

Chapter 4

Single-stage MOS amplifiers



Single-stage MOS amplifiers

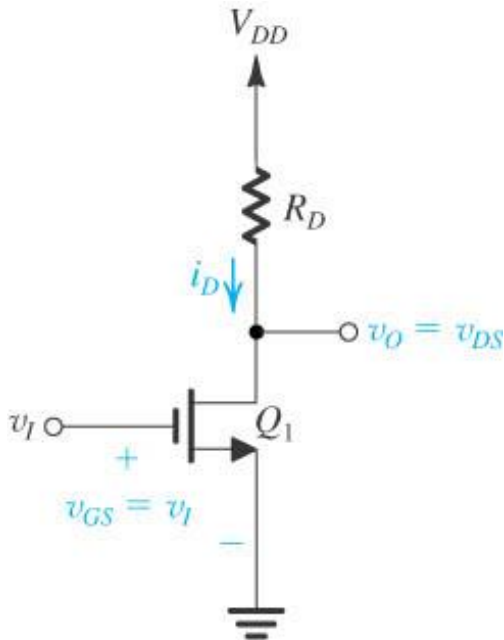
Outline

- NMOS as an amplifier: example of common-source circuit
 - NMOS amplifier example
 - Introduction to biasing and small-signal operation
- Biasing in MOS amplifier circuits
 - Fixing V_{GS}
 - Fixing V_G and adding a source resistance
 - Using RDG
 - Using a constant-current source
- Small-signal operation and models
 - Introduction to small-signal operation: common-source circuit
 - Small-signal models for NMOS transistors
- Other common single-stage MOS amplifiers
 - Common-source circuit
 - Common-source with source resistance
 - Common-drain circuit
 - Common-gate circuit
- Frequency response of MOSFET
 - HF response of NMOS transistor
 - HF response of common-source amplifier
 - LF response of common-source amplifier

NMOS amplifier example

Common-source amplifier

- common = grounded
- i_D can be defined in two ways
 - Kirchhoff's voltage law in V_{DD} - R_D - v_o loop: $v_o = V_{DD} - R_D i_D$
 - Output of transistor:

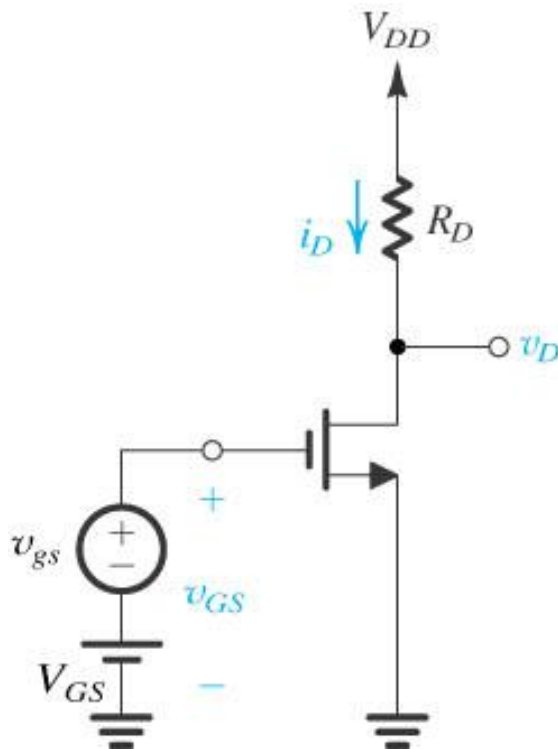


$$\begin{cases} i_D = k'_n \frac{W}{L} \left[(v_{GS} - V_T) v_o - \frac{1}{2} v_o^2 \right] & \text{if } v_{DS} \leq v_{GS} - V_T \\ i_D = \frac{1}{2} k'_n \frac{W}{L} (v_{GS} - V_T)^2 & \text{otherwise} \end{cases}$$

Small-signal operation

Common-source amplifier

- V_{GS} is the constant voltage used to bias the transistor
- v_{gs} is the AC signal to be amplified



$$v_{GS} = V_{GS} + v_{gs}$$

$$\Rightarrow i_D = \frac{1}{2} k'_n \frac{W}{L} (V_{GS} + v_{gs} - V_t)^2$$

$$\Rightarrow i_D = \frac{1}{2} k'_n \frac{W}{L} (V_{GS} - V_t)^2 + k'_n \frac{W}{L} (V_{GS} - V_t) v_{gs} + \frac{1}{2} k'_n \frac{W}{L} v_{gs}^2$$

$v_{gs} \ll V_{GS}$

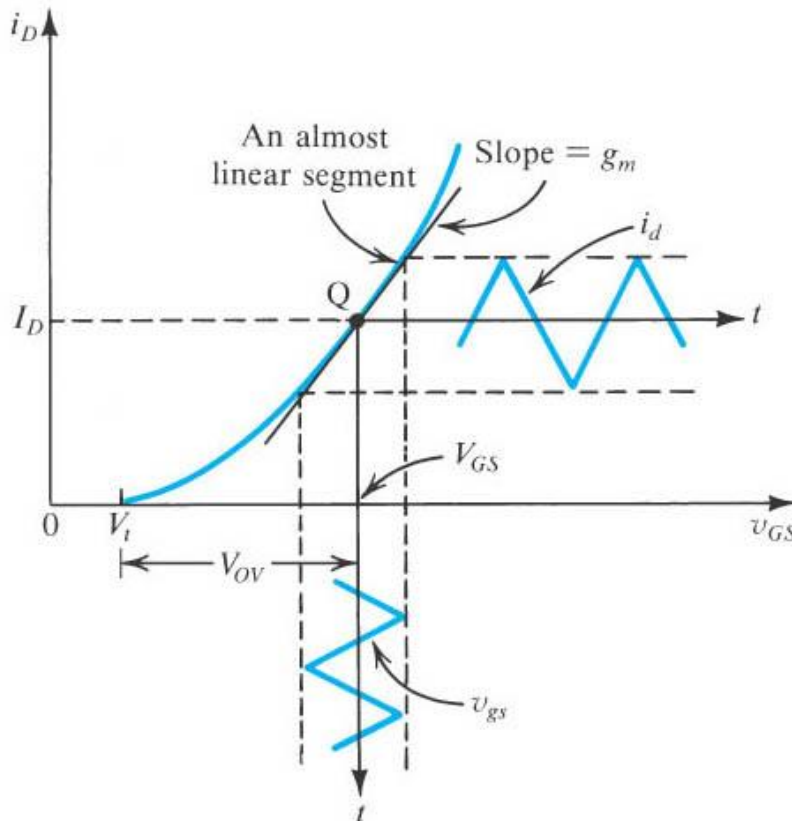
$$\Rightarrow i_D = I_D + i_d \quad \text{with} \quad i_d = k'_n \frac{W}{L} (V_{GS} - V_t) v_{gs}$$

Small-signal operation

Common-source amplifier

- MOSFET transconductance: $g_m = \frac{i_d}{v_{gs}} = k'_n \frac{W}{L} (V_{GS} - V_t)$

⇒ Equal to slope of i_D - v_{GS} characteristic at bias point: $g_m = \left. \frac{\partial i_D}{\partial v_{GS}} \right|_{v_{GS}=V_{GS}}$



Biasing: setting the bias point Q



Small-signal operation: signal variations around the point Q

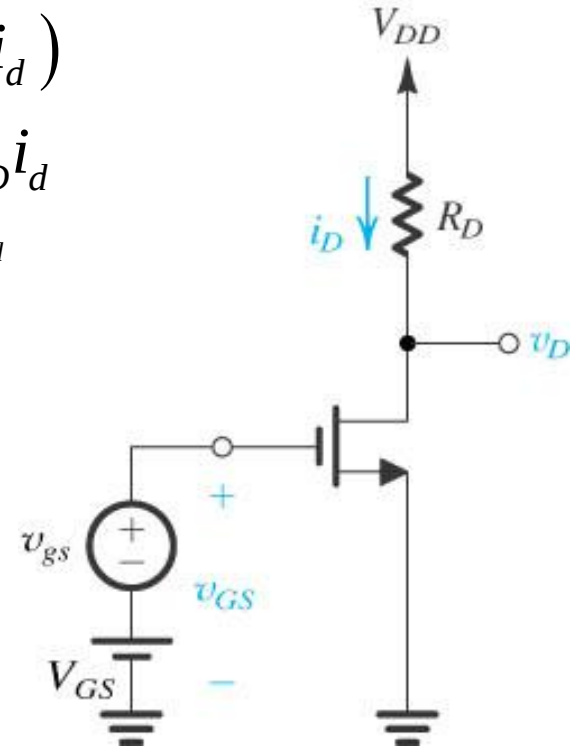
Small-signal operation

Common-source amplifier

- Output voltage:
$$\begin{aligned} v_D &= V_{DD} - R_D i_D \\ &= V_{DD} - R_D (I_D + i_d) \\ &= \underbrace{V_{DD} - R_D I_D}_{V_D} - R_D i_d = v_d \end{aligned}$$

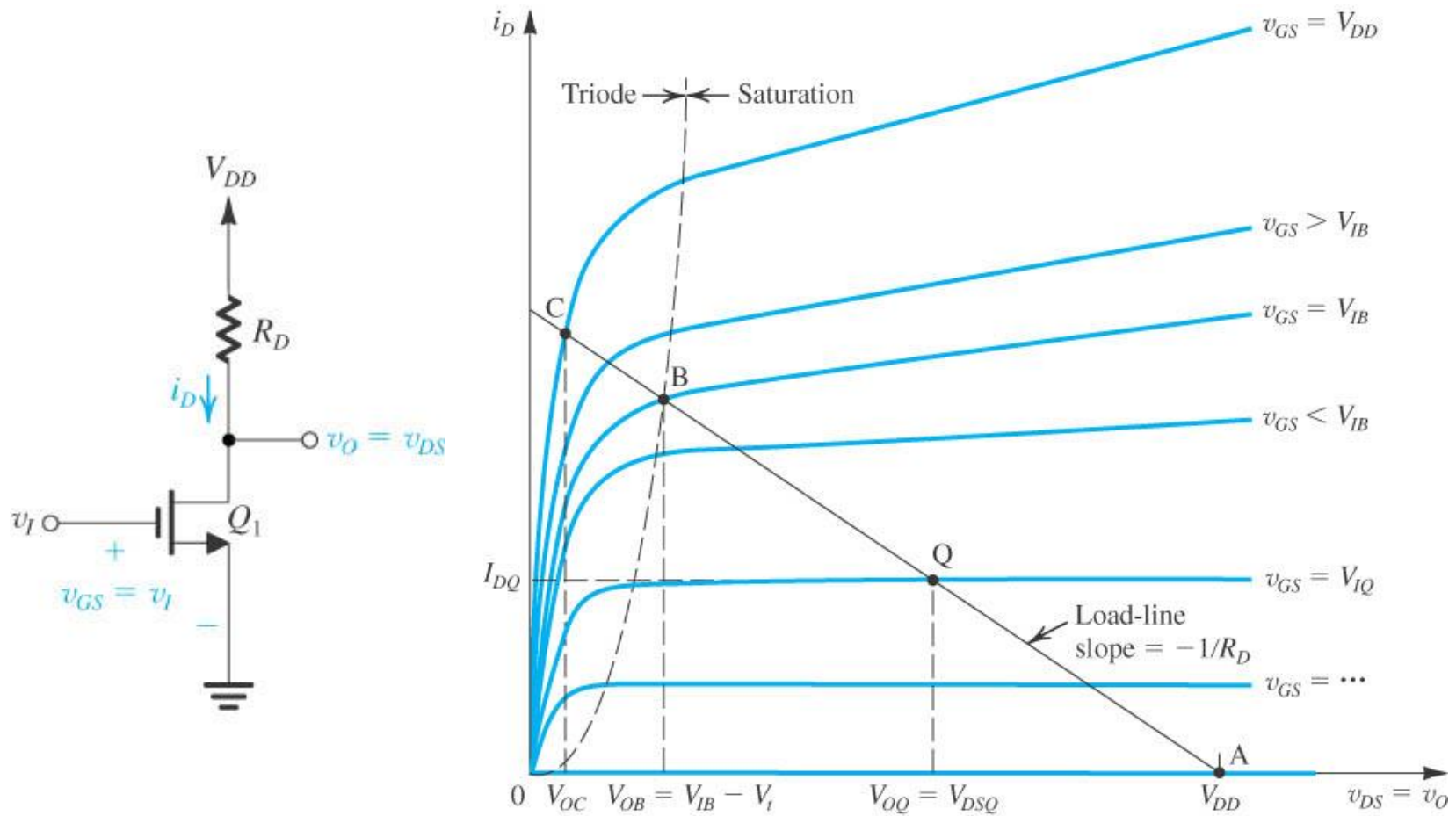
$$\Rightarrow v_d = -R_D i_d = -g_m R_D v_{gs}$$

