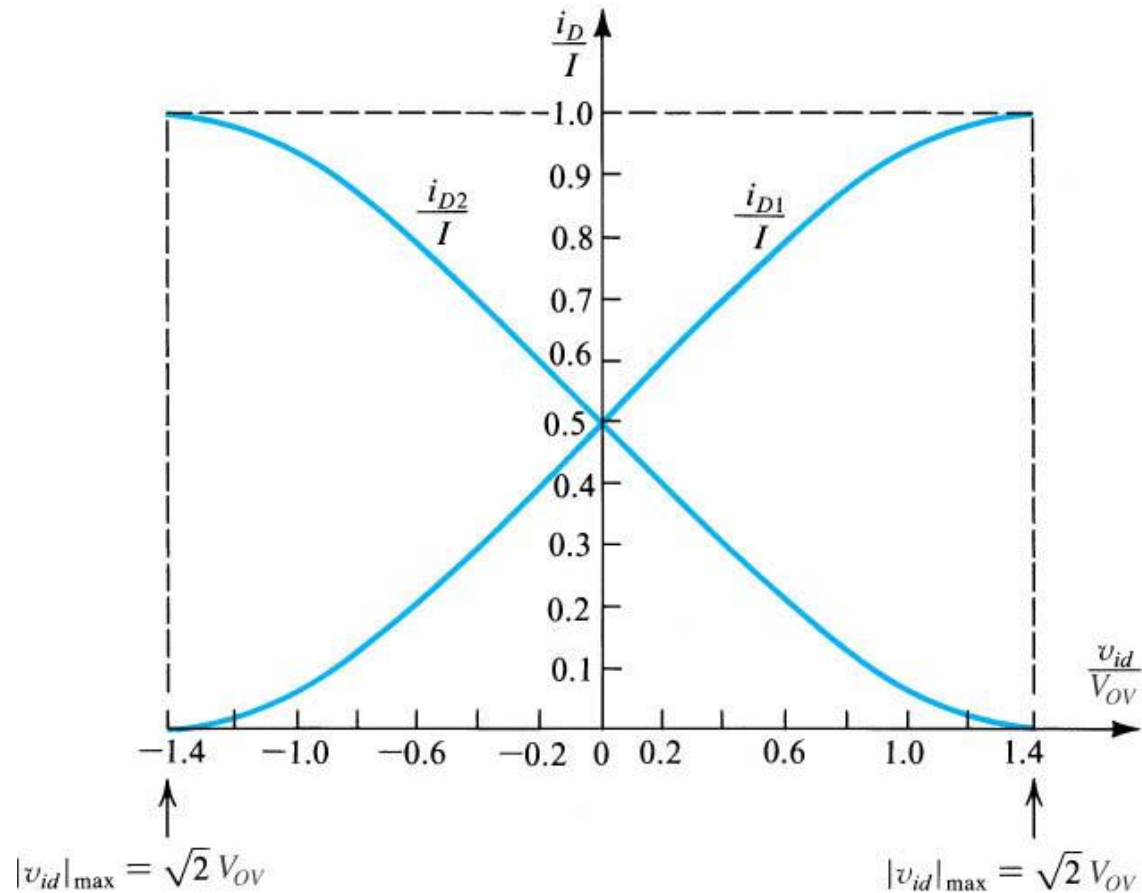
$$\Rightarrow \frac{I}{2} = \frac{1}{2} k'_n \frac{W}{L} \underbrace{(V_{GS} - V_t)^2}_{=V_{OV}^2}$$

$$\Rightarrow \begin{cases} i_{D1} = \frac{I}{2} + \left( \frac{I}{V_{OV}} \right) \frac{v_{id}}{2} \sqrt{1 - \left( \frac{v_{id}/2}{V_{OV}} \right)^2} \\ i_{D2} = \frac{I}{2} - \left( \frac{I}{V_{OV}} \right) \frac{v_{id}}{2} \sqrt{1 - \left( \frac{v_{id}/2}{V_{OV}} \right)^2} \end{cases}$$

# Ideal MOS differential pair

## Differential input voltage: large-signal behavior

- currents  $i_{D1}$  and  $i_{D2}$



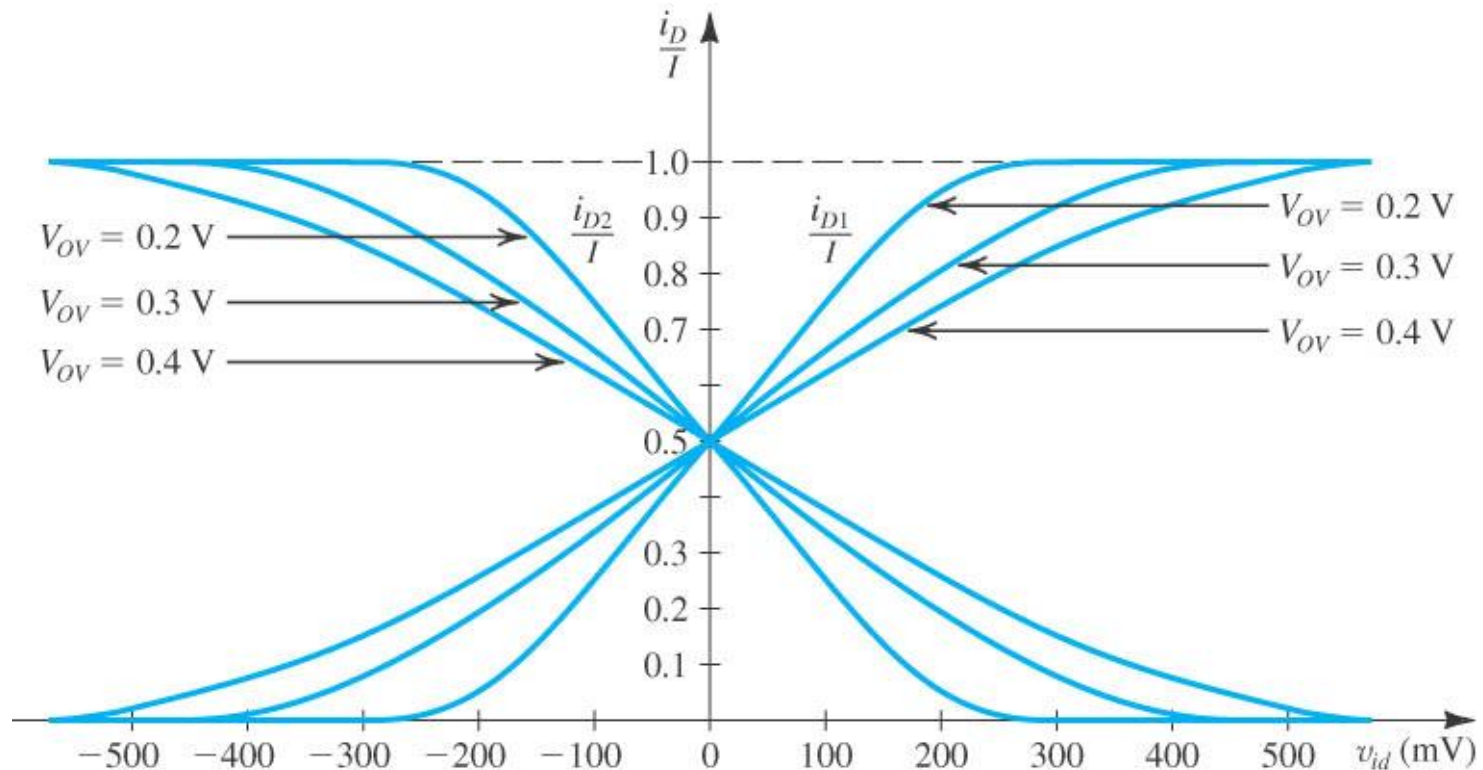
# Ideal MOS differential pair

## Differential input voltage: large-signal behavior

- $V_{OV}$  can be changed (e.g. by changing  $W$  or  $L$  for unchanged  $I$ )

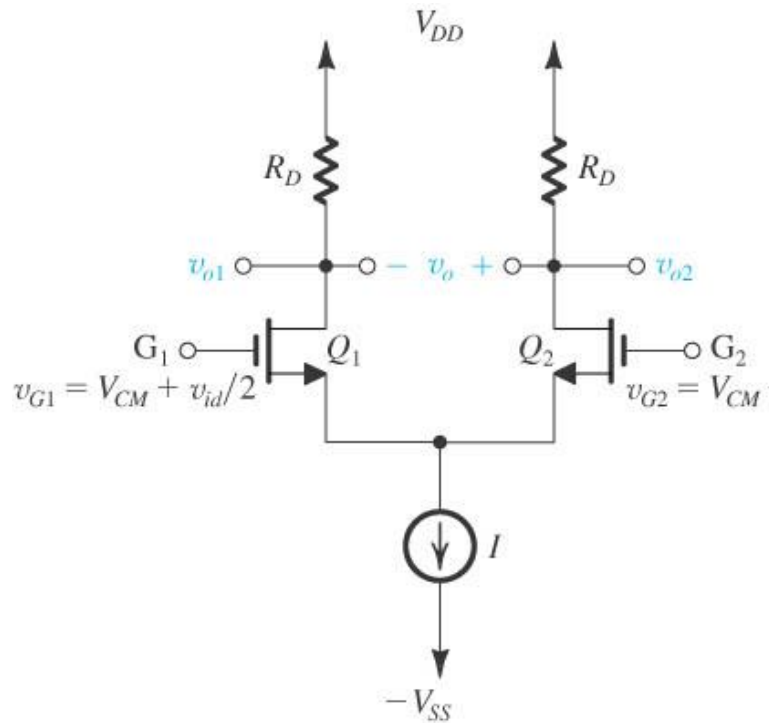
$$\frac{I}{2} = \frac{1}{2} k'_n \frac{W}{L} V_{OV}^2$$

⇒ Controls slope of currents

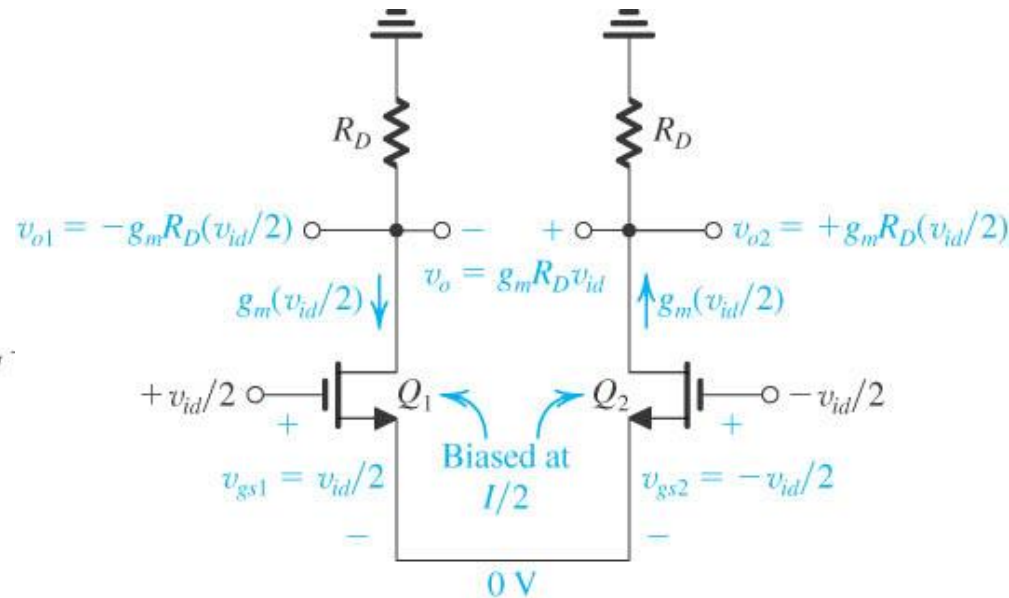


# Ideal MOS differential pair

## Differential input: small-signal behavior



$$\begin{cases} v_{G1} = v_{CM} + \frac{v_{id}}{2} \\ v_{G2} = v_{CM} - \frac{v_{id}}{2} \end{cases}$$



$$\begin{cases} v_{o1} = -R_D i_{d1} = -R_D g_m v_{gs1} \\ v_{o2} = -R_D i_{d2} = -R_D g_m v_{gs2} \end{cases}$$

$$\Rightarrow v_o = g_m R_D v_{id}$$

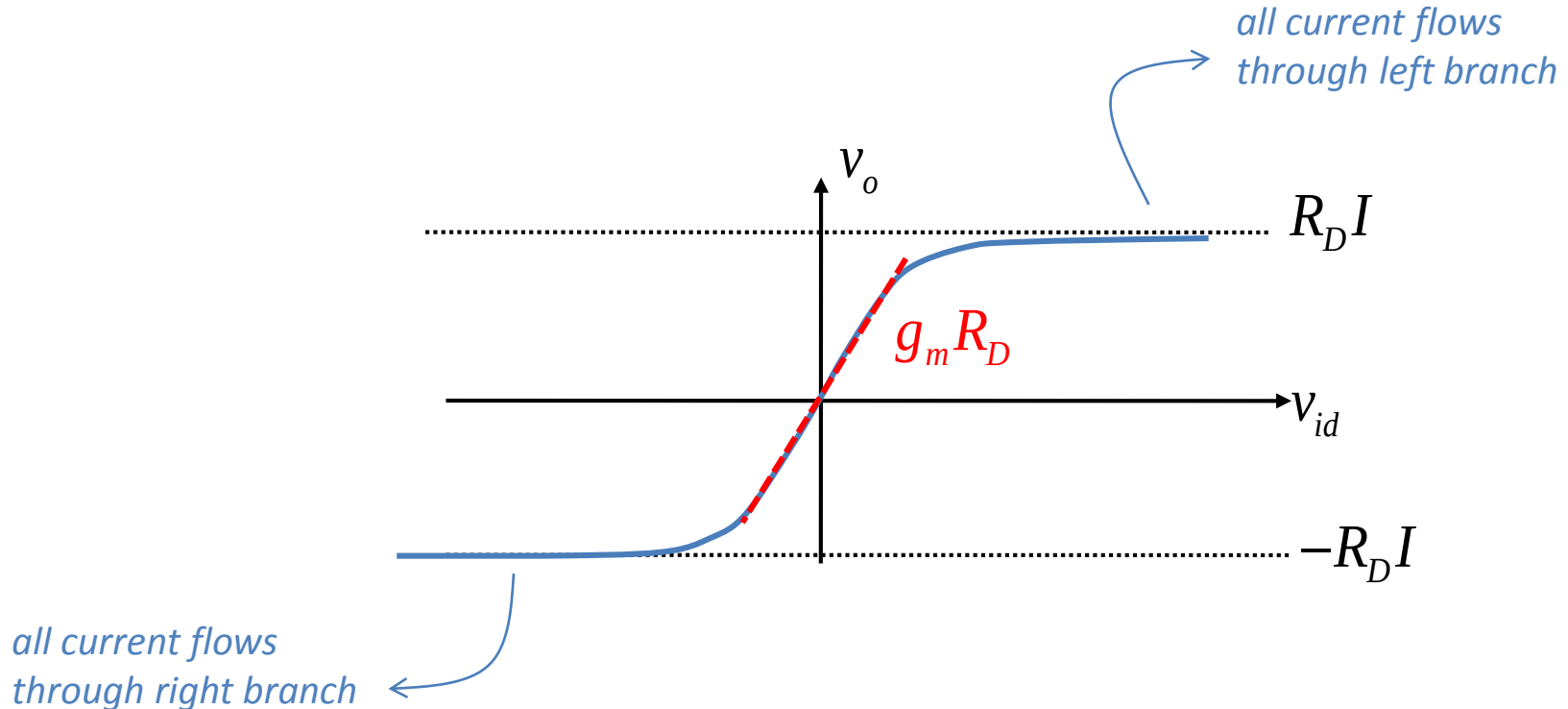
# Ideal MOS differential pair

## Differential input: small-signal behavior

$$v_o = g_m R_D v_{id}$$

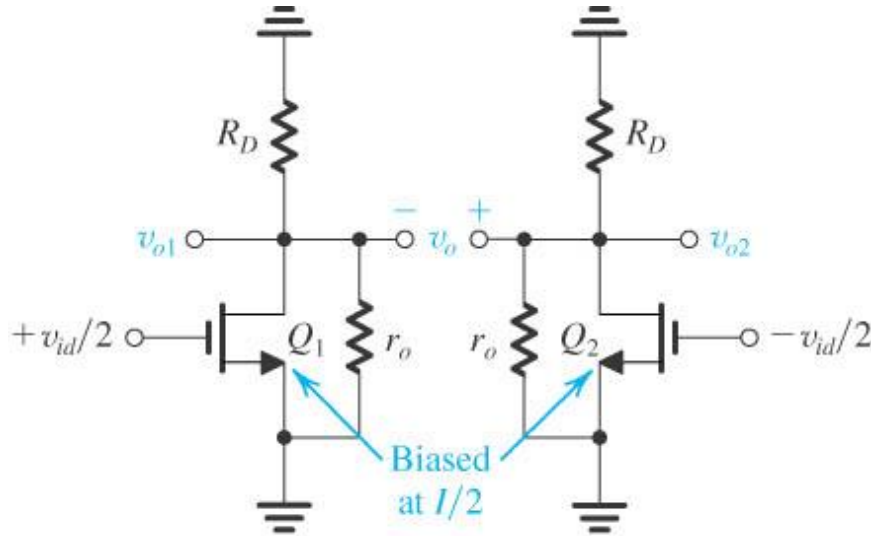
$$\Rightarrow A_d = g_m R_D$$

$$\text{with } g_m = \frac{I}{V_{GS} - V_t} = \frac{I}{V_{OV}}$$



# Ideal MOS differential pair

## effect of MOSFET's $r_o$



$$v_{o1} = -g_m (R_D \parallel r_o) (v_{id} / 2)$$

$$v_{o2} = +g_m (R_D \parallel r_o) (v_{id} / 2)$$

$$\Rightarrow v_o = g_m (R_D \parallel r_o) v_{id}$$

- NMOS output resistance plays role in final gain
- Resistance of current source for biasing plays no role in gain  
 $\Rightarrow$  prove this as an exercise!



# Nonidealities of MOS differential pair

## Common-mode rejection ratio

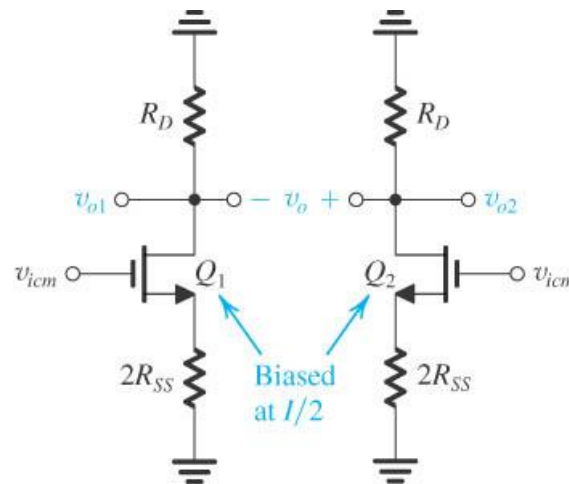
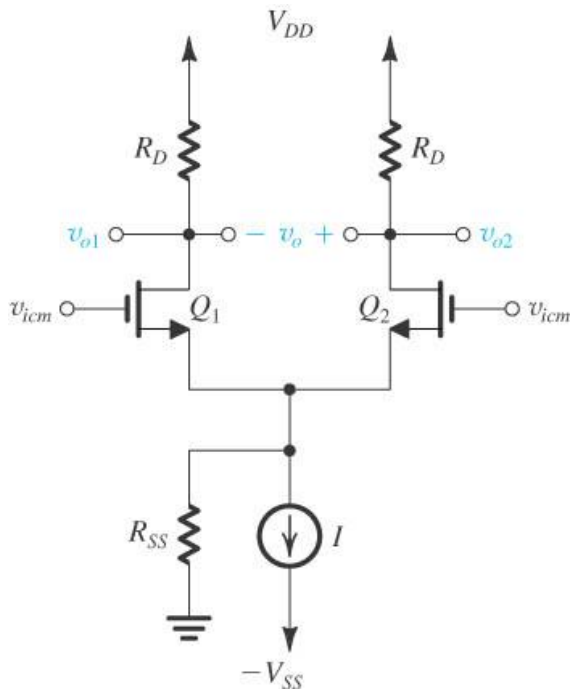
$$\text{CMRR} = \left| \frac{A_d}{A_{cm}} \right|$$

- For an ideal MOS differential pair:

$$A_{cm} = \frac{v_{o2} - v_{o1}}{v_{icm}} = 0$$

$$A_d = \frac{v_{o2} - v_{o1}}{v_{id}} = g_m R_D$$

$$\Rightarrow \text{CMRR} = \infty$$



# Nonidealities of MOS differential pair

## Effect of $R_D$ mismatch

- Q1 has load  $R_D$  and Q2 has load  $R_D + \Delta R_D$

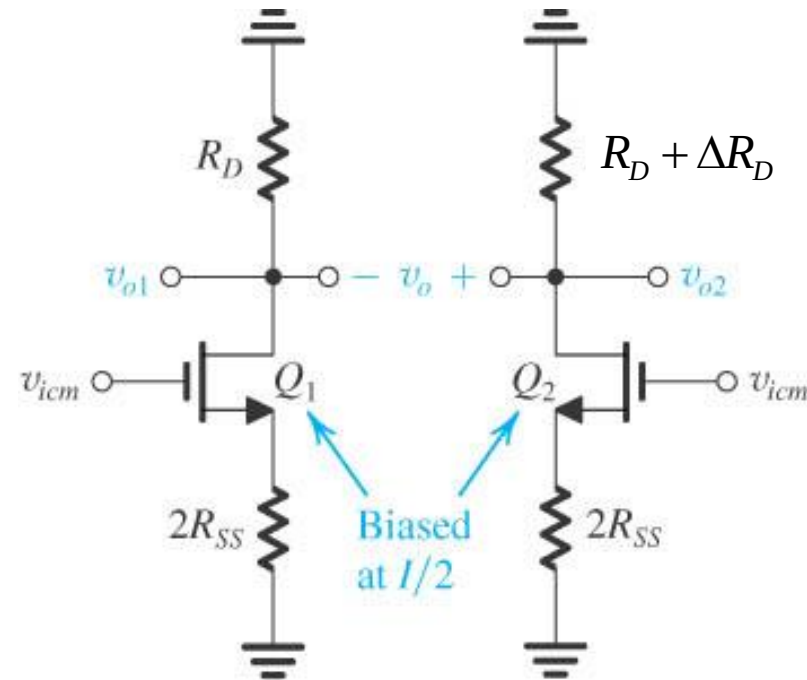
- $$v_{o1} \cong -\frac{R_D}{2R_{SS}} v_{icm}$$

$$v_{o2} \cong -\frac{R_D + \Delta R_D}{2R_{SS}} v_{icm}$$

$$\Rightarrow A_{cm} = -\frac{\Delta R_D}{2R_{SS}} \quad A_d = g_m R_D$$

$$CMRR = \frac{g_m R_D}{\Delta R_D / 2R_{SS}}$$

$$\Rightarrow CMRR = \frac{2g_m R_{SS}}{(\Delta R_D / R_D)}$$



# Nonidealities of MOS differential pair

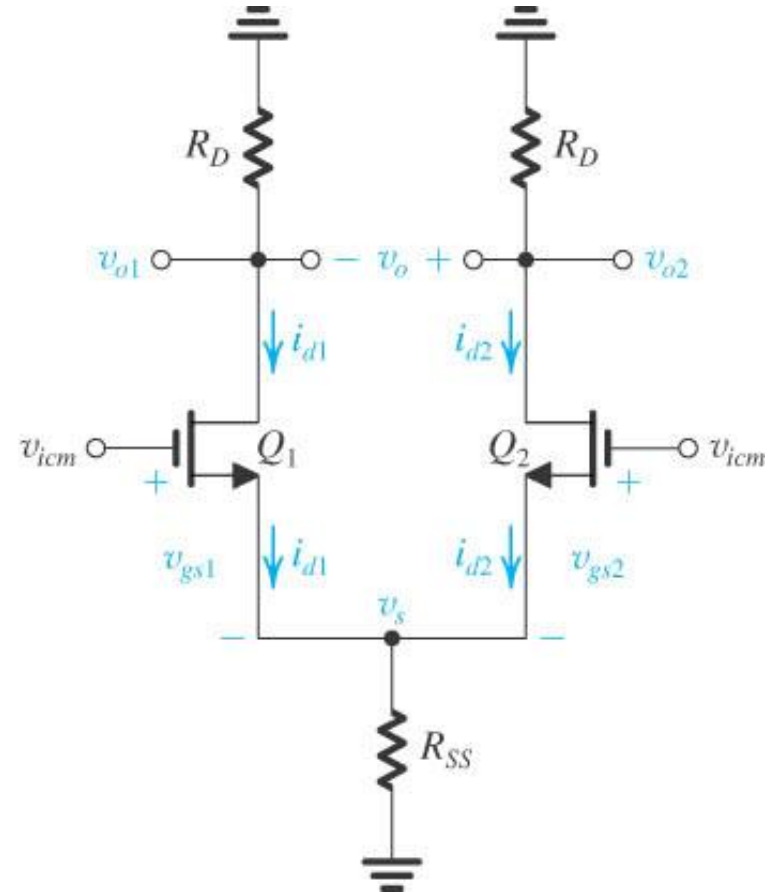
## Effect of $g_m$ mismatch

$$\begin{cases} i_{d1} = g_{m1} v_{gs1} \\ i_{d2} = g_{m2} v_{gs2} \end{cases} \Rightarrow \frac{i_{d1}}{i_{d2}} = \frac{g_{m1}}{g_{m2}}$$

$$i_{d1} + i_{d2} = \frac{v_S}{R_{SS}} \quad R_{SS} \text{ large} \Rightarrow v_S \cong v_{icm}$$

$$\Rightarrow i_{d1} + i_{d2} \cong \frac{v_{icm}}{R_{SS}}$$

$$\Rightarrow \begin{cases} i_{d1} = \frac{g_{m1} v_{icm}}{(g_{m1} + g_{m2}) R_{SS}} \\ i_{d2} = \frac{g_{m2} v_{icm}}{(g_{m1} + g_{m2}) R_{SS}} \end{cases}$$