$$\Rightarrow A_v = \frac{v_d}{v_{gs}} = -g_m R_D$$



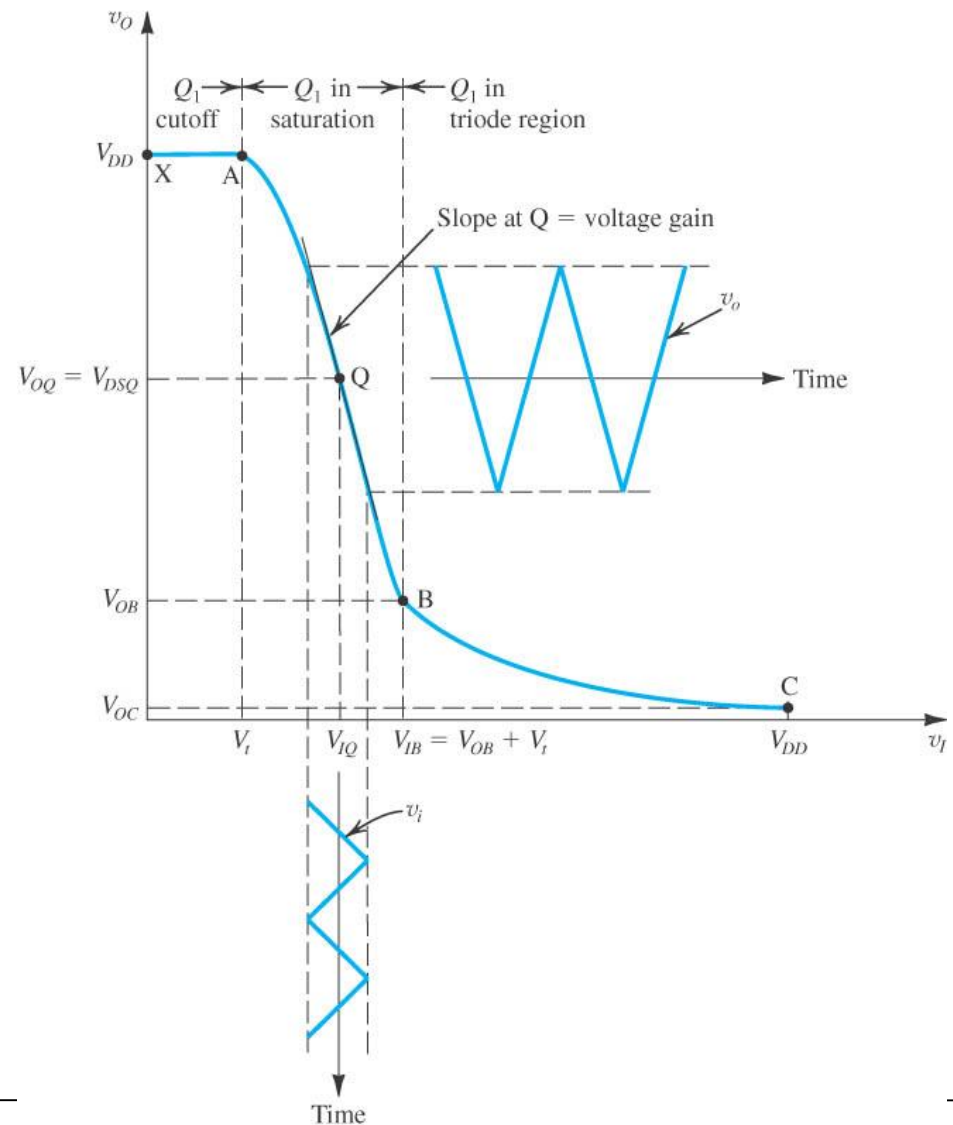
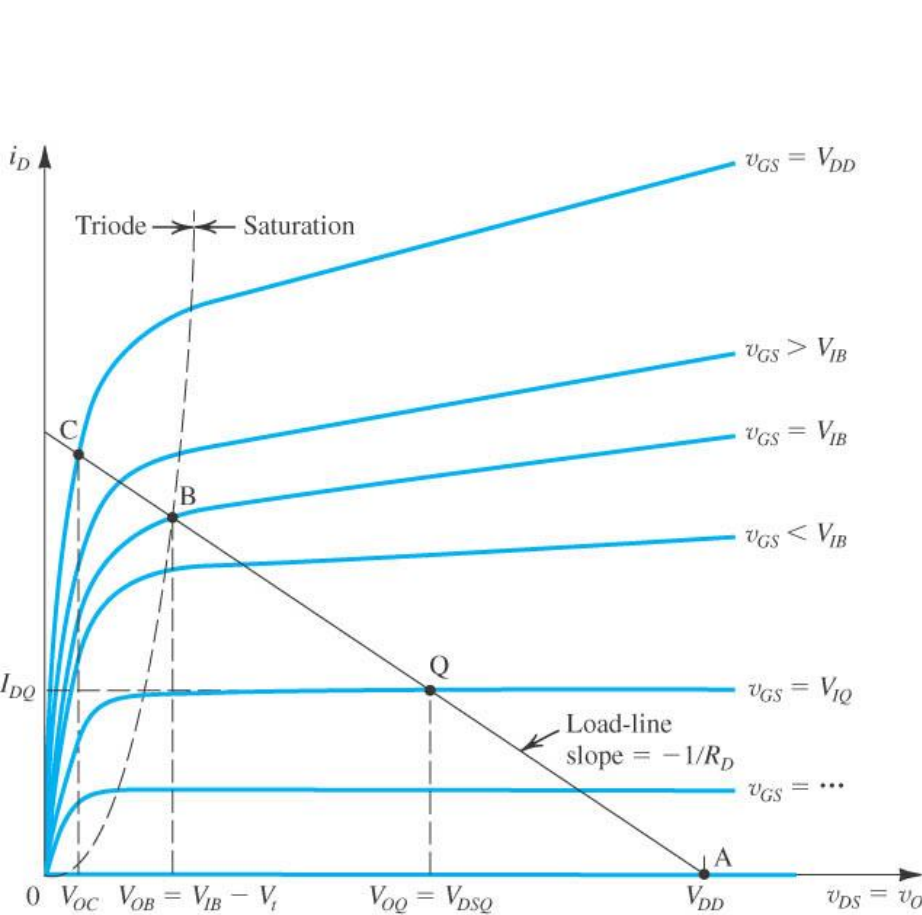
NMOS amplifier example

Common-source amplifier



NMOS amplifier example

Common-source amplifier

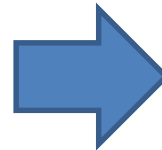
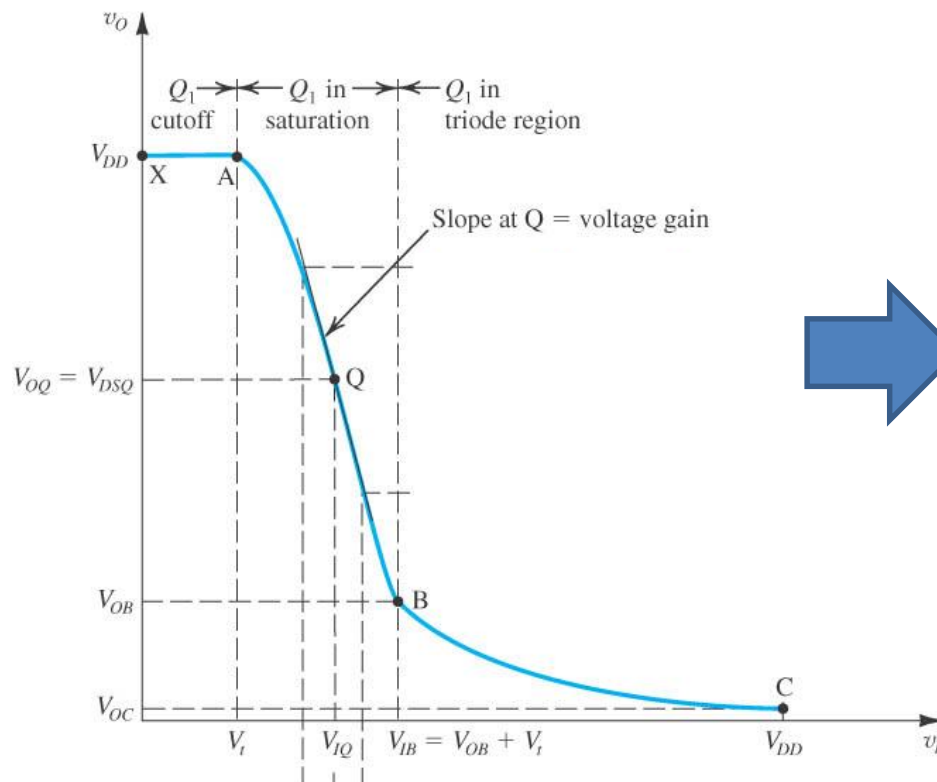


NMOS amplifier example

Biasing and small scale signals

- If input: small AC signal v_I (no DC component)
=> Add a DC component so that $v_{GS} = V_{IQ} + v_I$
=> if v_I small, transfer characteristic +/- linear and $v_o = V_{oQ} + A v_I$

↪ gain of circuit



Biasing => setting the DC signals such that the transistor circuit operates in a linear region for small AC input

Note: choosing the bias point Q appropriately allows for larger AC input signals

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Biasing MOS amplifier circuits

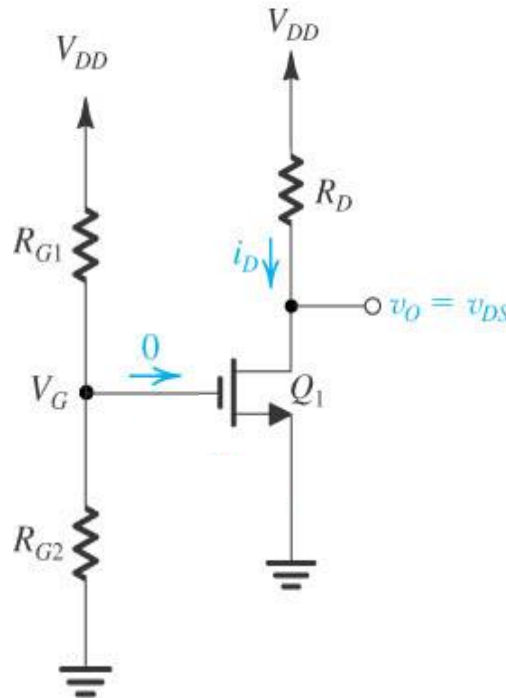
About notations...

- Real voltages and currents in the transistor are noted with lowercase letters and uppercase indices
 - E.g. v_{GS} , i_D , v_{DS}
- DC voltages and currents for biasing a transistor are usually noted with uppercase letters and uppercase indices
 - E.g. V_{GS} , I_D , V_{DS} , ...
- Voltage and current variations (AC signals) are noted with lowercase letters and lowercase indices
 - E.g. v_{gs} , i_d , v_{ds}
- In the frequency analysis (LF and HF), AC signals are noted with uppercase letters and lowercase indices
 - E.g. V_{gs} , I_d , V_{ds}

Biasing MOS amplifier circuits

Biasing by fixing V_{GS}

- V_{GS} fixed (e.g. by voltage divider with power supply V_{DD})



$$V_{GS} = V_G - V_S = \frac{R_{G2}}{R_{G1} + R_{G2}} V_{DD}$$

$\Rightarrow I_D$ is fixed through transistor equation $\Rightarrow I_D = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_{GS} - V_t)^2$

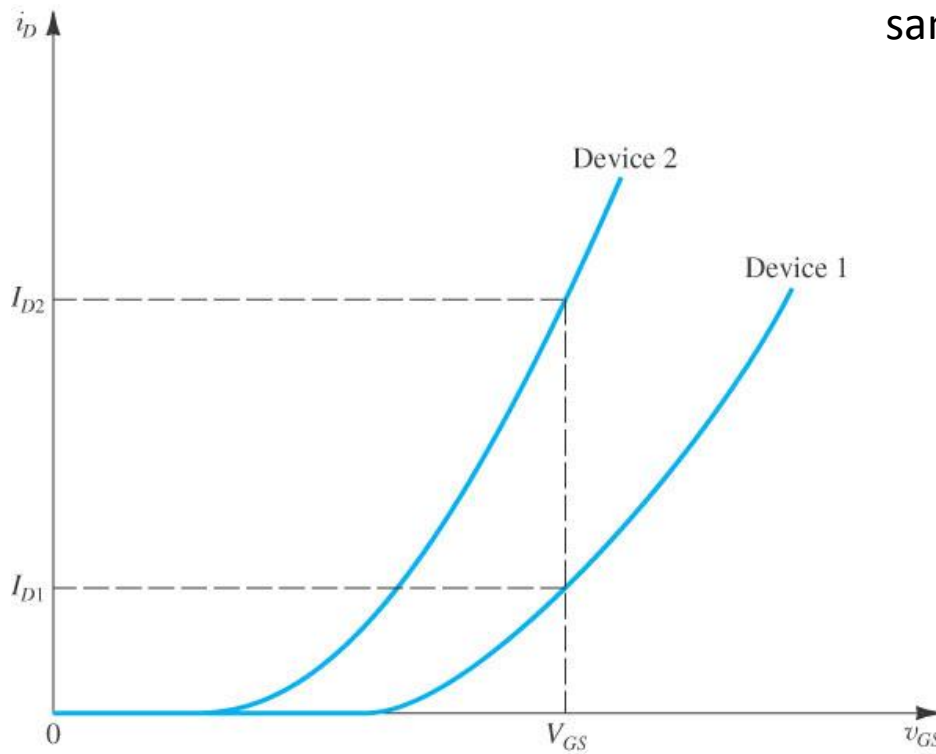
Biasing MOS amplifier circuits

Biasing by fixing V_{GS}

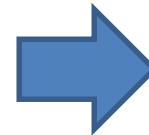
$$I_D = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_{GS} - V_t)^2$$



can vary a lot between transistors,
even for devices supposedly of the
same size !



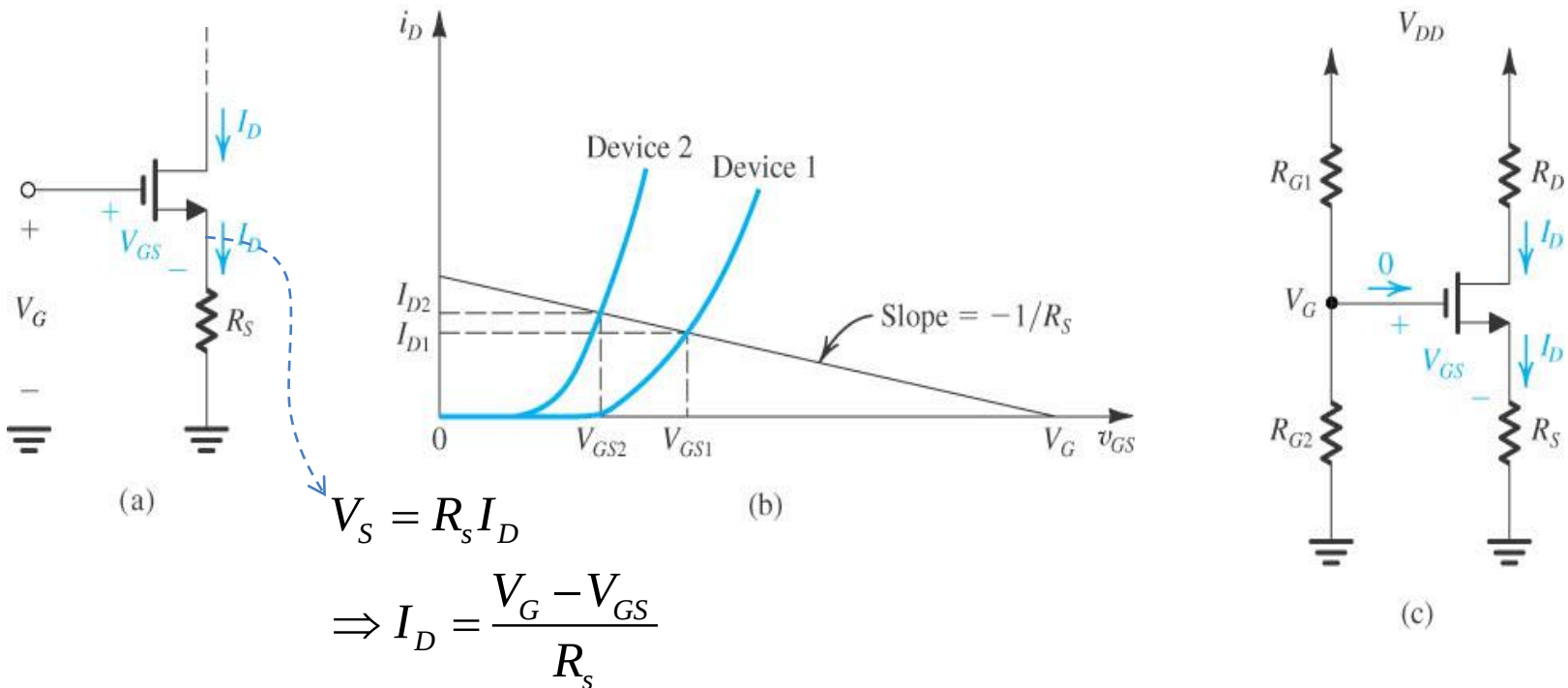
For the same value of V_{GS} ,
 I_D varies a lot between
different transistors



Fixing V_{GS} not a good way
to set I_D !

Biasing MOS amplifier circuits

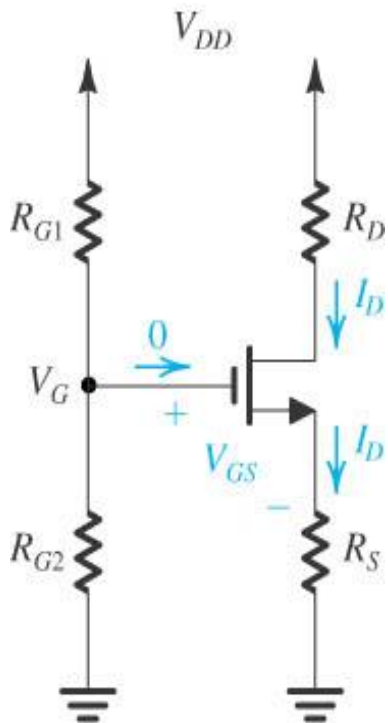
Fixing V_G and adding a source resistance



- If R_S is chosen large enough, I_D becomes much more stable w.r.t. manufacturing tolerance
- V_G again set with voltage divider

Biasing MOS amplifier circuits

Fixing V_G and adding a source resistance



⇒ Adding a source resistance makes the transistor biasing much more stable against transistor manufacturing tolerance

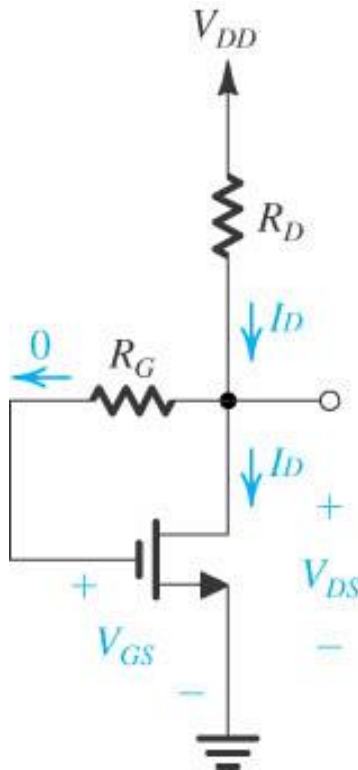
⇒ I_D does not change so much between transistors

⇒ Key idea: set I_D (and let V_{GS} vary depending on transistor)

⇒ Choosing large values for R_{G1} and R_{G2} ensures that the circuit has large input impedance

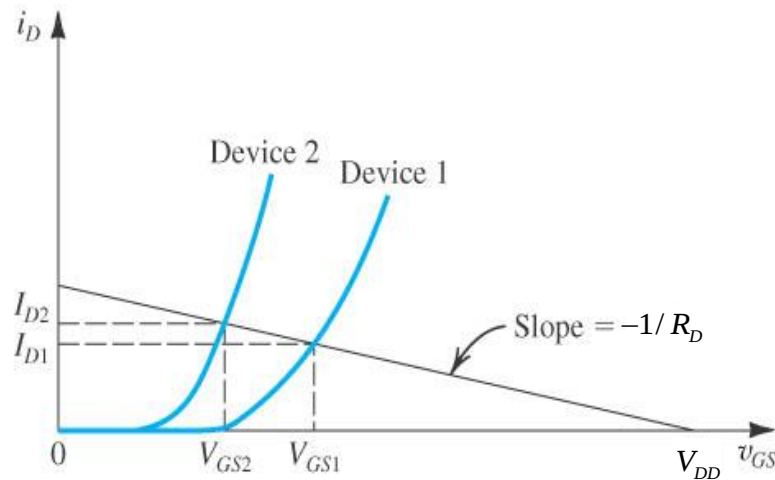
Biasing MOS amplifier circuits

Using a drain-to-gate feedback resistor



$$V_G = V_{DD} - R_D I_D$$

$$\Rightarrow I_D = \frac{V_{DD} - V_{GS}}{R_D}$$



- Similar to previous biasing scheme
- R_G chosen large (usually $M\Omega$ range) to force $I_G=0$

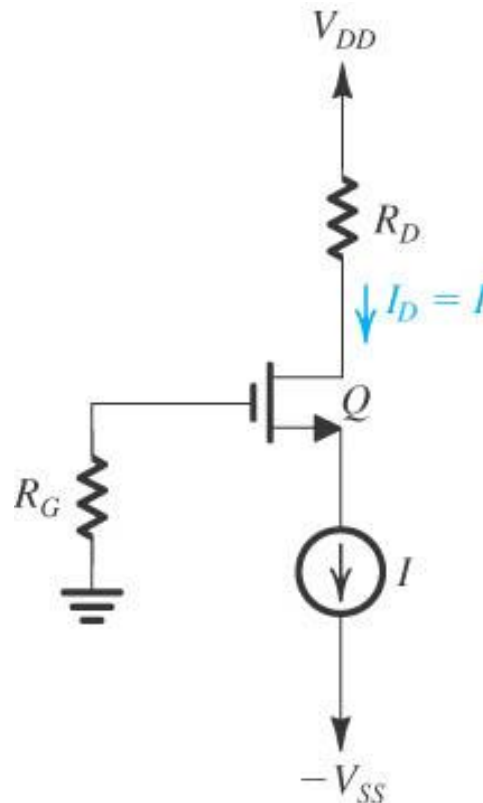
Biasing MOS amplifier circuits

Using a constant-current source

- Set I_D with a constant-current source

$\Rightarrow V_G = 0$ in this circuit

$\Rightarrow V_S$ is such that V_{GS} matches I_D



I_D fixed

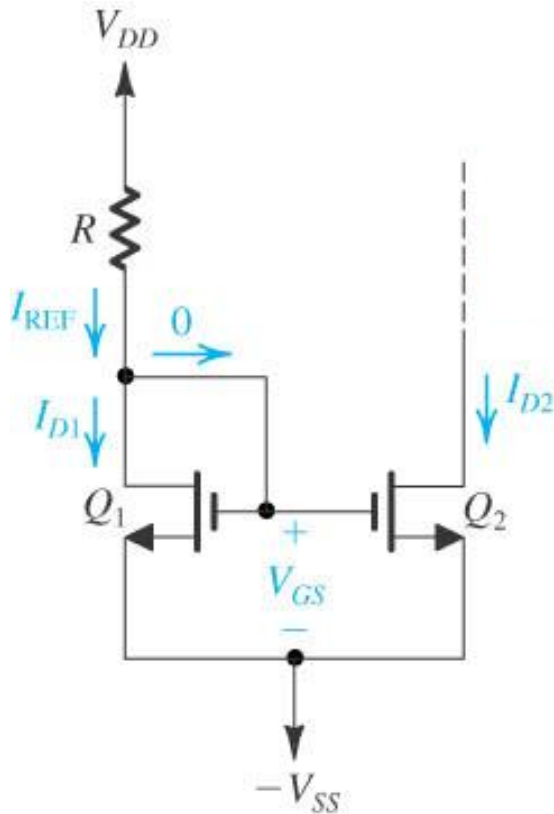
$$V_{GS} = V_t + \sqrt{2I_D/(k'_n W/L)}$$