# Nonidealities of MOS differential pair

## Effect of $g_m$ mismatch

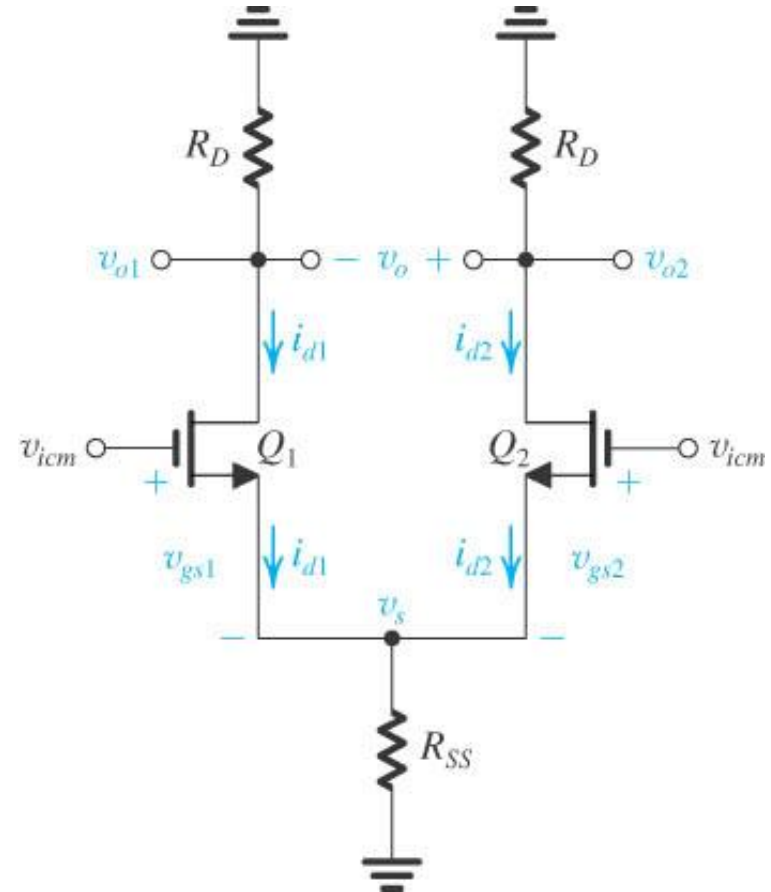
$$\Delta g_m \text{ small} \Rightarrow \begin{cases} g_{m1} + g_{m2} \cong 2g_m \\ g_{m1} - g_{m2} = \Delta g_m \end{cases}$$

$$\Rightarrow \begin{cases} i_{d1} = \frac{g_{m1} v_{icm}}{2g_m R_{SS}} \\ i_{d2} = \frac{g_{m2} v_{icm}}{2g_m R_{SS}} \end{cases}$$

$$\Rightarrow v_o = v_{o2} - v_{o1} = -i_{d2} R_D + i_{d1} R_D$$

$$\Rightarrow v_o = \frac{\Delta g_m R_D}{2g_m R_{SS}} v_{icm}$$

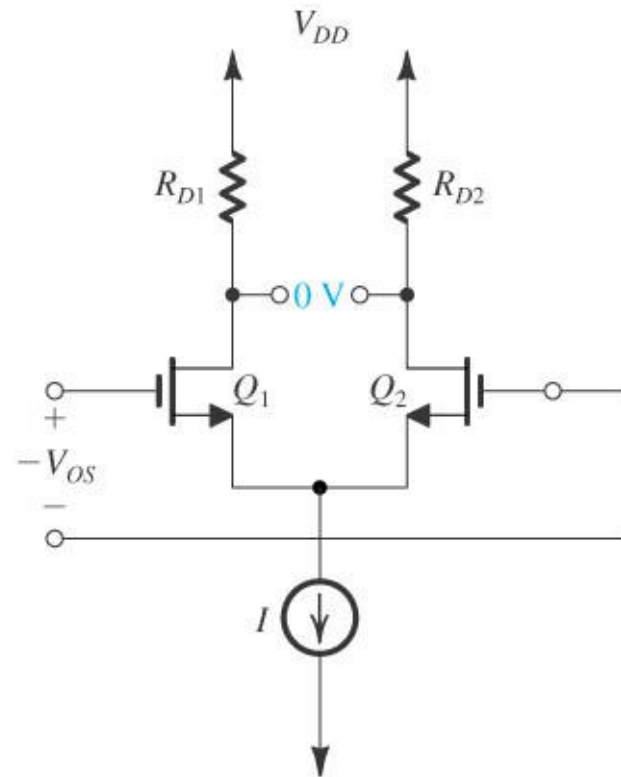
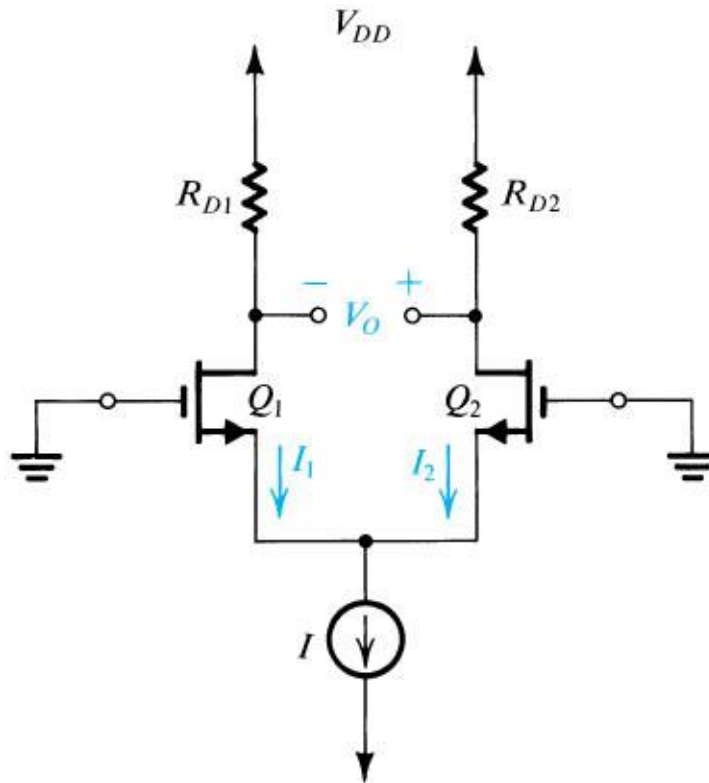
$$\Rightarrow \text{CMRR} = \frac{2g_m R_{SS}}{(\Delta g_m / g_m)}$$



# Nonidealities: input offset voltage

## Mismatches in circuit cause output offset voltage

- Imagine both inputs grounded
  - ⇒ Nonidealities will cause output offset voltage  $V_O$
  - ⇒ We divide  $V_O$  by  $A_d$  to get input offset voltage  $V_{OS}$



# Nonidealities: input offset voltage

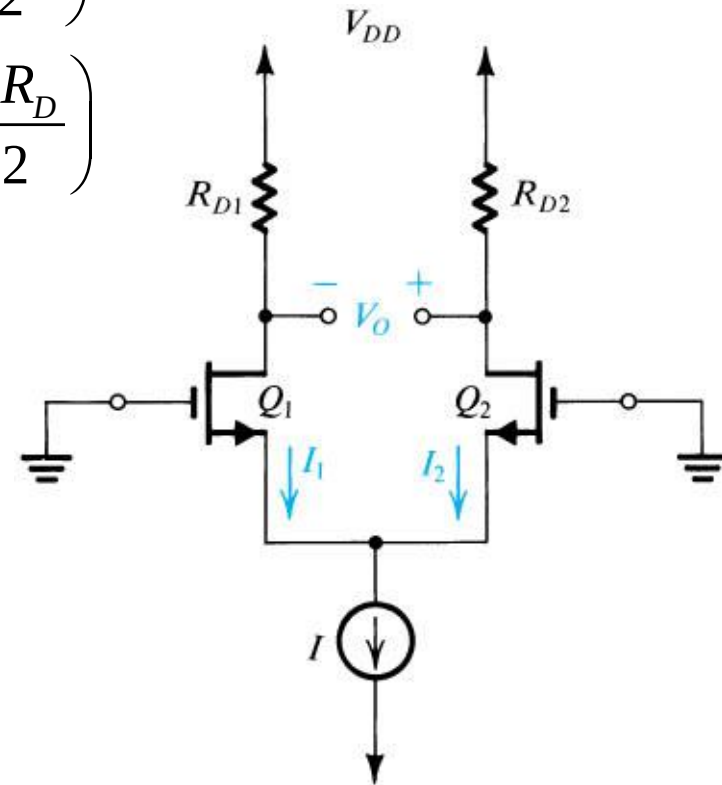
## Effect of $R_D$ mismatch on $V_{OS}$

$$\begin{cases} R_{D1} = R_D + \frac{\Delta R_D}{2} \\ R_{D2} = R_D - \frac{\Delta R_D}{2} \end{cases} \Rightarrow \begin{cases} V_{D1} = V_{DD} - I_D \left( R_D + \frac{\Delta R_D}{2} \right) \\ V_{D2} = V_{DD} - I_D \left( R_D - \frac{\Delta R_D}{2} \right) \end{cases}$$

$$\Rightarrow V_O = V_{D2} - V_{D1} = \frac{I}{2} \Delta R_D$$

$$V_{OS} = V_O / A_d$$

$$\Rightarrow V_{OS} = \left( \frac{V_{OV}}{2} \right) \left( \frac{\Delta R_D}{R_D} \right)$$



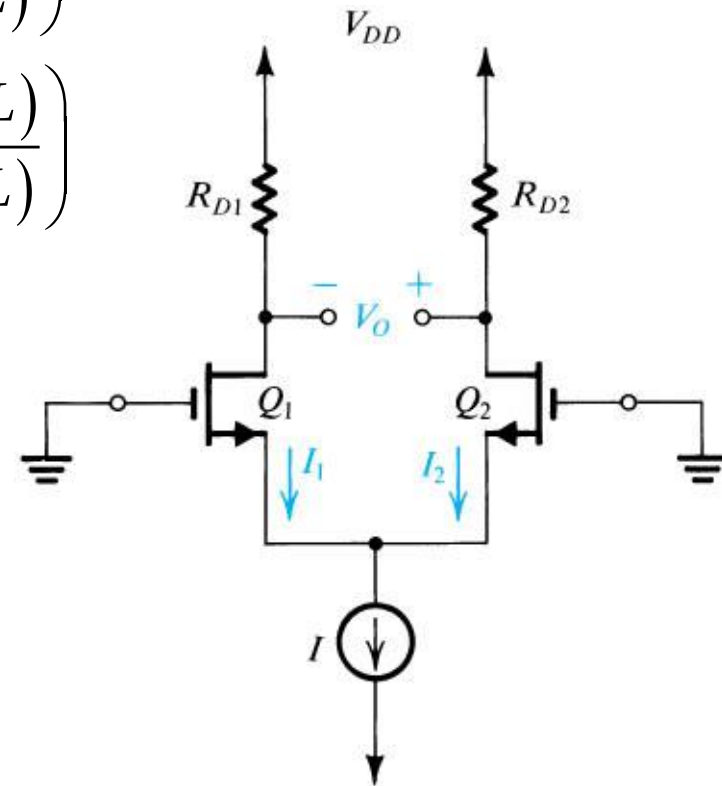
# Nonidealities: input offset voltage

## Effect of (W/L) mismatch on $V_{OS}$

$$\begin{cases} \left(\frac{W}{L}\right)_1 = \frac{W}{L} + \frac{1}{2}\Delta\left(\frac{W}{L}\right) \\ \left(\frac{W}{L}\right)_2 = \frac{W}{L} - \frac{1}{2}\Delta\left(\frac{W}{L}\right) \end{cases} \Rightarrow \begin{cases} I_1 = \frac{I}{2} + \frac{I}{2}\left(\frac{\Delta(W/L)}{2(W/L)}\right) \\ I_2 = \frac{I}{2} - \frac{I}{2}\left(\frac{\Delta(W/L)}{2(W/L)}\right) \end{cases}$$

$$\Rightarrow V_O = R_D (I_1 - I_2)$$

$$\Rightarrow V_{OS} = \left(\frac{V_{OV}}{2}\right)\left(\frac{\Delta(W/L)}{(W/L)}\right)$$