Saturation region iif $V_{DS} \geq V_{GS} - V_t$
iif $V_D \geq -V_t$

Biasing MOS amplifier circuits

... making a current source with 2 NMOS transistors

Current mirror

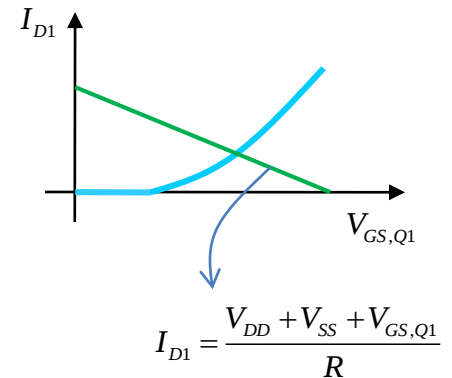


- Drain Q1 connected to gate Q1:
 $V_{D,Q1} = V_{G,Q1}$
 $\Rightarrow V_{DS,Q1} > V_{GS,Q1} - V_t$
 $\Rightarrow$ saturation region
- I_{REF} fixed by R , V_{DD} , V_{SS} and NMOS:

$$\begin{cases} I_{REF} = \frac{V_{DD} + V_{SS} + V_{GS,Q1}}{R} \\ I_{REF} = \frac{1}{2} k'_n \frac{W_{Q1}}{L_{Q1}} (V_{GS,Q1} - V_t)^2 \end{cases}$$

- $V_{GS,Q1} = V_{GS,Q2}$
 $\Rightarrow I_{D2} = \frac{1}{2} k'_n \frac{W_{Q2}}{L_{Q2}} (V_{GS,Q2} - V_t)^2$

$$\Rightarrow I_{D2} = I_{REF} \frac{W_{Q2} / L_{Q2}}{W_{Q1} / L_{Q1}}$$



Single-stage MOS amplifiers

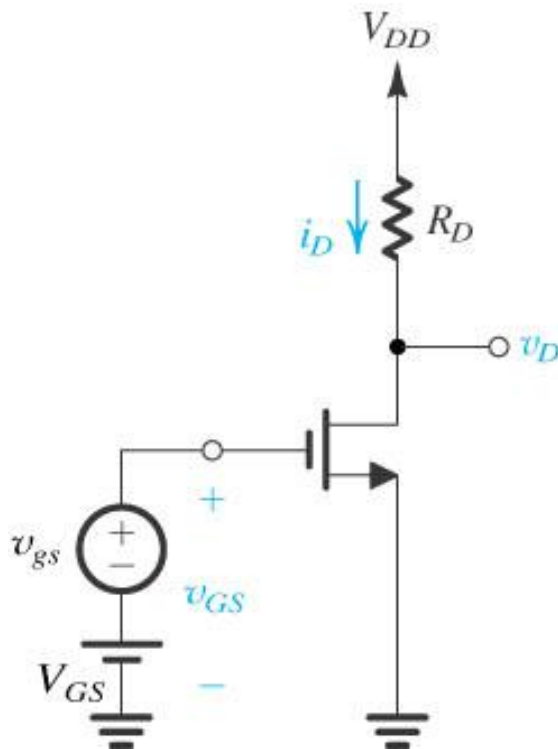
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Small-signal operation

Example of the common-source circuit

- V_{GS} is the constant voltage used to bias the transistor
- v_{gs} is the AC signal to be amplified



$$v_{GS} = V_{GS} + v_{gs}$$

$$\Rightarrow i_D = \frac{1}{2} k'_n \frac{W}{L} (V_{GS} + v_{gs} - V_t)^2$$

$$\Rightarrow i_D = \frac{1}{2} k'_n \frac{W}{L} (V_{GS} - V_t)^2 + k'_n \frac{W}{L} (V_{GS} - V_t) v_{gs} + \frac{1}{2} k'_n \frac{W}{L} v_{gs}^2$$

~~$v_{gs} \ll V_{GS}$~~

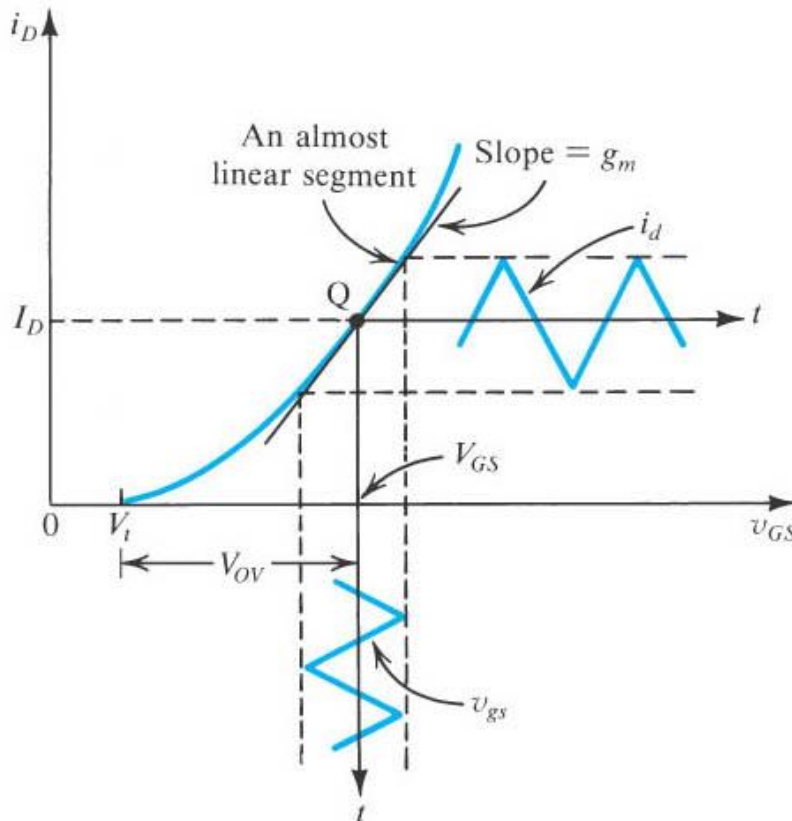
$$\Rightarrow i_D = I_D + i_d \quad \text{with} \quad i_d = k'_n \frac{W}{L} (V_{GS} - V_t) v_{gs}$$

Small-signal operation

Example of the common-source circuit

- MOSFET transconductance: $g_m = \frac{i_d}{v_{gs}} = k'_n \frac{W}{L} (V_{GS} - V_t)$

⇒ Equal to slope of i_D - v_{GS} characteristic at bias point: $g_m = \left. \frac{\partial i_D}{\partial v_{GS}} \right|_{v_{GS}=V_{GS}}$



Biasing: setting the bias point Q

Small-signal operation: signal variations around the point Q

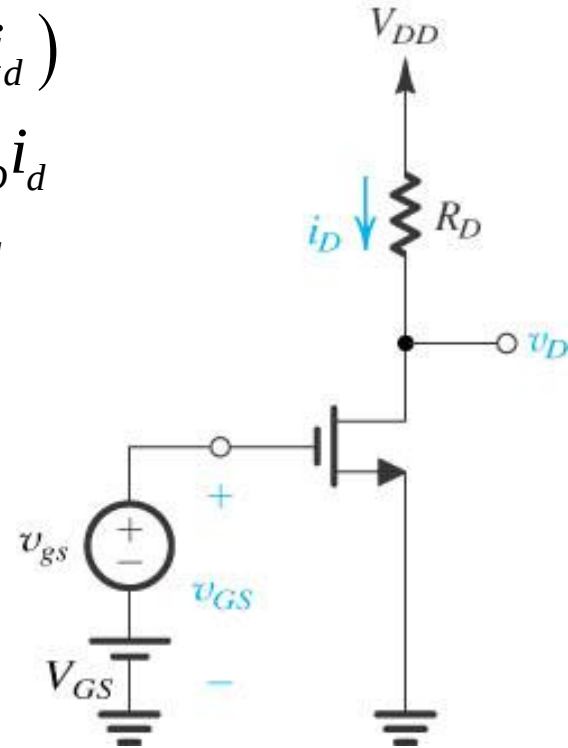
Small-signal operation

Example of the common-source circuit

- Output voltage:
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$$\Rightarrow v_d = -R_D i_d = -g_m R_D v_{gs}$$

$$\Rightarrow A_v = \frac{v_d}{v_{gs}} = -g_m R_D$$



Small signal equivalent-circuit models

For small- signal operation, MOSFET transistor...

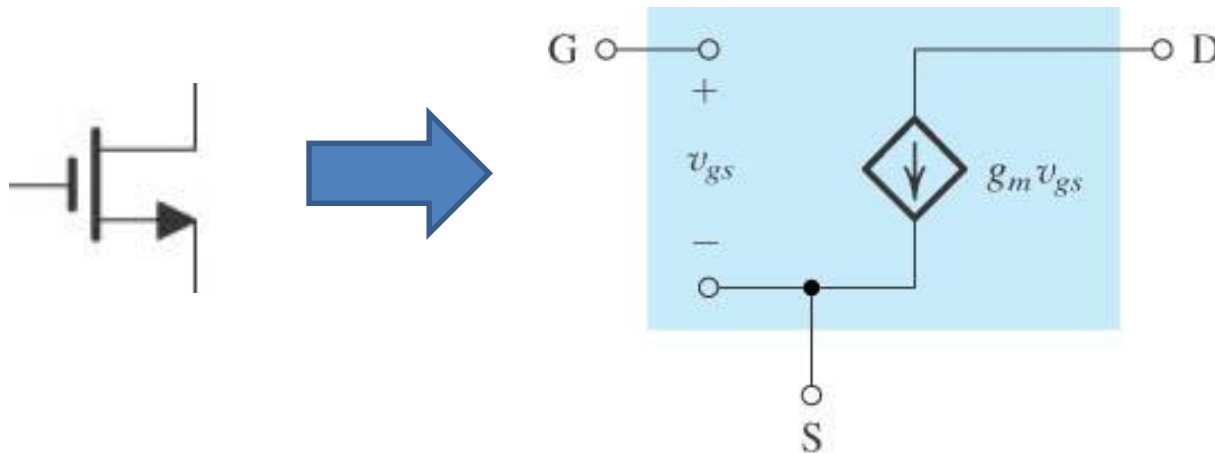
... behaves like a voltage-controlled current source

⇒ Input voltage v_{gs} between gate and source

⇒ Input resistance is very high (+/- infinite)

⇒ Output current $g_m v_{gs}$ at drain terminal

⇒ when analyzing (small) AC signal component, NMOS can be replaced with small-signal equivalent