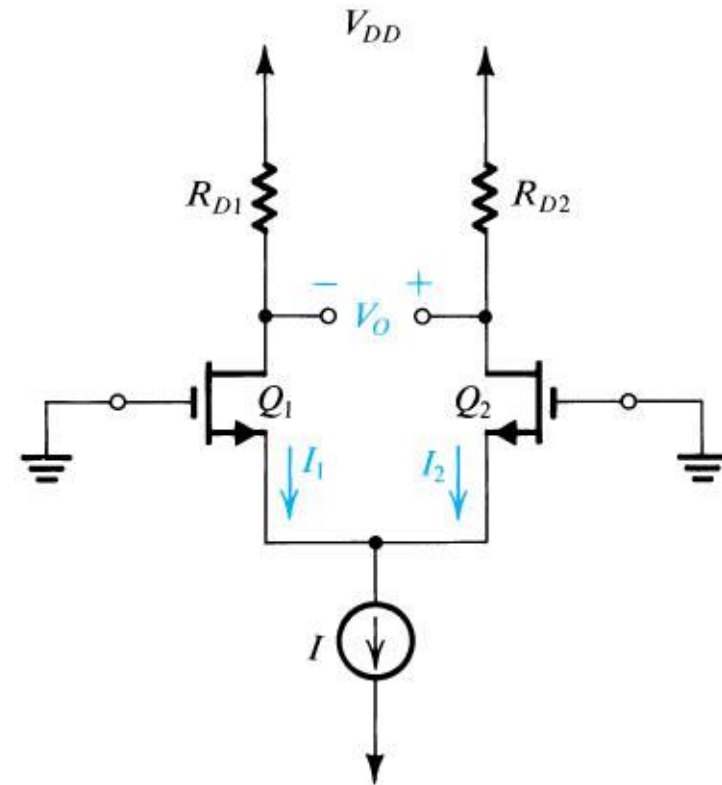
# Nonidealities: input offset voltage

## Effect of $V_t$ mismatch on $V_{os}$

$$\begin{cases} V_{t1} = V_t + \frac{\Delta V_t}{2} \\ V_{t2} = V_t - \frac{\Delta V_t}{2} \end{cases}$$

$$\begin{aligned} I_1 &= \frac{1}{2} k'_n \frac{W}{L} \left( V_{GS} - V_t - \frac{\Delta V_t}{2} \right)^2 \\ &= \frac{1}{2} k'_n \frac{W}{L} (V_{GS} - V_t)^2 \left[ 1 - \frac{\Delta V_t}{2(V_{GS} - V_t)} \right]^2 \\ &\approx \frac{1}{2} k'_n \frac{W}{L} (V_{GS} - V_t)^2 \left( 1 - \frac{\Delta V_t}{(V_{GS} - V_t)} \right) \end{aligned}$$

$$I_2 \approx \frac{1}{2} k'_n \frac{W}{L} (V_{GS} - V_t)^2 \left( 1 + \frac{\Delta V_t}{(V_{GS} - V_t)} \right)$$





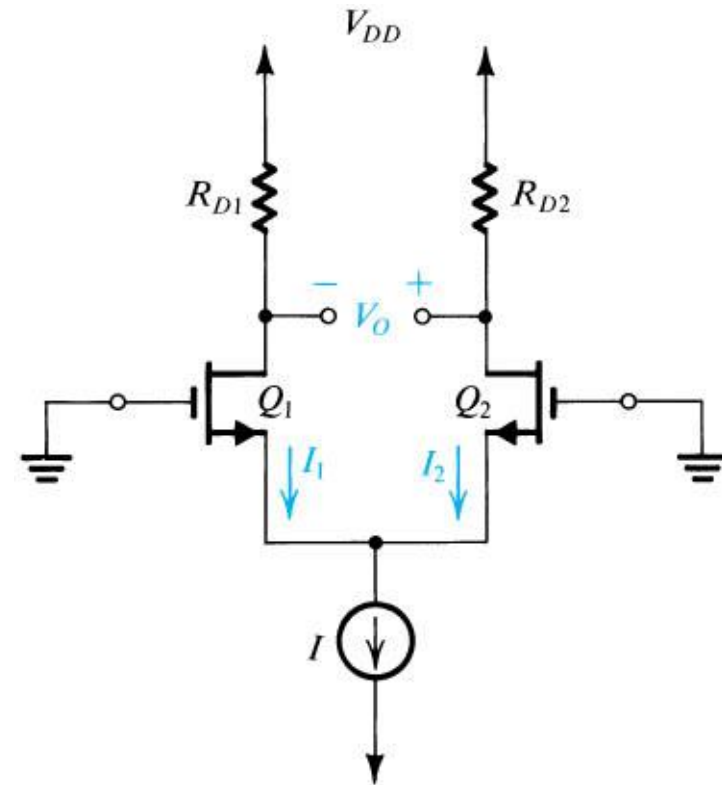
# Nonidealities: input offset voltage

## Effect of $V_t$ mismatch on $V_{OS}$

$$\Rightarrow V_O = R_D (I_1 - I_2)$$

$$\Rightarrow V_O = -R_D I \frac{\Delta V_t}{V_{OV}}$$

$$\Rightarrow V_{OS} = \Delta V_t$$



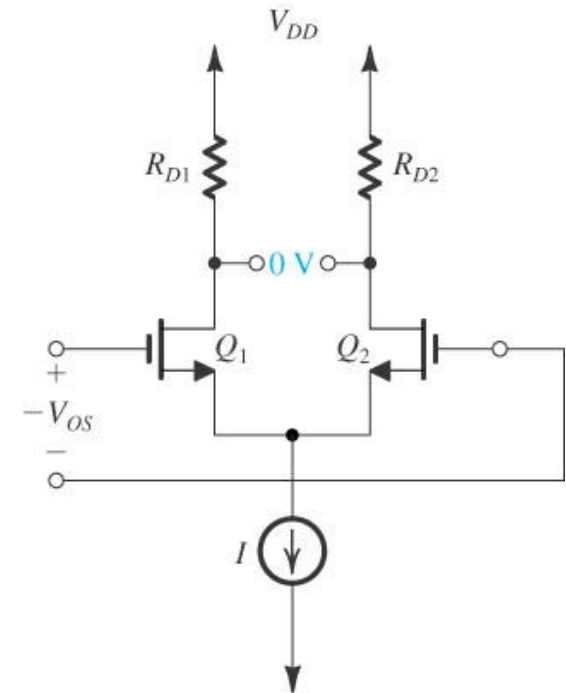
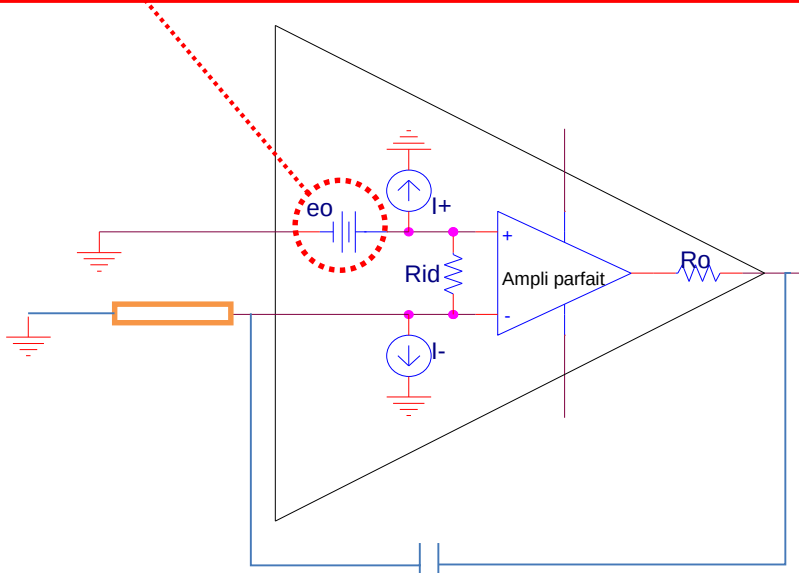
# Nonidealities: input offset voltage

## Combined effect of mismatches on $V_{OS}$

- Circuit is linear and three sources of offset voltage are uncorrelated

⇒ superposition can be applied

$$V_{OS} = \left( \frac{V_{OV}}{2} \right) \left( \frac{\Delta R_D}{R_D} \right) + \left( \frac{V_{OV}}{2} \right) \left( \frac{\Delta(W/L)}{(W/L)} \right) + \Delta V_t$$

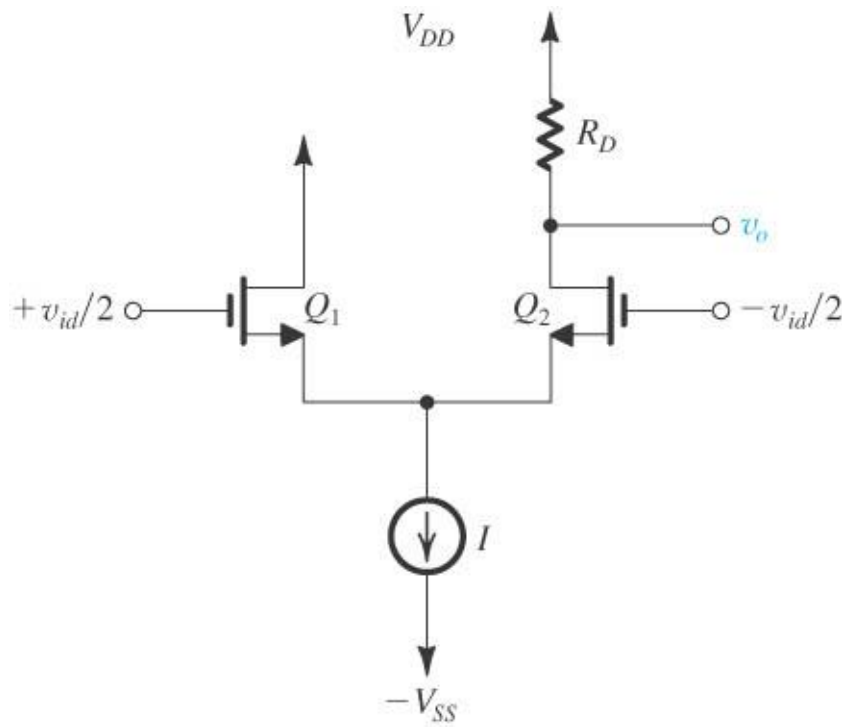


⇒ Input offset voltage in op-amp is due to asymmetries in NMOS differential pair

# Differential amplifier with active load

## Differential to single-ended conversion

- Multistage circuits usually use single-ended voltages
  - Voltage w.r.t. ground of circuit
- ⇒ Differential output cannot always be used



$$v_o = g_m R_D \frac{v_{id}}{2}$$

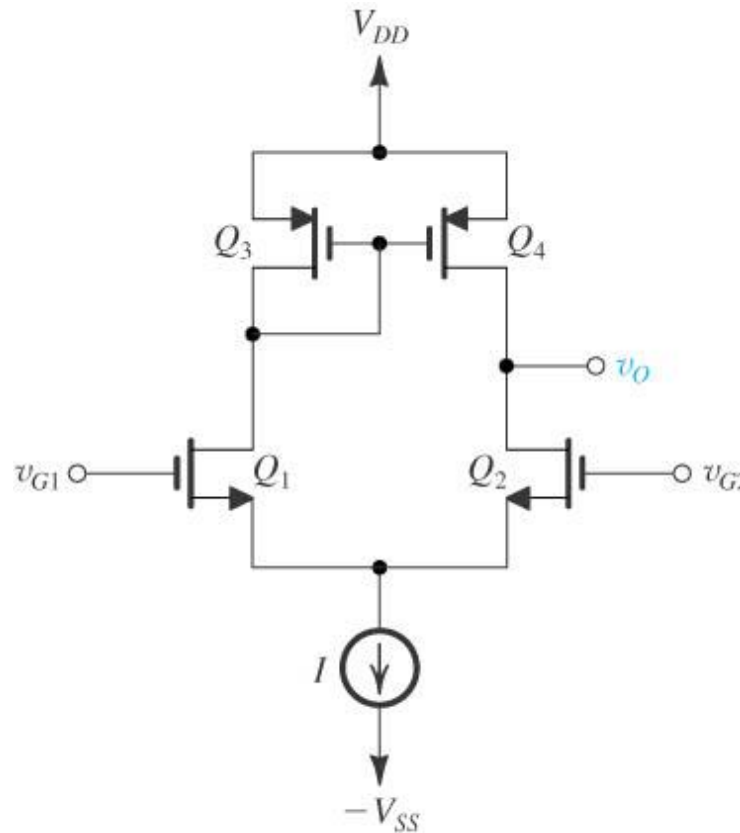
⇒ we lose a factor 2 (6dB)

⇒ « wasting » the signal

# Differential amplifier with active load

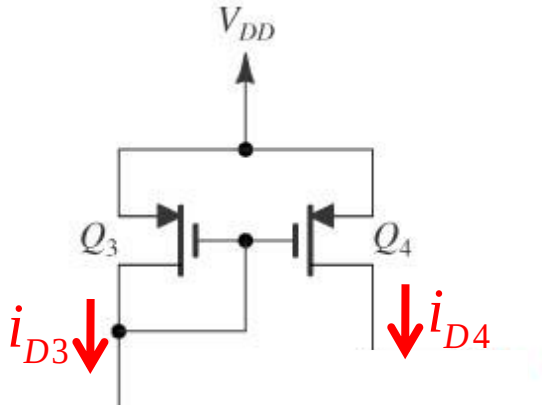
## Active-loaded differential MOS pair

- differential pair  $Q_1$  and  $Q_2$  loaded in a PMOS current mirror by  $Q_3$  and  $Q_4$



# Reminder

## PMOS current mirror



- Current through Q3 determines current going through Q4

$$\text{Q3 saturation} \Leftrightarrow v_{SD3} \geq v_{SG3} - V_t$$

$$\Leftrightarrow v_{D3} \geq v_{G3} - V_t$$

$$\Leftrightarrow 0 \geq -V_t$$

$\Rightarrow$  Q3 always in saturation region

$$i_{D3} = \frac{1}{2} k'_n \frac{W}{L} (v_{SG3} - V_t)^2$$

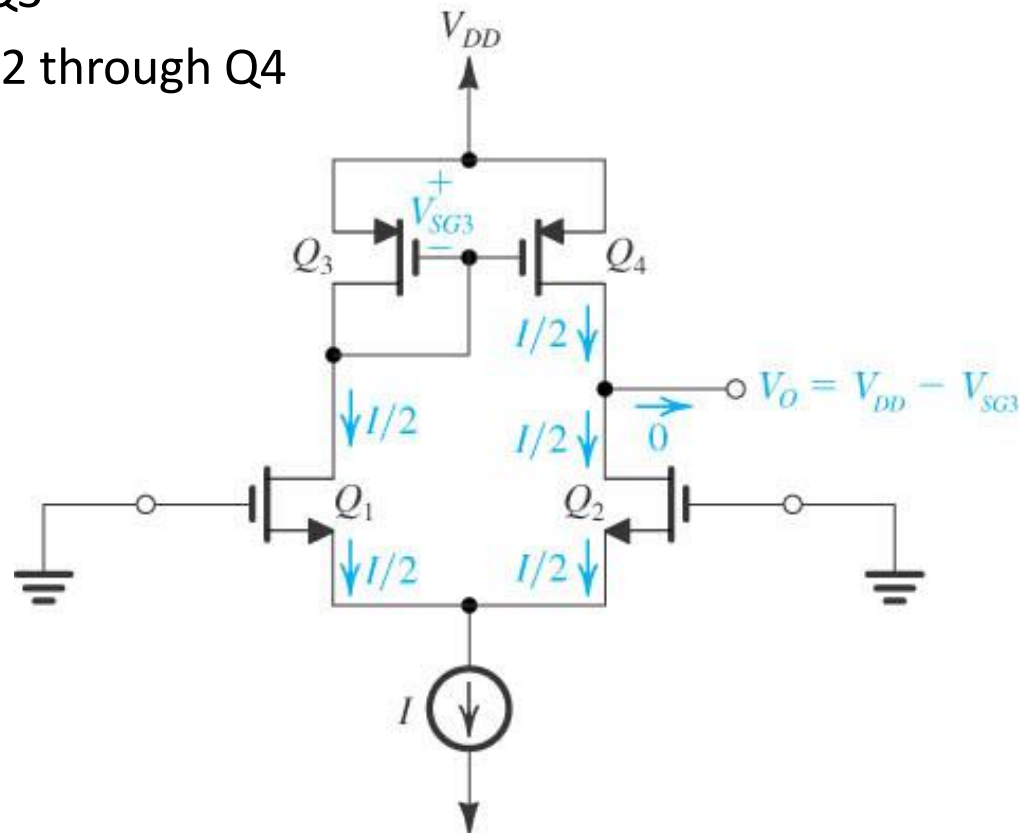
$$= \frac{1}{2} k'_n \frac{W}{L} (v_{SG4} - V_t)^2$$

$$\Rightarrow i_{D3} = i_{D4}$$

# Differential amplifier with active load

## Active-loaded differential MOS pair

- Q1 and Q2 have same common-mode voltage
  - ⇒ current  $I/2$  flowing through Q1 and Q2
  - ⇒ current  $I/2$  flowing through Q3
  - ⇒ mirror current  $\Rightarrow$  current  $I/2$  through Q4
  - ⇒ output current is zero
  - ⇒ equilibrium state

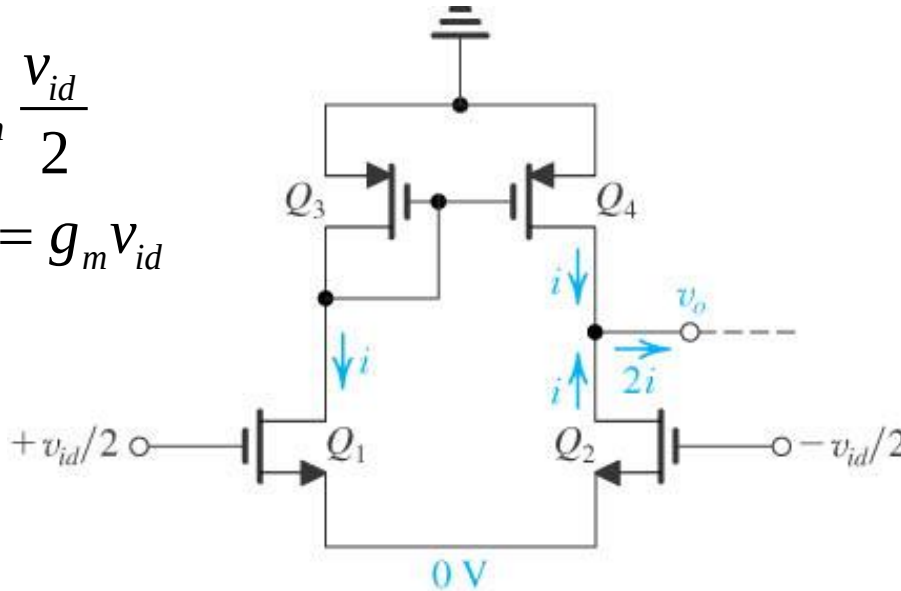


# Differential amplifier with active load

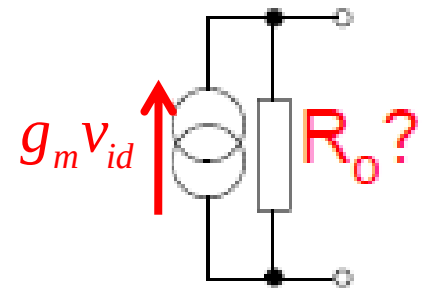
## Active-loaded differential MOS pair

- Differential voltage causes
  - increase of  $i$  in  $Q_1$  and decrease of  $i$  in  $Q_2$
  - current  $i$  going through  $Q_3$
  - $\Rightarrow$  current  $i$  going through  $Q_4$
  - $\Rightarrow$  output current of  $2i$

$$i = g_m \frac{v_{id}}{2}$$
$$\Rightarrow 2i = g_m v_{id}$$



**Norton equivalent circuit:**



$$v_o = R_o g_m v_{id}$$

$$\Rightarrow A_d = R_o g_m$$

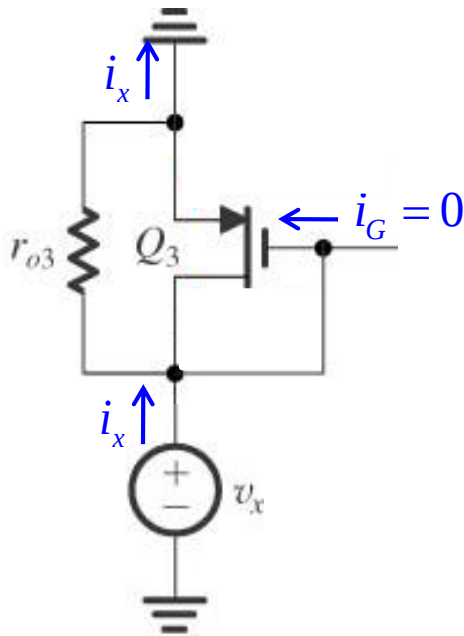
## Output resistance $R_o$

- [illegible]



# Differential amplifier with active load

Output resistance  $R_o$  – step 1: compute  $R_{o3}$



$$v_x = r_{o3} (i_x + g_{m3} v_{SG3})$$

$$\Downarrow v_{SG3} = 0 - v_x$$

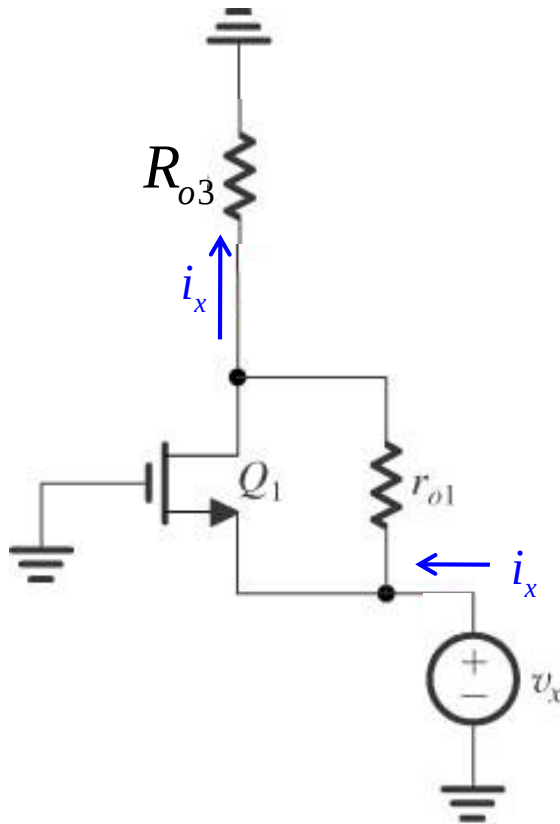
$$v_x = r_{o3} (i_x - g_{m3} v_x)$$

$$\Rightarrow R_{o3} = \frac{v_x}{i_x} = \frac{r_{o3}}{1 + r_{o3} g_{m3}}$$

$$\Rightarrow R_{o3} \approx \frac{1}{g_{m3}}$$

# Differential amplifier with active load

Output resistance  $R_o$  – step 2: compute  $R_{o1}$



$$v_x = R_{o3} i_x + r_{o1} (i_x + g_{m1} v_{GS1})$$

$$\Downarrow \quad v_{SG3} = 0 - v_x$$

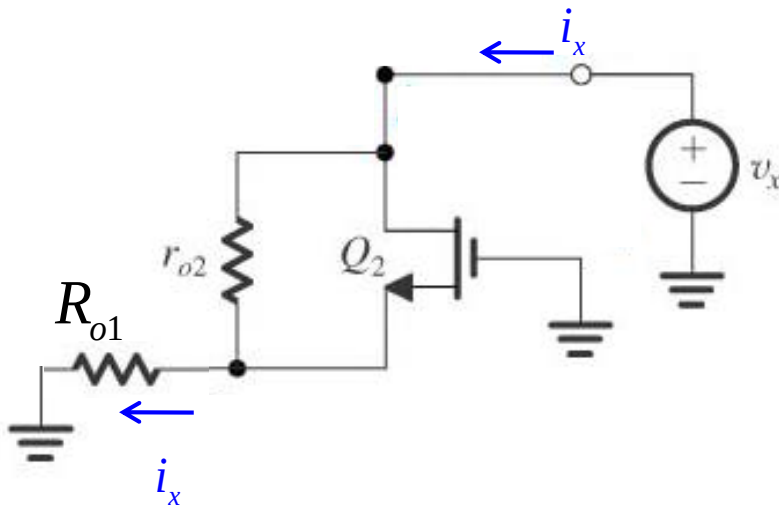
$$v_x = R_{o3} i_x + r_{o1} (i_x - g_{m1} v_x)$$

$$\Rightarrow R_{o1} = \frac{v_x}{i_x} = \frac{R_{o3} + r_{o1}}{1 + r_{o1} g_{m1}} = \frac{1/g_{m3} + r_{o1}}{1 + r_{o1} g_{m1}}$$

$$\Rightarrow R_{o1} \approx \frac{1}{g_{m1}}$$

# Differential amplifier with active load

## Output resistance $R_o$ – step 3: compute $R_{o2}$



$$v_x = R_{o1}i_x + r_{o2}(i_x - g_{m2}v_{GS2})$$

$$\Downarrow v_{GS2} = 0 - v_S = 0 - R_{o1}i_x$$

$$v_x = R_{o1}i_x + r_{o2}(i_x + g_{m2}R_{o1}i_x)$$

$$\Rightarrow R_{o2} = \frac{v_x}{i_x} = R_{o1} + r_{o2} + r_{o2}g_{m2}R_{o1}$$

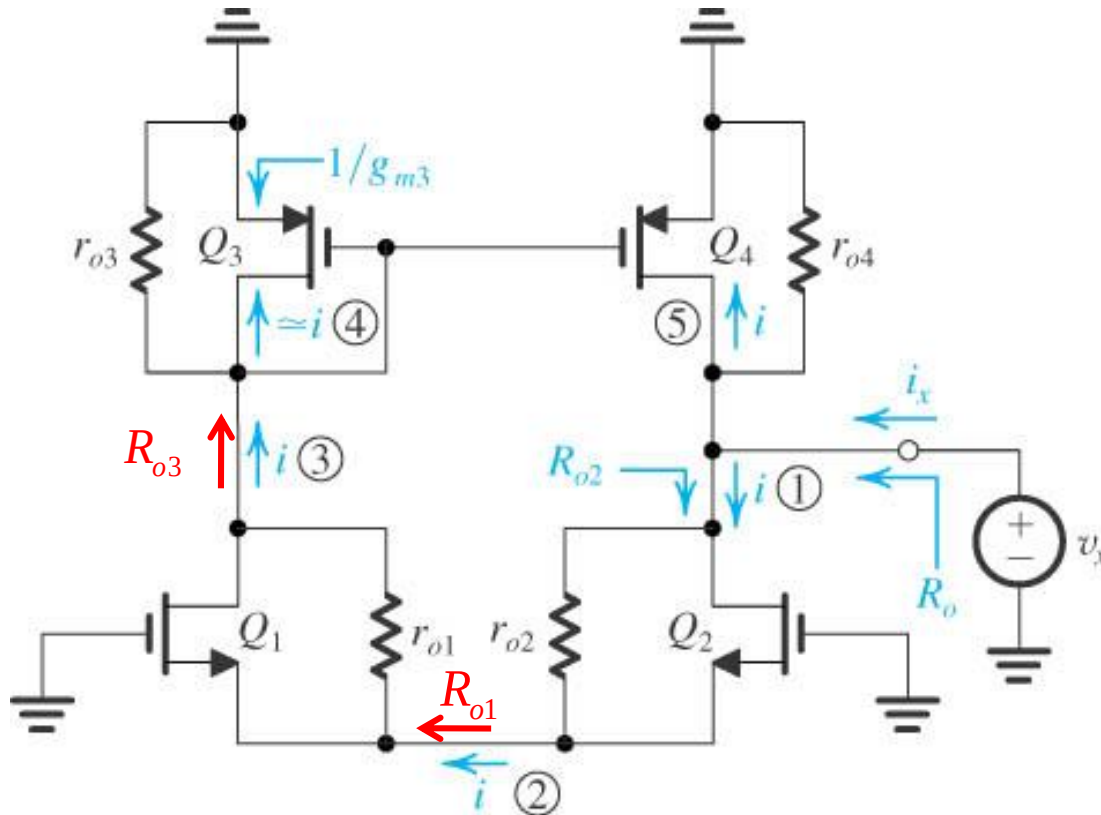
$$\Rightarrow R_{o2} \approx r_{o2} + (1 + g_{m2}r_{o2})\frac{1}{g_{m1}}$$