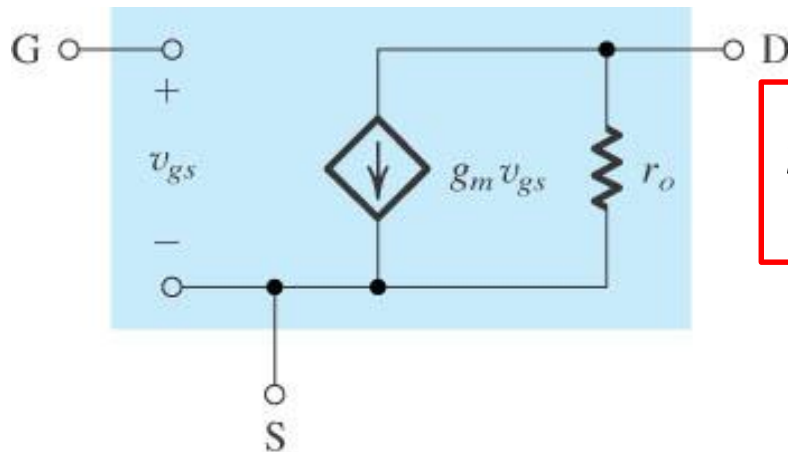
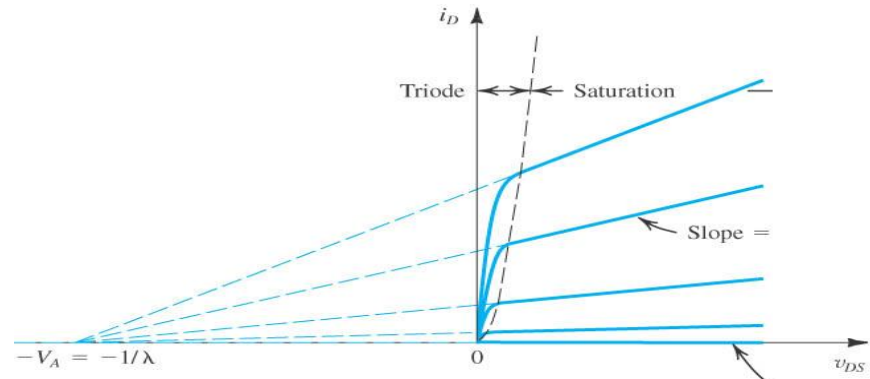
NMOS small-signal model

... taking into account output resistance

... due to channel pinching

⇒ Variations in v_{DS} result in different i_D

⇒ Imperfect current source



$$r_o = \frac{V_A}{I_D}$$

$$g_m = k'_n (W/L)(V_{GS} - V_t)$$



g_m and r_o depend on biasing point !!!

NMOS small-signal model

g_m can be expressed several ways

$$g_m = \frac{i_d}{v_{gs}} = k'_n \frac{W}{L} (V_{GS} - V_t)$$

$$I_D = \frac{1}{2} k'_n \frac{W}{L} (V_{GS} - V_t)^2 \Rightarrow (V_{GS} - V_t) = \sqrt{2 \frac{L}{W} \frac{1}{k'_n} I_D}$$

$$\Rightarrow g_m = \sqrt{2 k'_n \frac{W}{L} I_D}$$

$$I_D = \frac{1}{2} k'_n \frac{W}{L} (V_{GS} - V_t)^2 \Rightarrow k'_n \frac{W}{L} = \frac{2I_D}{(V_{GS} - V_t)^2}$$

$$\Rightarrow g_m = \frac{2I_D}{V_{GS} - V_t}$$

Small-signal models

Converting a circuit to its small-signal equivalent

- DC voltage sources => short-circuits



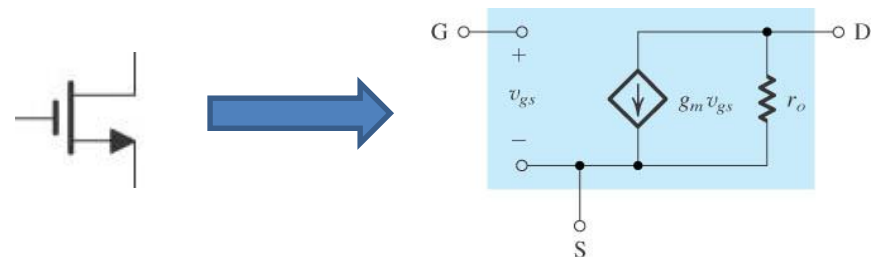
- DC current sources => open-circuits



- Capacitors => short-circuits

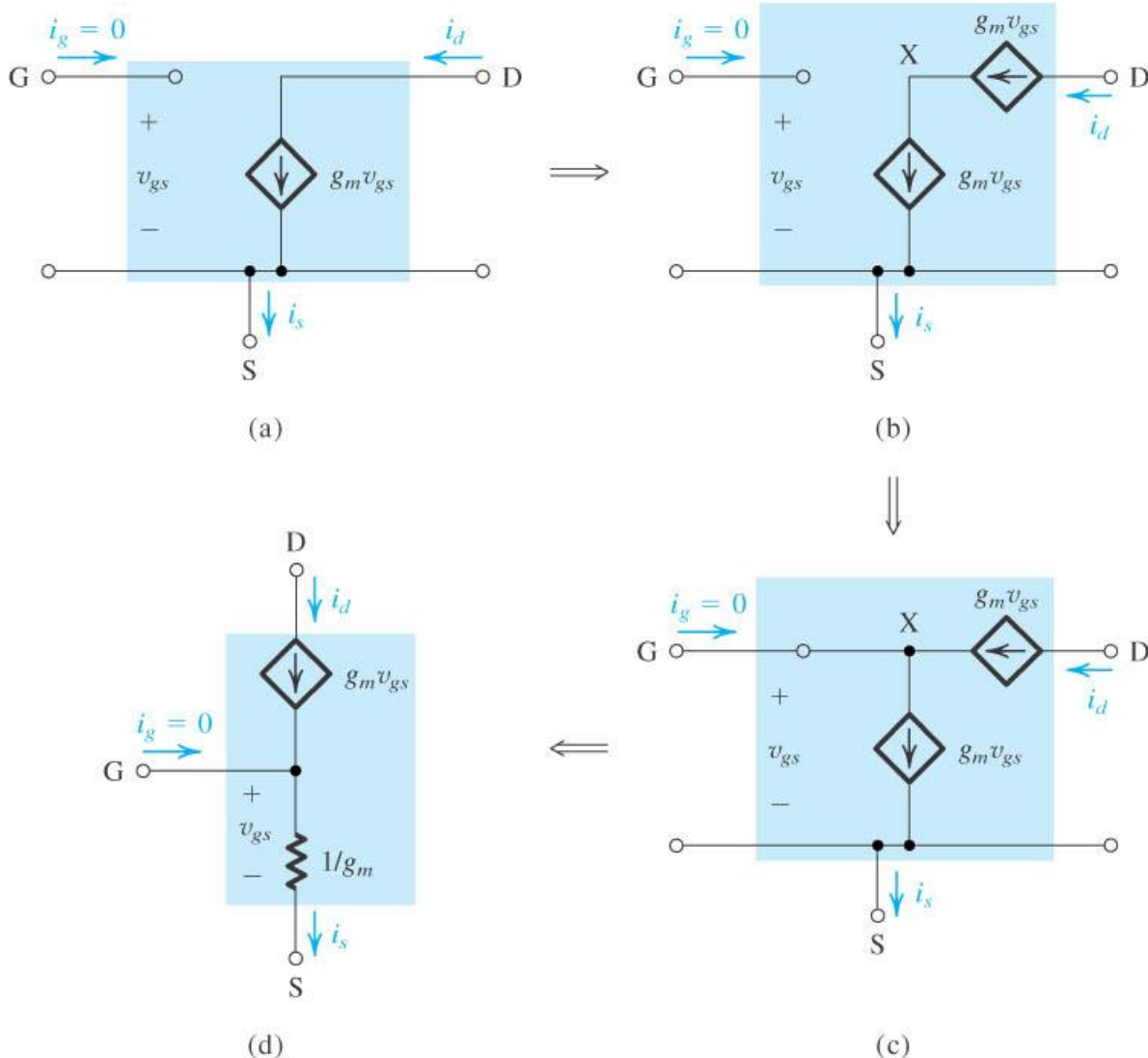


- Transistors => equivalent mode



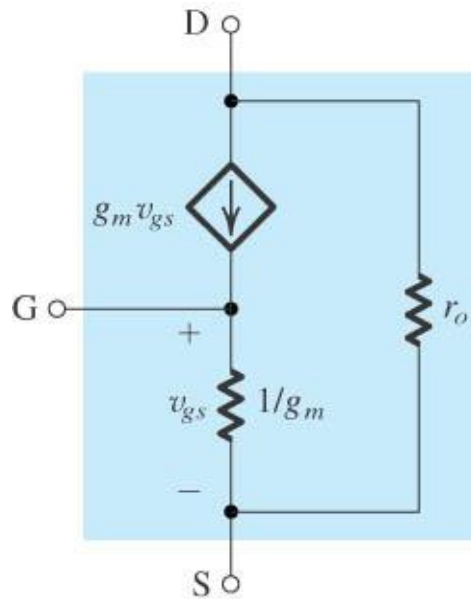
Small-signal models

T equivalent-circuit model for NMOS transistor



Small-signal models

T model with output resistance



$$i_g = 0 \Rightarrow r_{in} = \infty$$

$$g_m = k'_n (W / L) (V_{GS} - V_t)$$

$$r_o = \frac{V_A}{I_D}$$

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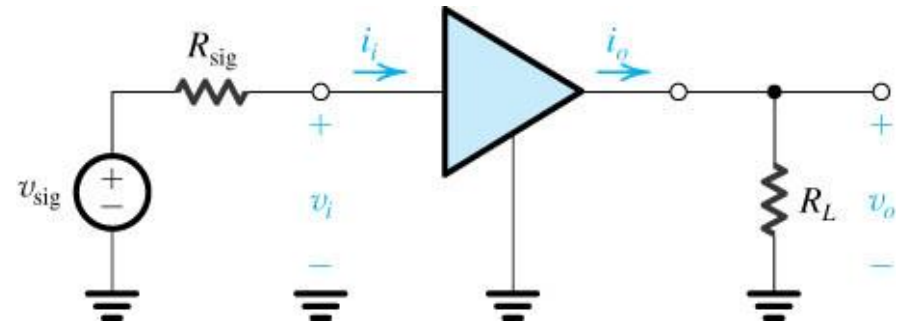
Amplifiers

Definitions and general characteristics

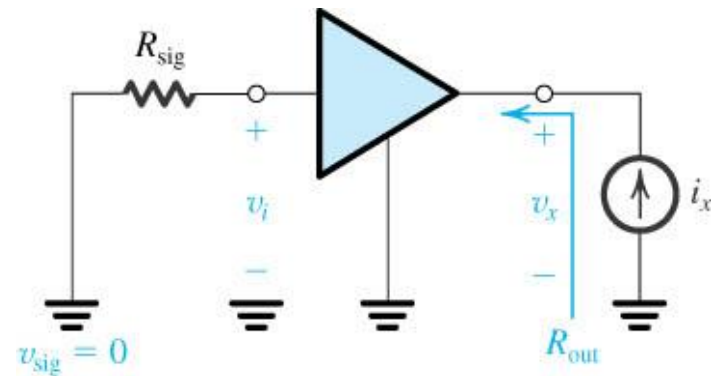
- Open-circuit voltage gain: $A_{vo} = \left. \frac{v_o}{v_i} \right|_{R_L = \infty}$

- Voltage gain: $A_v = \frac{v_o}{v_i}$

- Input resistance: $R_i = \frac{v_i}{i_i}$



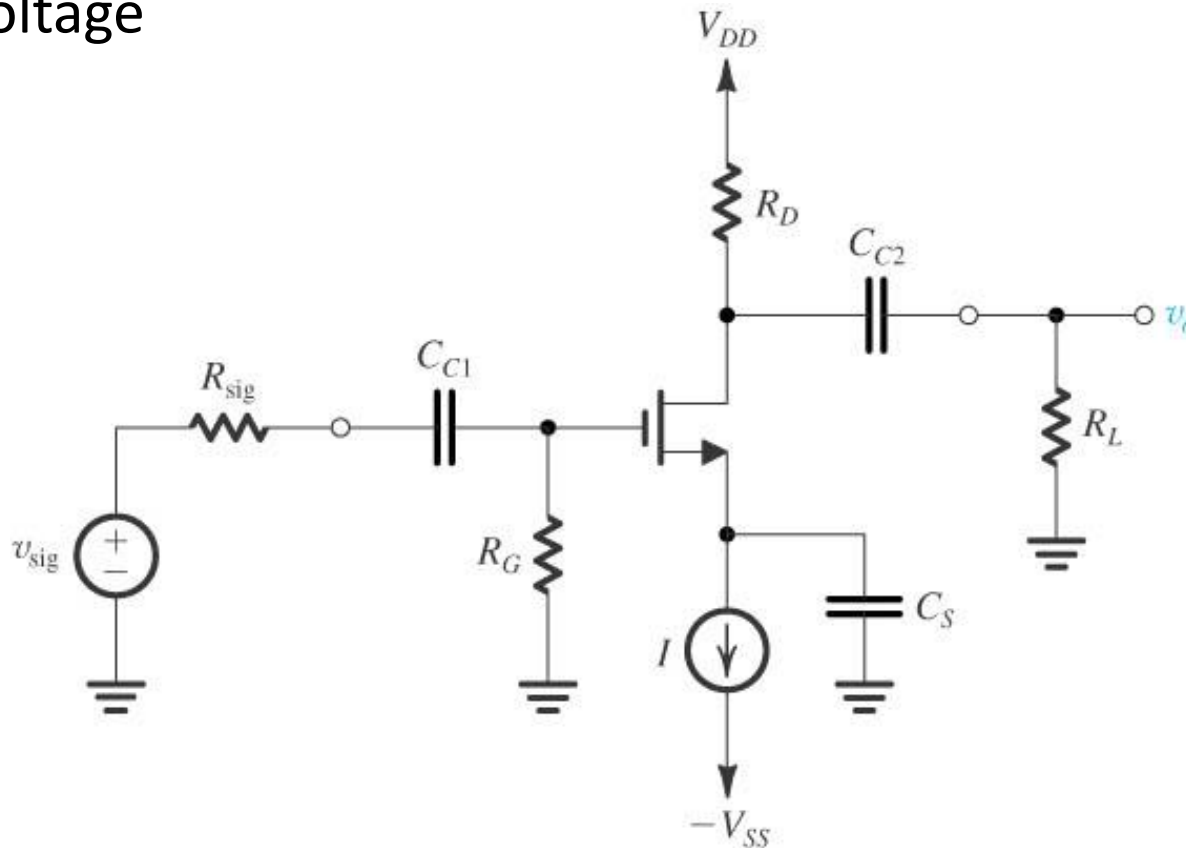
- Output resistance: $R_o = \left. \frac{v_x}{i_x} \right|_{v_{sig}=0}$



Common-source amplifier

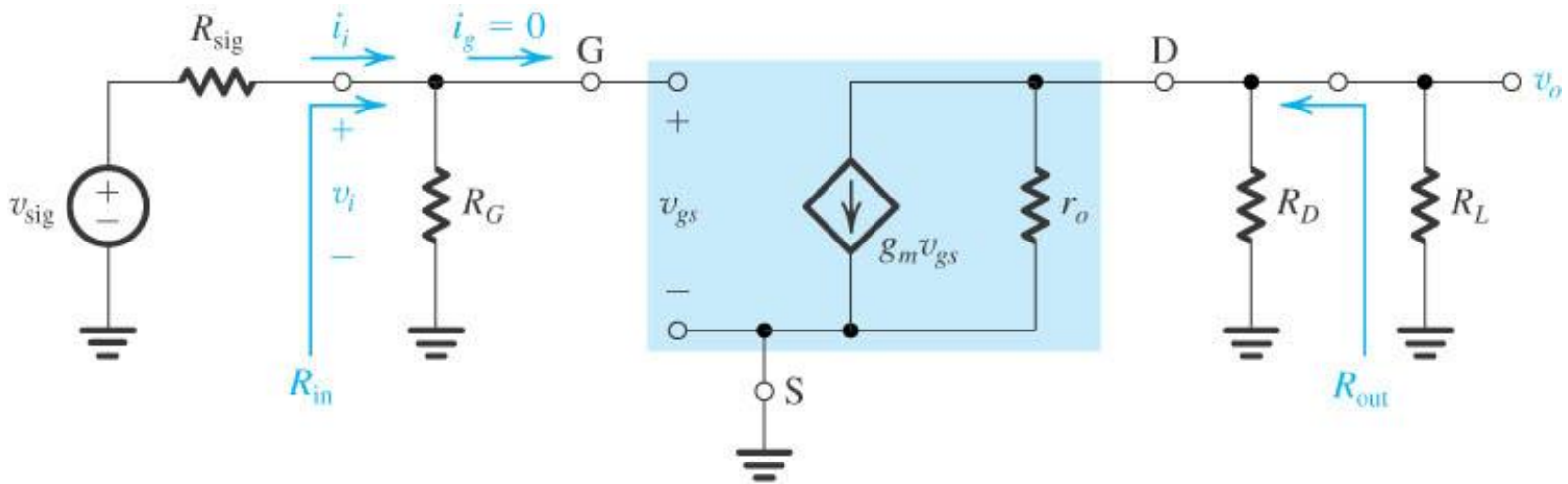
Biased with constant-current source

- Bypass capacitor C_S for connecting source to ground (in AC)
- Coupling capacitors C_{C1} and C_{C2} in order not to disturb the DC bias voltage



Common-source amplifier

Replace circuit with small-signal equivalent



$$i_g = 0 \Rightarrow v_i = \frac{R_G}{R_G + R_{sig}} v_{sig}$$

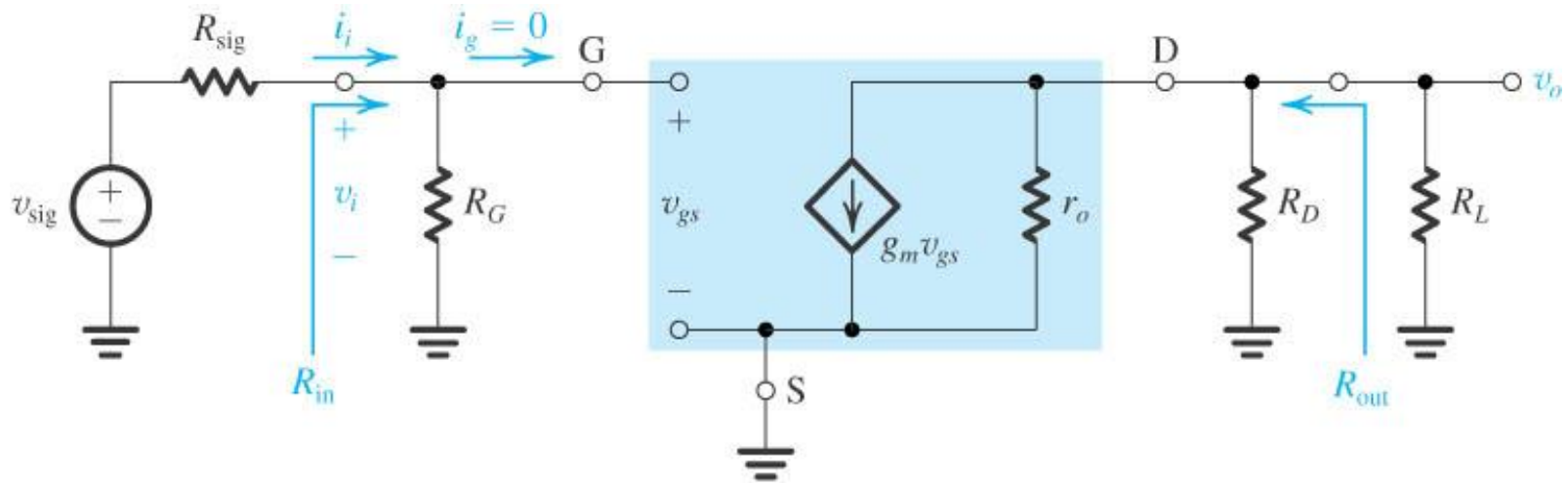
$$v_{gs} = v_i \Rightarrow v_o|_{R_L=\infty} = -(r_o \parallel R_D) g_m v_i$$

$$\Rightarrow A_{vo} = -(r_o \parallel R_D) g_m$$

$$\Rightarrow A_v = -(r_o \parallel R_D \parallel R_L) g_m$$

Common-source amplifier

Replace circuit with small-signal equivalent



$$R_i = \frac{v_i}{i_i}$$

$$\Rightarrow R_i = R_G$$

$$R_o = \left. \frac{v_x}{i_x} \right|_{v_{sig}=0}$$

$$\Rightarrow R_o = (r_o \parallel R_D)$$

Common-source amplifier

Summary

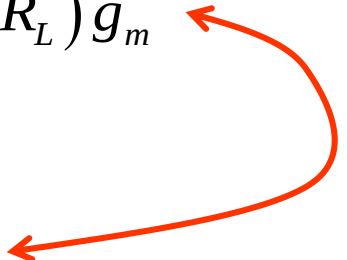
$$\Rightarrow A_{vo} = -(r_o \parallel R_D) g_m$$

$$\Rightarrow A_v = -(r_o \parallel R_D \parallel R_L) g_m$$

$$\Rightarrow R_i = R_G$$

$$\Rightarrow R_o = (r_o \parallel R_D)$$

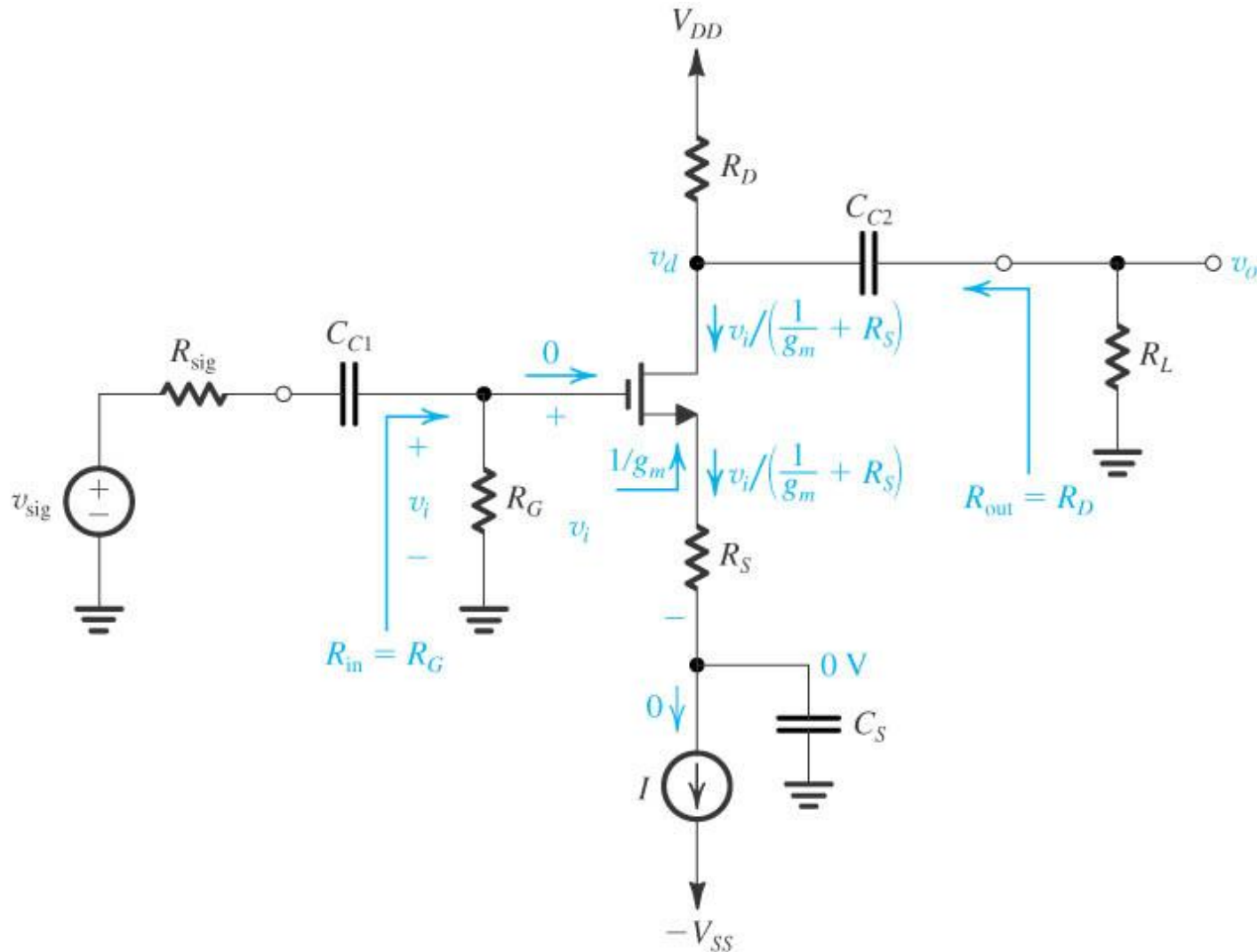
R_D large or small ??