$$\Downarrow g_{m1} = g_{m2}$$

$$\Rightarrow R_{o2} \approx 2r_{o2}$$

# Differential amplifier with active load

## Output resistance $R_o$ – step 4: compute $R_o$



$$i_x = i + i + \frac{v_x}{r_{o4}}$$

$$\Downarrow i = v_x / R_{o2}$$

$$i_x = 2 \frac{v_x}{R_{o2}} + \frac{v_x}{r_{o4}}$$

$$\Rightarrow i_x \approx \frac{v_x}{r_{o2}} + \frac{v_x}{r_{o4}}$$

$$\Rightarrow R_o = r_{o2} \parallel r_{o4}$$

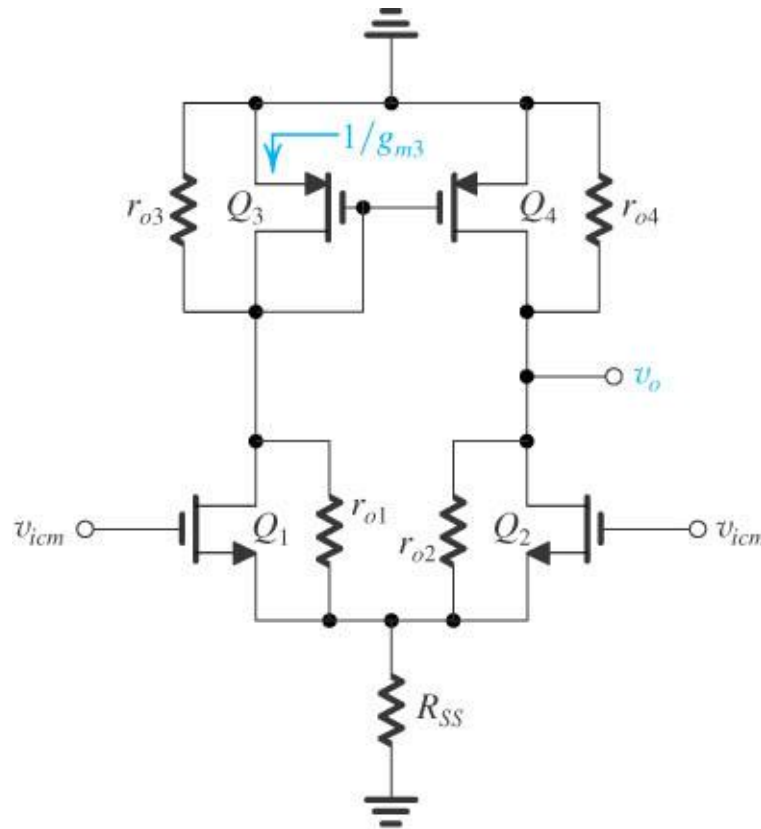
$$\Rightarrow A_d = R_o g_m$$

$$\Rightarrow A_d = (r_{o2} \parallel r_{o4}) g_m$$

# Differential amplifier with active load

## Common-mode gain and CMRR

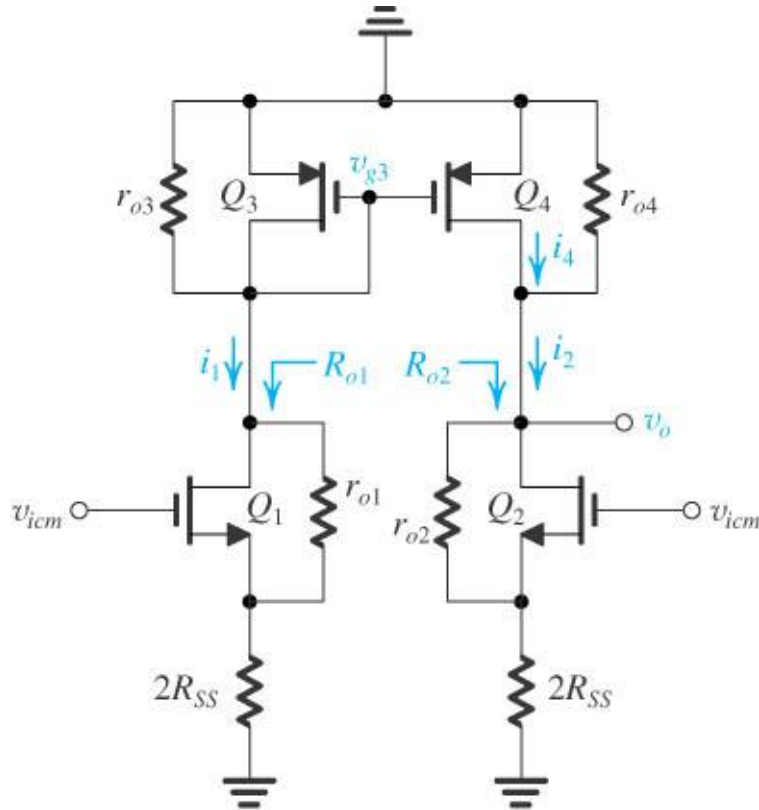
- Effect of current source resistance  $R_{SS}$



Equal current in both branches  
 $\Rightarrow$  Split  $R_{SS}$  into  $2R_{SS}$  on each branch

# Differential amplifier with active load

## Common-mode gain and CMRR



$$R_{SS} \text{ large} \Rightarrow i_1 = i_2 \approx \frac{v_{icm}}{2R_{SS}}$$

$$R_{o1} = R_{o2} = r_o + 2R_{SS} + 2g_m r_o R_{SS}$$

$$v_{g3} = -i_1 \left( \frac{1}{g_{m3}} \parallel r_{o3} \right)$$

$$\Rightarrow i_4 = -g_{m4} v_{g3}$$

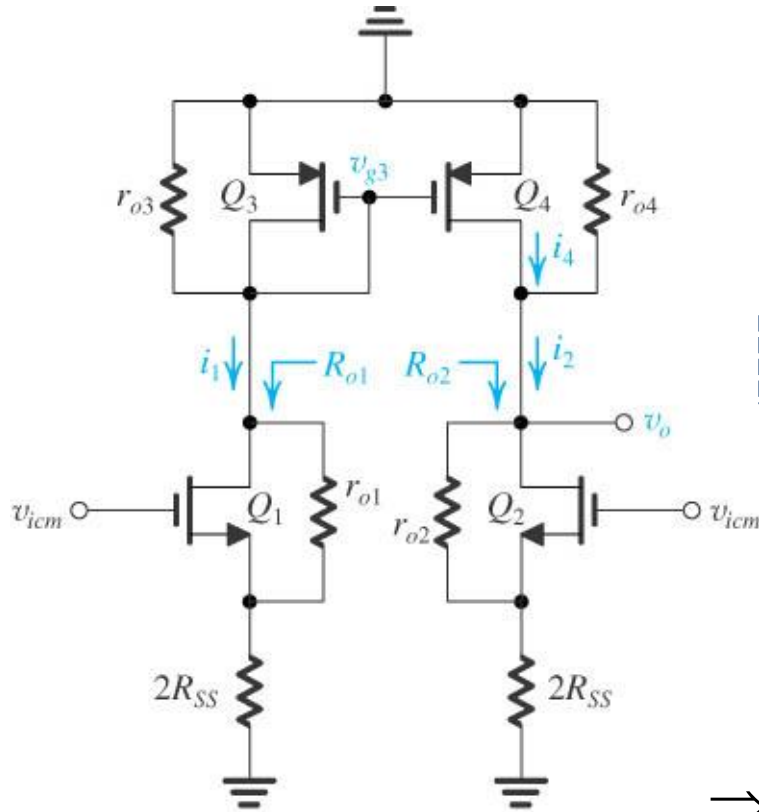
$$= i_1 g_{m4} \left( \frac{1}{g_{m3}} \parallel r_{o3} \right)$$

$$\Rightarrow v_o = r_{o4} (i_4 - i_2)$$

$$= r_{o4} \left[ i_1 g_{m4} \left( \frac{1}{g_{m3}} \parallel r_{o3} \right) - i_2 \right]$$

# Differential amplifier with active load

## Common-mode gain and CMRR



$$R_{SS} \text{ large} \Rightarrow i_1 = i_2 \approx \frac{v_{icm}}{2R_{SS}}$$

$$v_o = r_{o4} \left[ g_{m4} \left( \frac{1}{g_{m3}} \parallel r_{o3} \right) - 1 \right] \frac{v_{icm}}{2R_{SS}}$$

$$g_{m3} = g_{m4}$$

$$\Rightarrow A_{cm} = \frac{v_o}{v_{icm}} = - \frac{1}{2R_{SS}} \frac{r_{o4}}{1 + g_{m3}r_{o3}}$$

$$\Rightarrow A_{cm} \approx - \frac{1}{2g_{m3}R_{SS}}$$

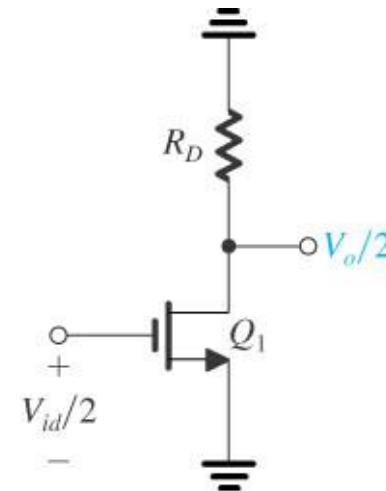
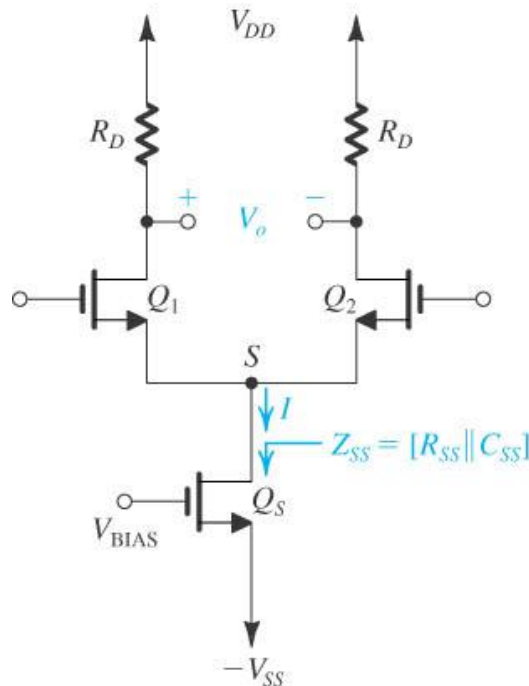
$$\Rightarrow \text{CMRR} = \left| \frac{A_d}{A_{cm}} \right| = [g_m (r_{o2} \parallel r_{o4})] [2g_{m3}R_{SS}]$$

$$\text{if } g_{m3} = g_m \text{ and } r_{o2} = r_{o4} = r_o \Rightarrow \text{CMRR} = (g_m r_o)(g_m R_{SS})$$

# Frequency response of differential amplifier

## Resistively-loaded MOS amplifier

- Capacitance  $C_{SS}$  contains:
  - $C_{db}$  and  $C_{gd}$  of  $Q_S$
  - $C_{sb}$  of  $Q_1$  and  $C_{sb}$  of  $Q_2$
- No change for differential input

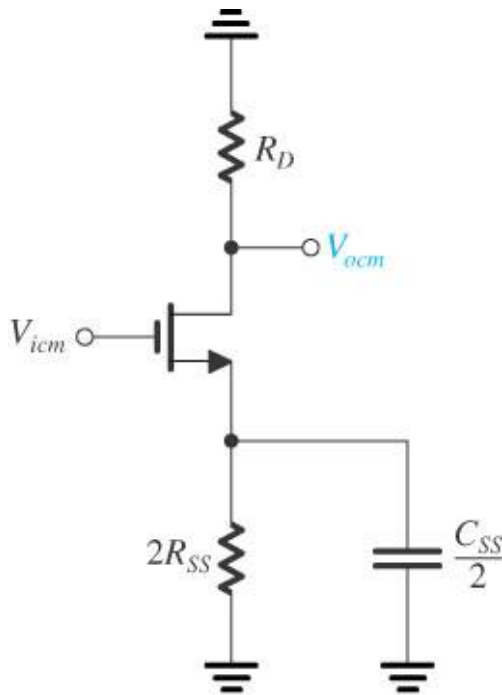




# Frequency response of differential amplifier

## Resistively-loaded MOS amplifier

- Common-mode half-circuit:
  - Frequency of  $C_{SS}/2$  and  $2R_{SS}$  is much lower than frequency due to other capacitances
  - $\Rightarrow$  Other capacitances ( $C_{gs}$ ,  $C_{gd}$ ,  $C_{db}$ ) of transistors ignored



$$\text{Reminder: } A_{cm} = -\frac{\Delta R_D}{2R_{SS}}$$

$$\Rightarrow A_{cm} = -\frac{\Delta R_D}{2Z_{SS}} = -\frac{\Delta R_D}{2} Y_{SS}$$

$$\Rightarrow A_{cm} = -\frac{\Delta R_D}{2} \left( \frac{1}{R_{SS}} + j\omega C_{SS} \right)$$