- Moderately high voltage gain
- High input resistance
- Relatively high output resistance

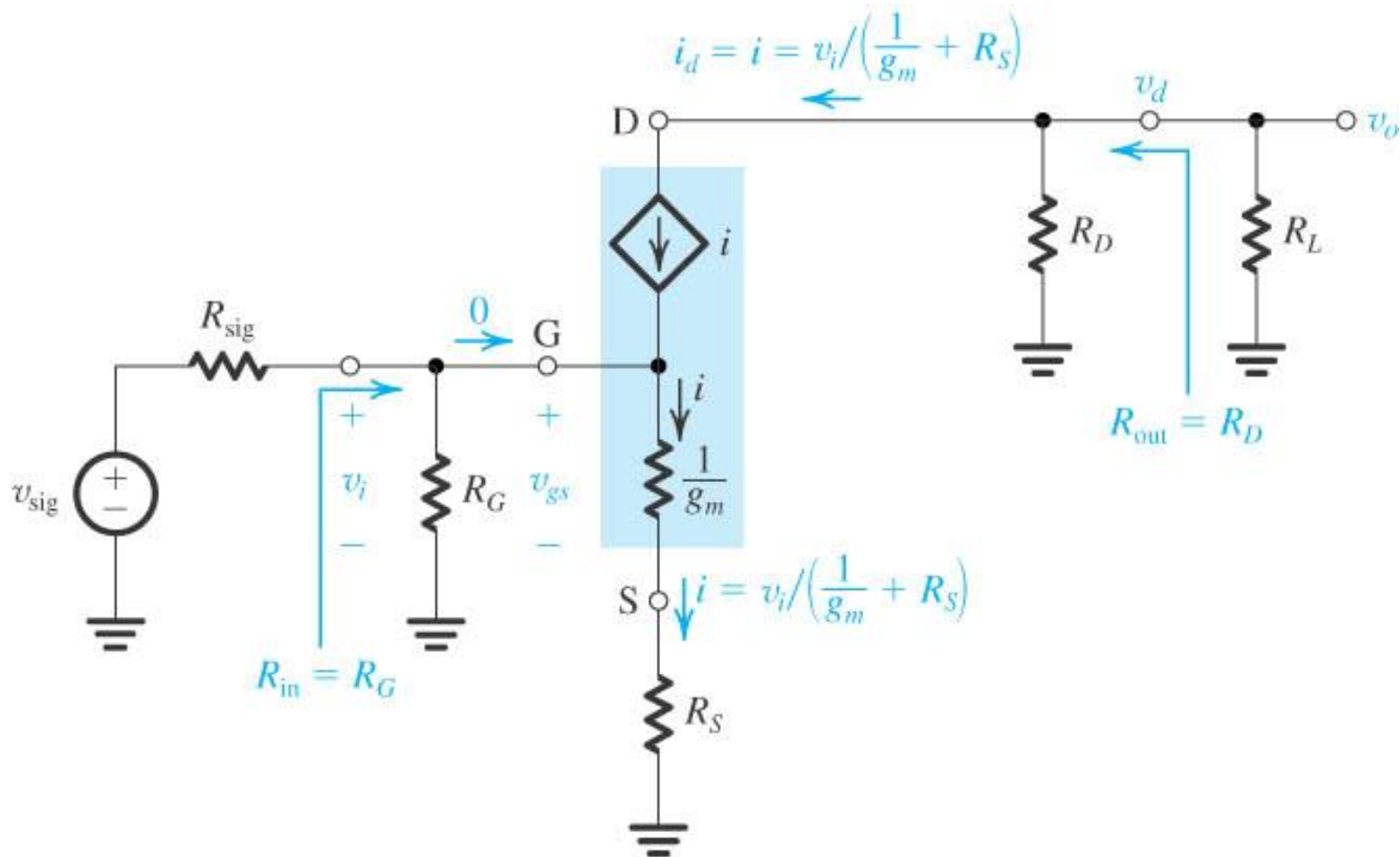
CS amplifier with source resistance

R_s at source terminal, biased with current source



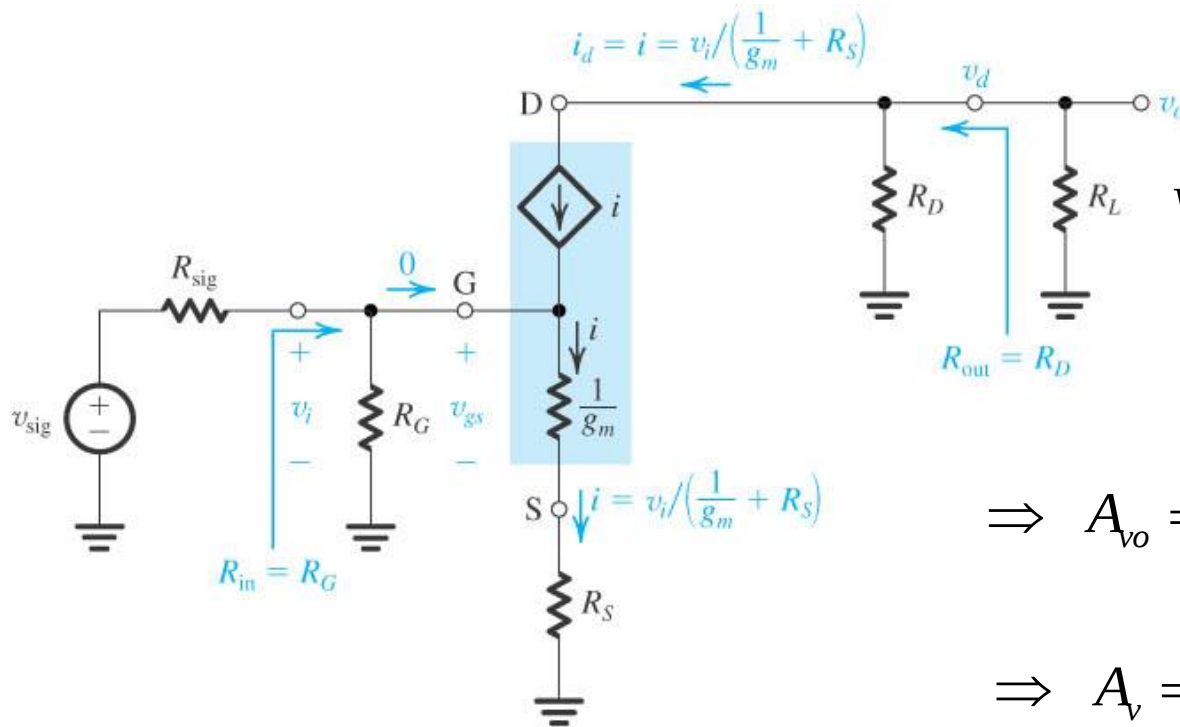
CS amplifier with source resistance

Use T model for small-signal equivalent (neglect r_o)



CS amplifier with source resistance

Use T model for small-signal equivalent



$$v_{gs} = \frac{1/g_m}{1/g_m + R_S} v_i$$

$$\Rightarrow i = g_m v_{gs} = \frac{1}{1/g_m + R_S} v_i$$

$$\Rightarrow A_{vo} = -\frac{g_m R_D}{1 + g_m R_S}$$

$$\Rightarrow A_v = -\frac{g_m (R_D \parallel R_L)}{1 + g_m R_S}$$

$$\Rightarrow R_i = R_G$$

$$R_o = \left. \frac{v_x}{i_x} \right|_{v_{sig}=0} \Rightarrow R_o = R_D$$

CS amplifier with source resistance

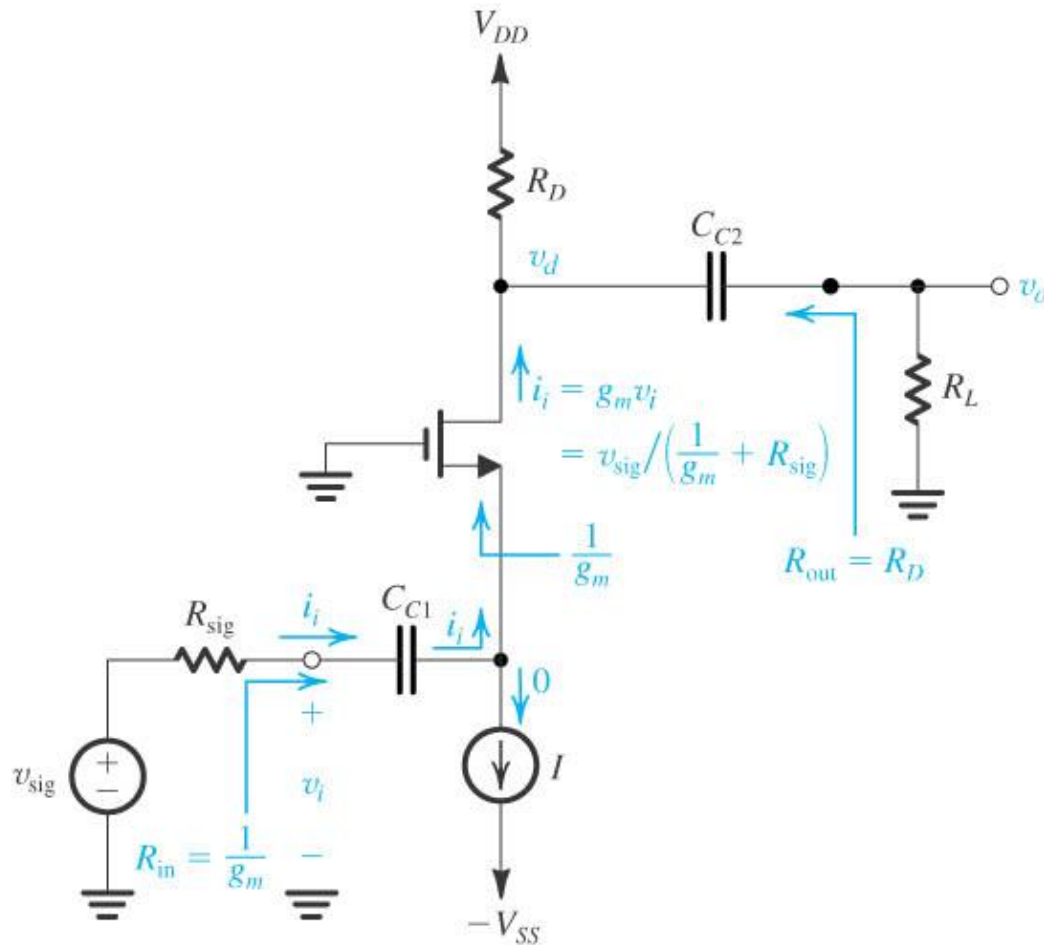
Summary

Compared to CS amplifier

- Decreases gain by factor $1 + g_m R_S$
- But remember ... adding a resistance at the source makes the biasing much more stable w.r.t. transistor variations
- Input and output resistance identical to CS amplifier