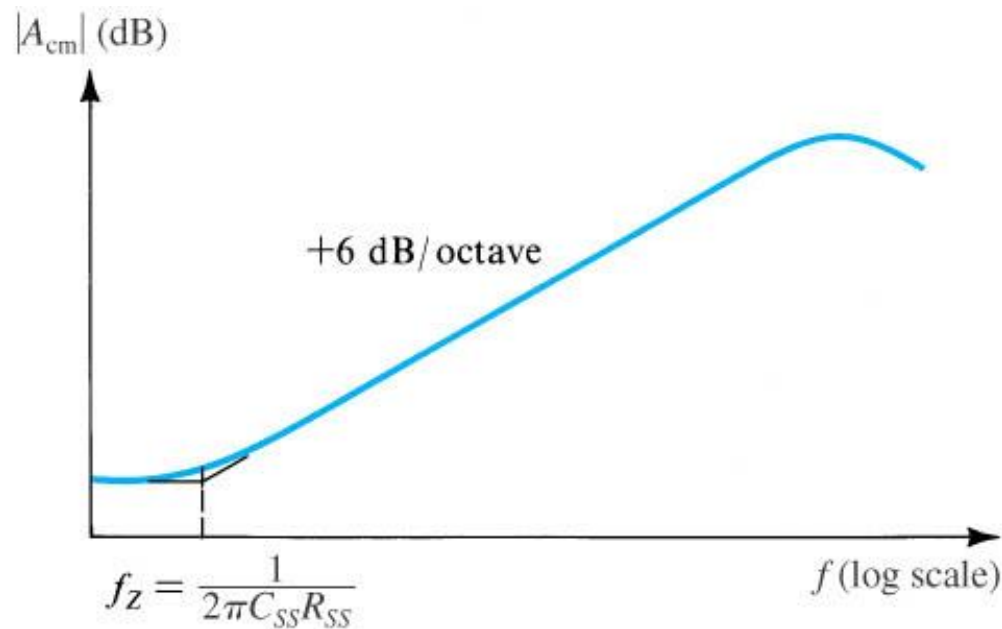
$$\Rightarrow A_{cm} = -\frac{\Delta R_D}{2R_{SS}} (1 + j\omega R_{SS} C_{SS})$$

$$\Rightarrow f_z = \frac{1}{2\pi R_{SS} C_{SS}}$$

# Frequency response of differential amplifier

## Resistively-loaded MOS amplifier

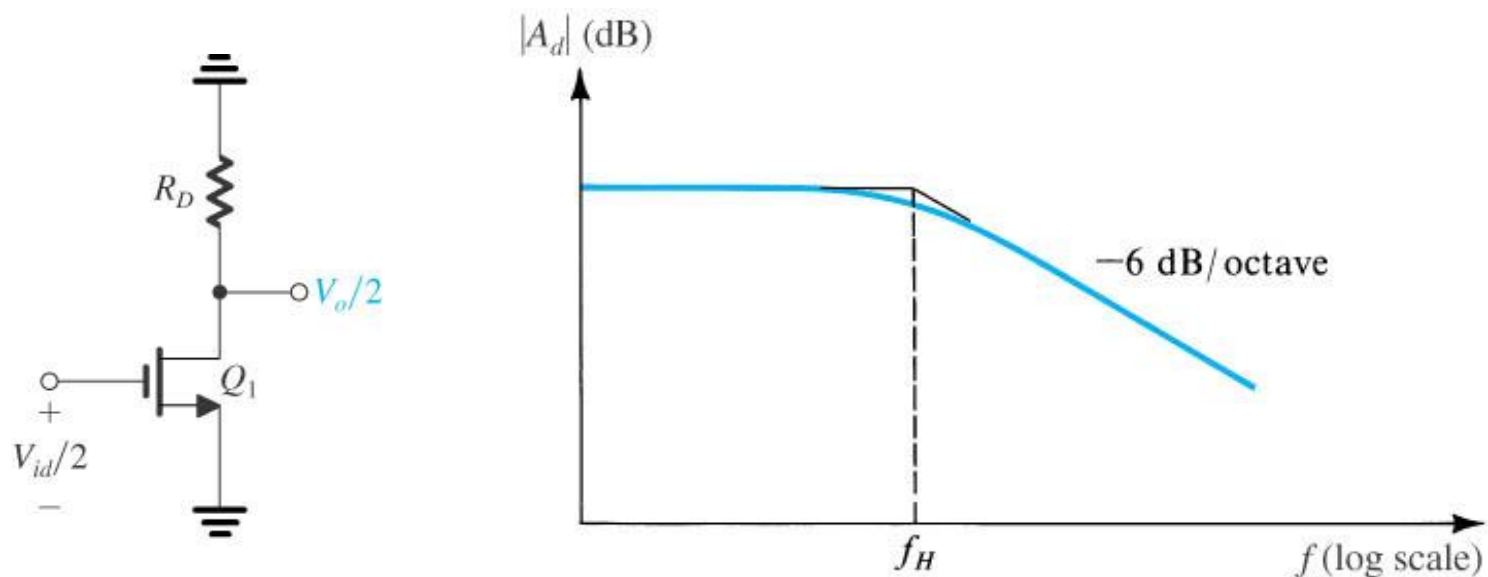
- $A_{cm}$  increases with frequency
  - $\Rightarrow$  At some point, other capacitances of transistor kick in
  - $\Rightarrow A_{cm}$  starts to decrease



# Frequency response of differential amplifier

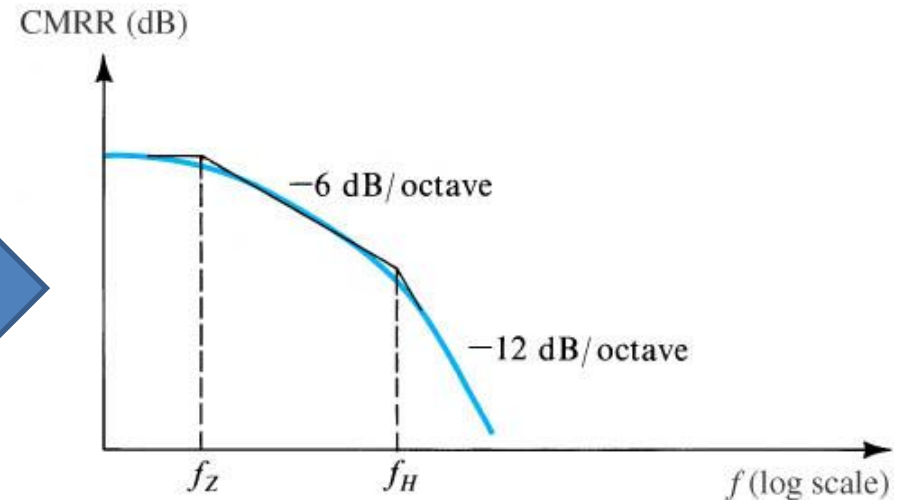
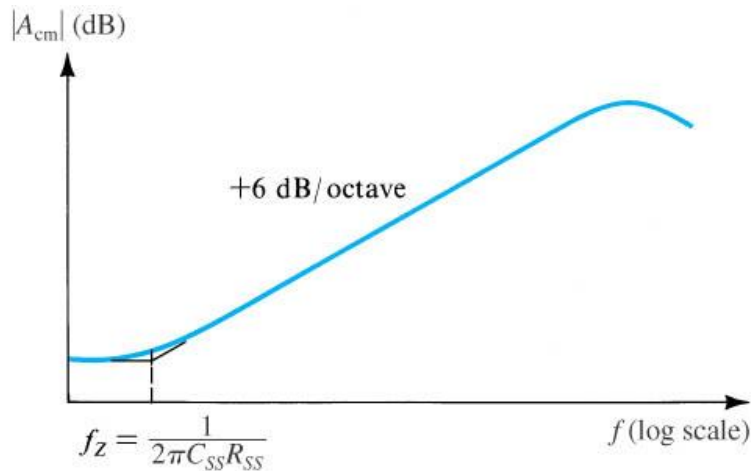
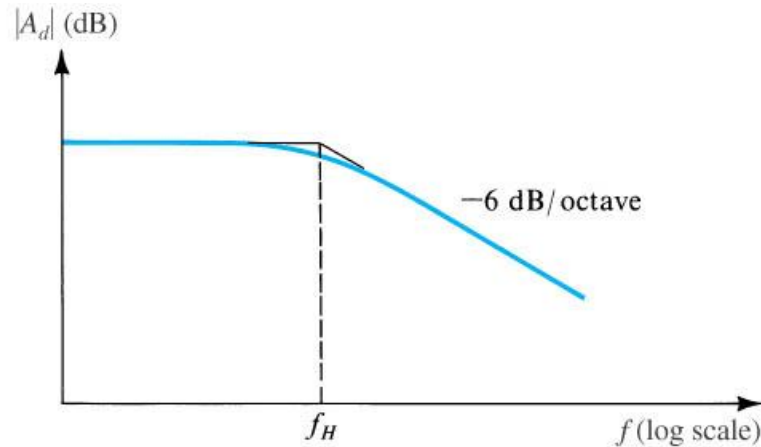
## Resistively-loaded MOS amplifier

- Differential signal
  - ⇒ common-source amplifier
  - ⇒ See frequency response of CS amplifier in previous chapter



# Frequency response of differential amplifier

## Resistively-loaded MOS amplifier



$\Rightarrow$  CMRR decreases (strongly) with frequency, starting at  $f_z$

# Frequency response of differential amplifier

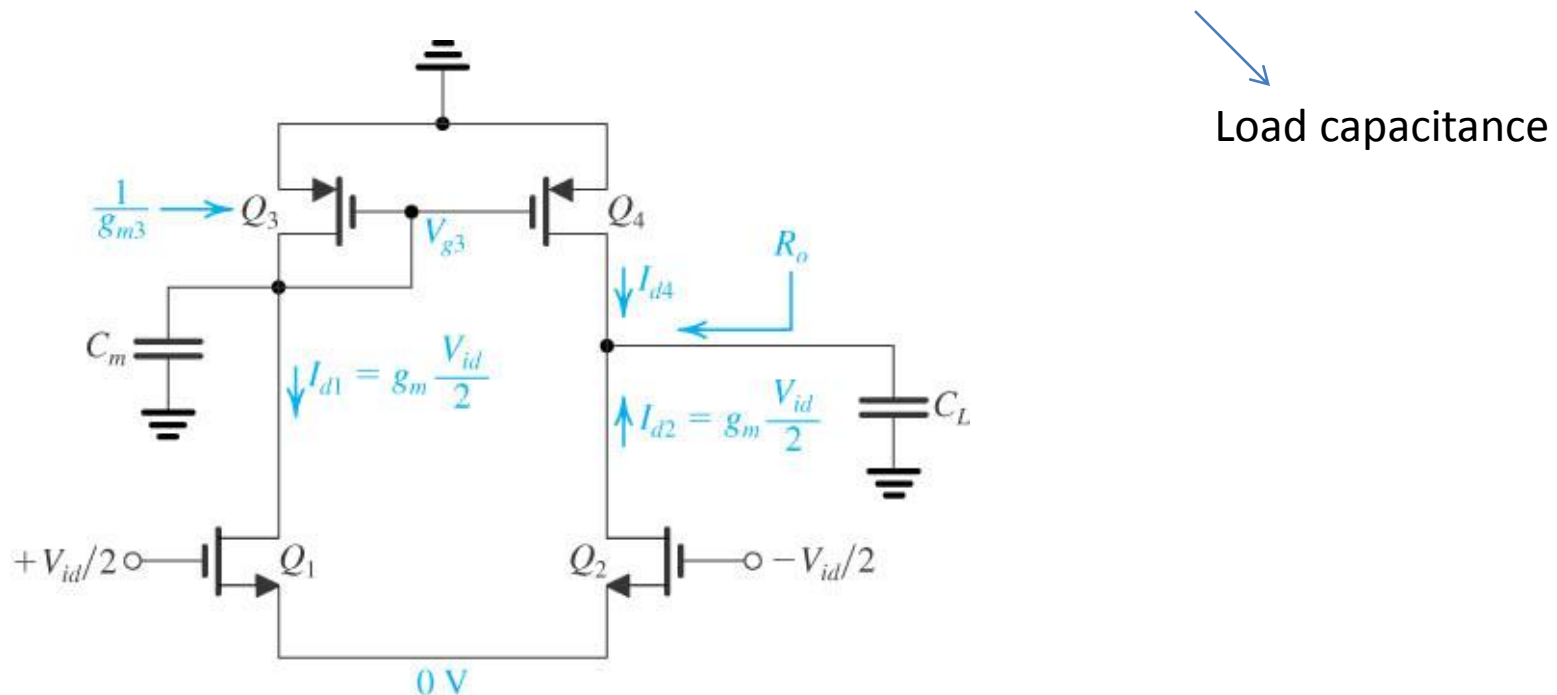
## Active-loaded MOS amplifier

- Capacitance  $C_m$  formed by several capacitances:

$$C_m = C_{gd1} + C_{db1} + C_{db3} + C_{gs3} + C_{gs4}$$

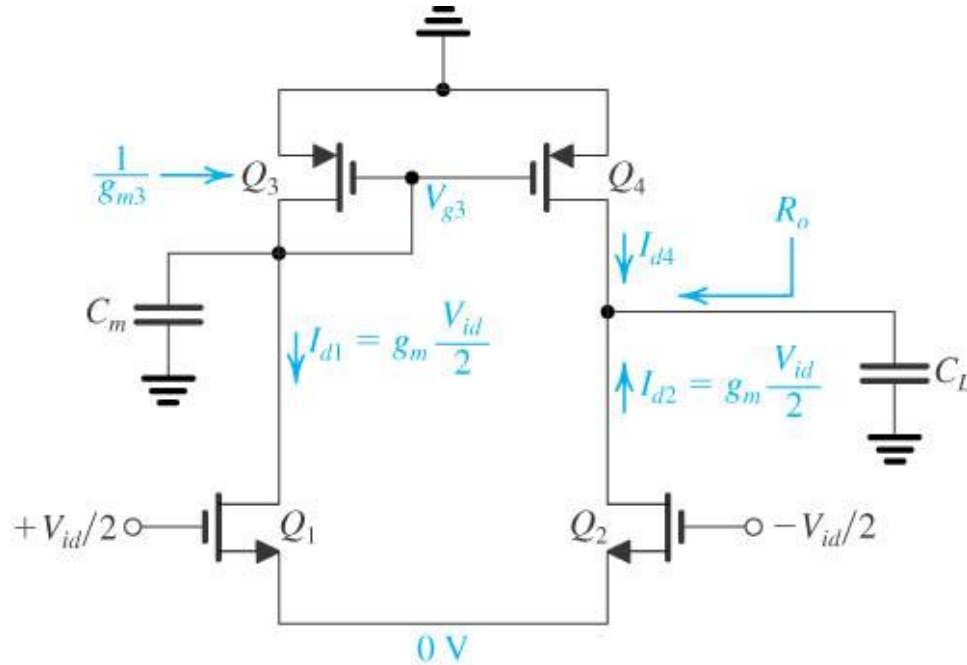
- Capacitance  $C_L$  formed by several capacitances:

$$C_L = C_{gd2} + C_{db2} + C_{gd4} + C_{db4} + C_x$$



# Frequency response of differential amplifier

## Active-loaded MOS amplifier



$$I_{d1} = g_m V_{id} / 2$$

$$V_{g3} = \left( \frac{1}{g_{m3}} \parallel C_m \right) I_{d1}$$

$$\Rightarrow V_{g3} = - \frac{g_m}{g_{m3} + j\omega C_m} \frac{V_{id}}{2}$$

$$I_{d4} = -g_{m4} V_{g3}$$

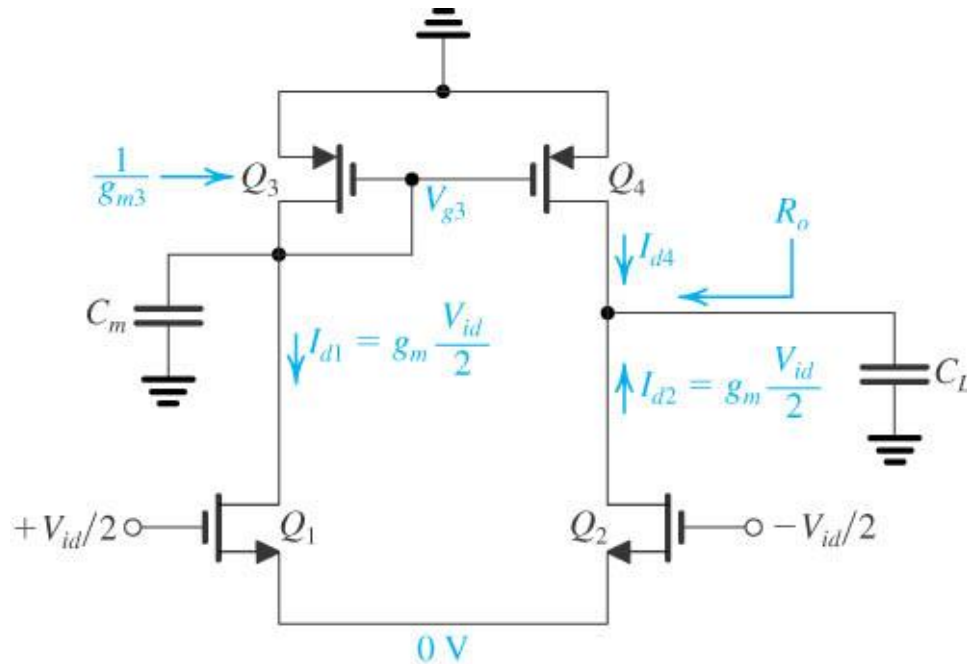
$$\boxed{g_{m3} = g_{m4}} \Rightarrow I_{d4} = \frac{g_m V_{id} / 2}{1 + j\omega C_m / g_{m3}}$$

$$I_o = I_{d4} + I_{d2}$$

$$\Rightarrow I_o = \frac{g_m V_{id} / 2}{1 + j\omega C_m / g_{m3}} + g_m (V_{id} / 2)$$

# Frequency response of differential amplifier

## Active-loaded MOS amplifier



$$I_o = \frac{g_m V_{id} / 2}{1 + j\omega C_m / g_{m3}} + g_m (V_{id} / 2)$$

$$V_o = I_o (R_o \parallel C_L)$$

$$\Rightarrow V_o = g_m R_o \left( \frac{V_{id}}{2} \right) \left[ 1 + \frac{1}{1 + \frac{j\omega C_m}{g_{m3}}} \right] \frac{1}{1 + j\omega C_L R_o}$$

# Frequency response of differential amplifier

## Active-loaded MOS amplifier

$$V_o = g_m R_o \left( \frac{V_{id}}{2} \right) \left[ 1 + \frac{1}{1 + \frac{j\omega C_m}{g_{m3}}} \right] \frac{1}{1 + j\omega C_L R_o} \Rightarrow A_d = (g_m R_o) \left( \frac{1}{1 + j\omega C_L R_o} \right) \left( \frac{1 + \frac{j\omega C_m}{2g_{m3}}}{1 + \frac{j\omega C_m}{g_{m3}}} \right)$$

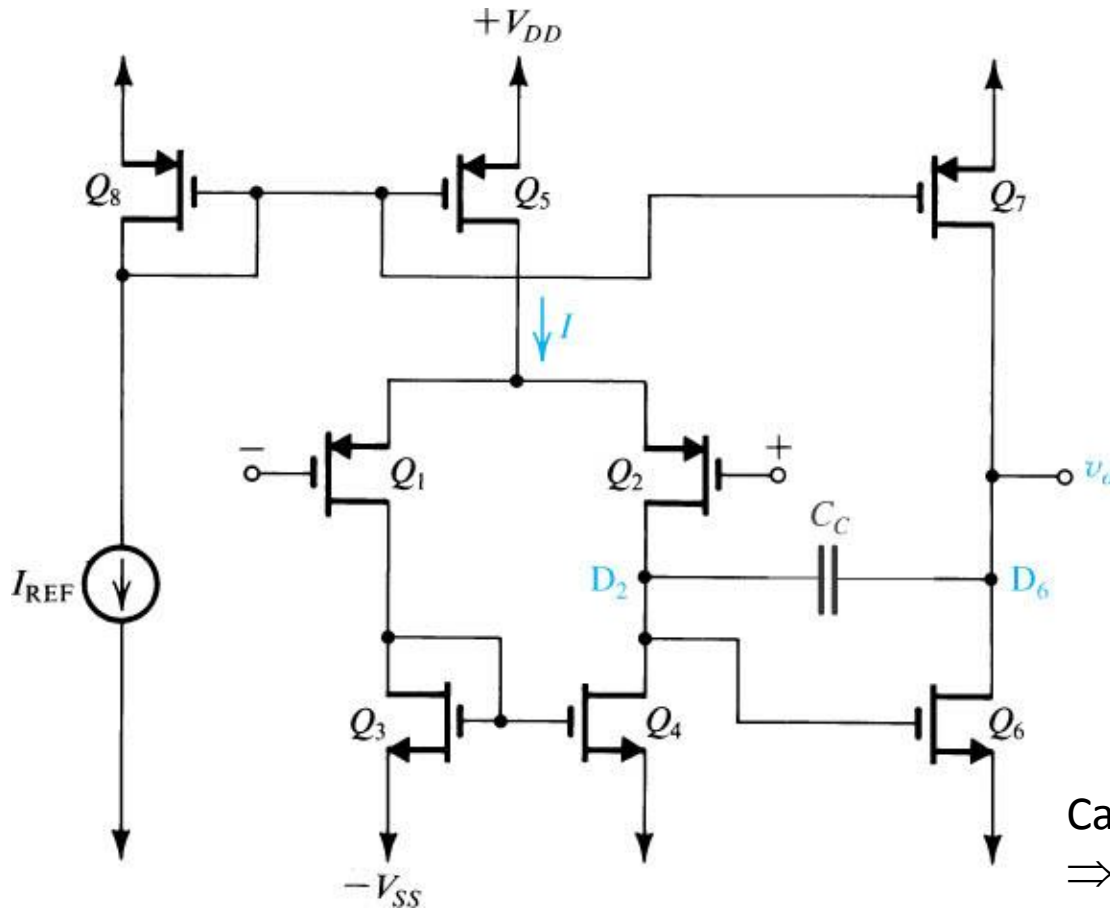
$$\Rightarrow f_{P1} = \frac{1}{2\pi C_L R_o} \quad \Rightarrow \text{Often dominant pole}$$

$$\begin{aligned} \Rightarrow f_{P2} &= \frac{g_{m3}}{2\pi C_m} \\ \Rightarrow f_z &= \frac{2g_{m3}}{2\pi C_m} \end{aligned} \quad \Rightarrow \text{Very high frequency!}$$

 Note that  $f_z$  due to current source is usually main cause of CMRR degradation



## Two-stage CMOS amplifier



- Q1-Q2 MOS differential amplifier with active load (Q3-Q4)
- Q6 common-source amplifier
- Current mirror Q8-Q5 for polarizing Q1-Q2
- Current mirror Q8-Q7 for polarising Q6

$$A_1 = -g_{m1} (r_{o2} \parallel r_{o4})$$

$$A_2 = -g_{m6} (r_{o6} \parallel r_{o7})$$

$$\Rightarrow A_{tot} = A_1 A_2$$

$$R_o = (r_{o6} \parallel r_{o7})$$

Capacitor  $C_c$  (see next slide)

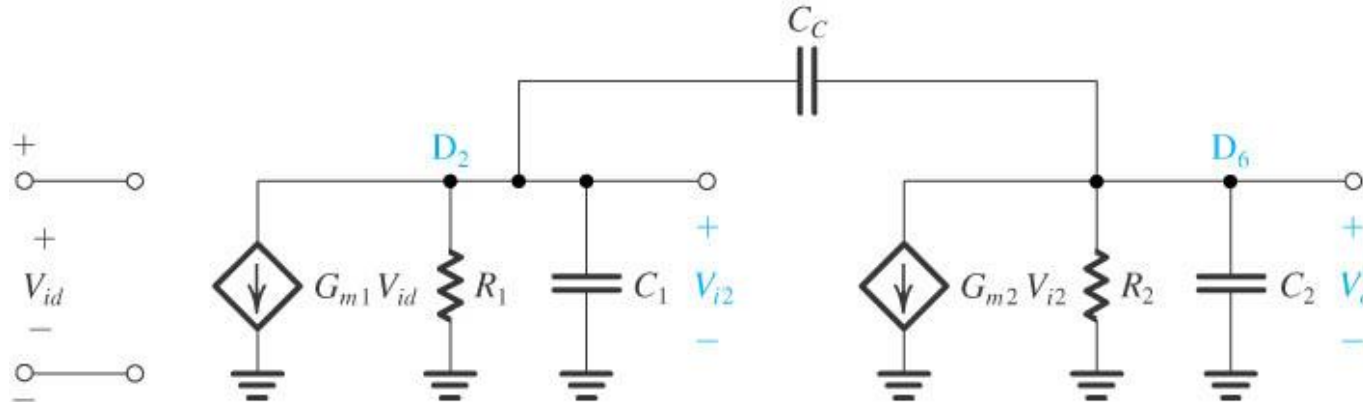
⇒ dominant pole

$\Rightarrow -20\text{dB/decade}$

$\Rightarrow$  stability

# Multistage amplifier

## Frequency response

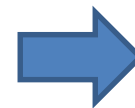


$$C_1 = C_{gd4} + C_{db4} + C_{gd2} + C_{db2} + C_{gs6}$$

$$C_2 = C_{db6} + C_{db7} + C_{gd7} + C_L$$

$$\Rightarrow f_z = \frac{G_{m2}}{2\pi C_C}$$

$$\Rightarrow f_{p1} \approx \frac{1}{2\pi R_1 [C_1 + C_C (1 + G_{m2} R_2)]}$$



Choose  $C_C$  so that  $f_{p1}$  lower than  $f_z$  and  $f_{p2}$ , in order to ensure -20dB/decade

$$\Rightarrow f_{p2} \approx \frac{G_{m2}}{C_2}$$