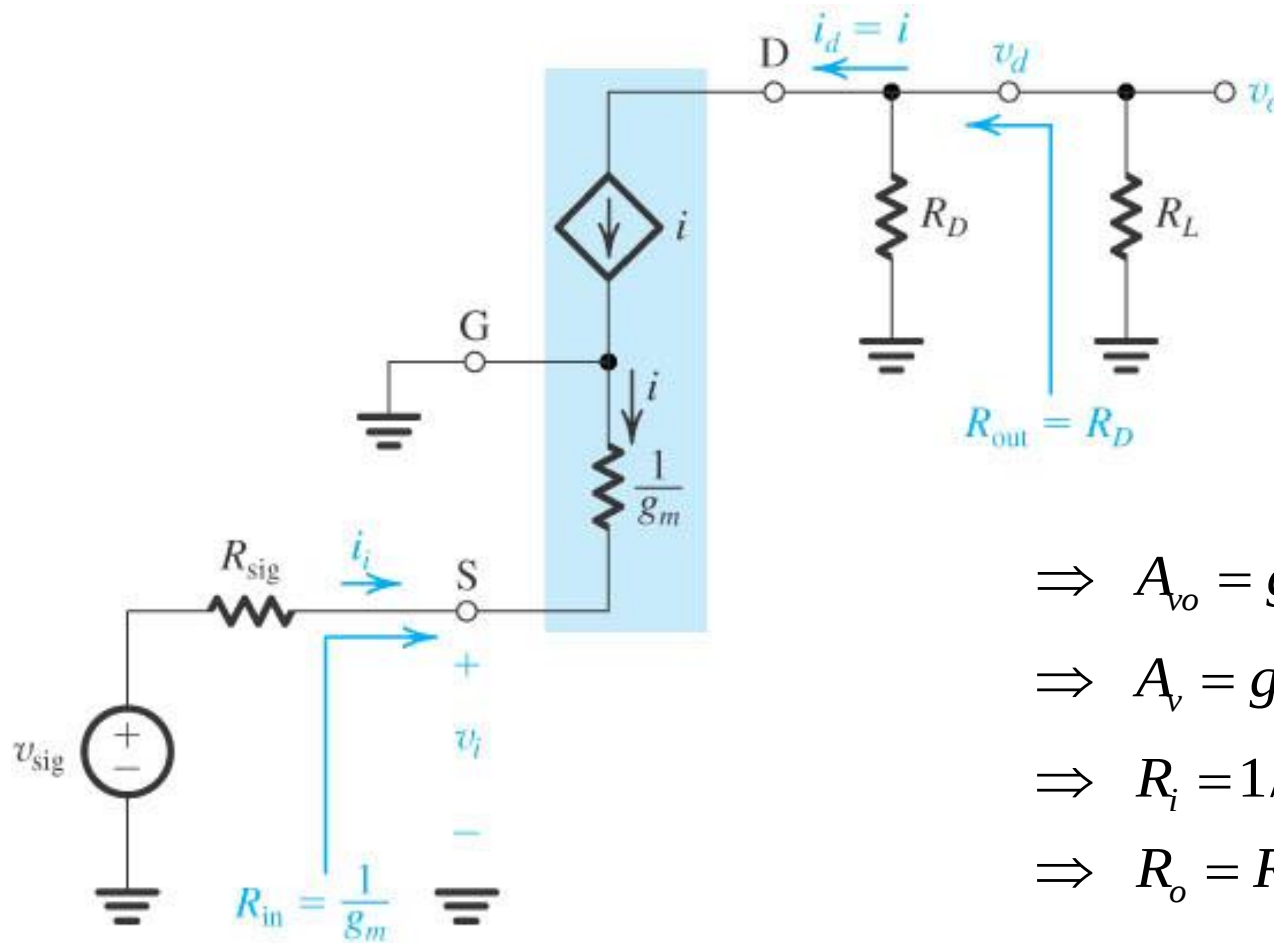
Common-gate amplifier

Input signal applied to source terminal



CG amplifier

Small-signal equivalent (neglecting r_o)



$$v_{gs} = -v_i$$

$$\Rightarrow i = -g_m v_i$$

$$\Rightarrow v_o = (R_D \parallel R_L) g_m v_i$$

$$\Rightarrow A_{vo} = g_m R_D$$

$$\Rightarrow A_v = g_m (R_D \parallel R_L)$$

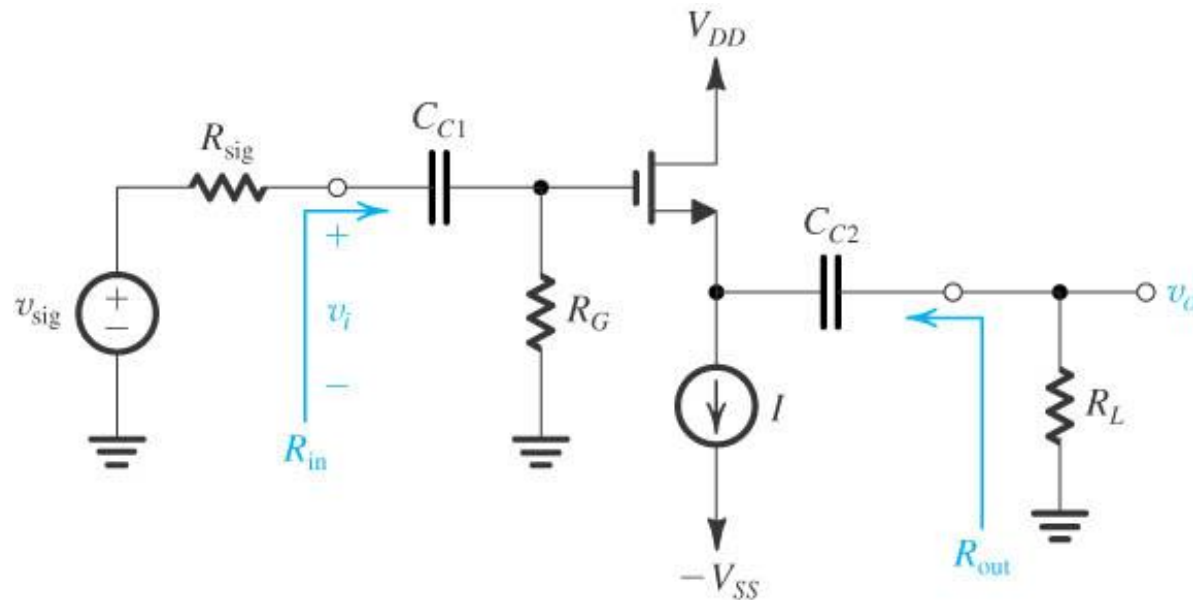
$$\Rightarrow R_i = 1/g_m$$

$$\Rightarrow R_o = R_D$$

Common-drain amplifier

... also called source-follower amplifier

- Drain is not connected to ground, but to V_{DD}
⇒ Small-signal equivalent is connected to ground



CD amplifier

Small-signal equivalent

- Use either small-signal model for transistor (do not neglect r_o)

CD amplifier

CD circuit characteristics

- Open-circuit gain = ?
- Gain = ?
- Input resistance = ?
- Output resistance = ?
- Why is it called source-follower amplifier? What is it used for?

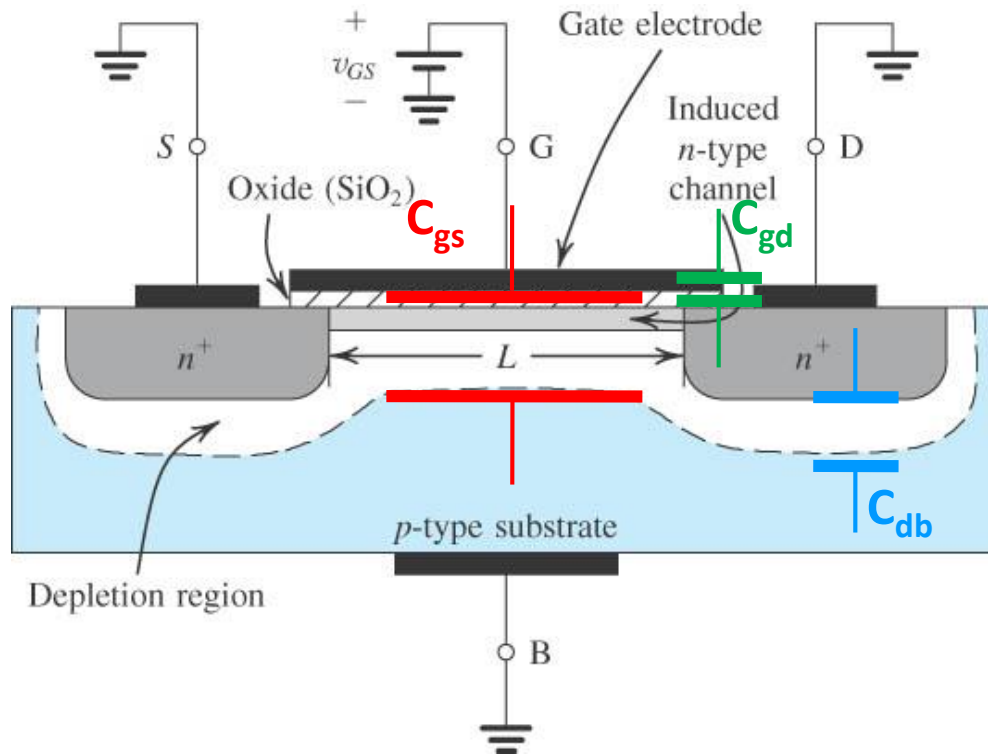
Single-stage MOS amplifiers

Outline

- NMOS as an amplifier: example of common-source circuit
 - NMOS amplifier example
 - Introduction to biasing and small-signal operation
- Biasing in MOS amplifier circuits
 - Fixing V_{GS}
 - Fixing V_G and adding a source resistance
 - Using RDG
 - Using a constant-current source
- Small-signal operation and models
 - Introduction to small-signal operation: common-source circuit
 - Small-signal models for NMOS transistors
- Other common single-stage MOS amplifiers
 - Common-source circuit
 - Common-source with source resistance
 - Common-drain circuit
 - Common-gate circuit
- Frequency response of MOSFET
 - HF response of NMOS transistor
 - HF response of common-source amplifier
 - LF response of common-source amplifier

High-frequency models for MOSFET

Several internal capacitances



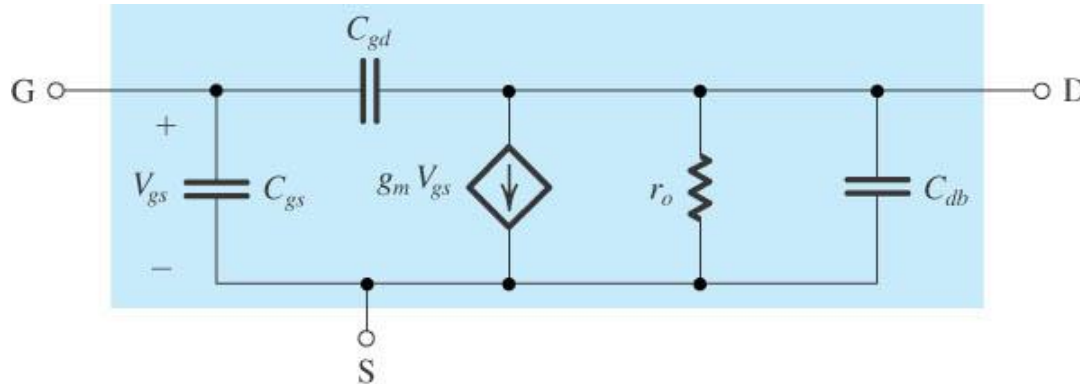
$$C_{gs} \approx (2/3)WLC_{ox}$$

$$C_{db} \propto V_{DB}^{-1/2}$$

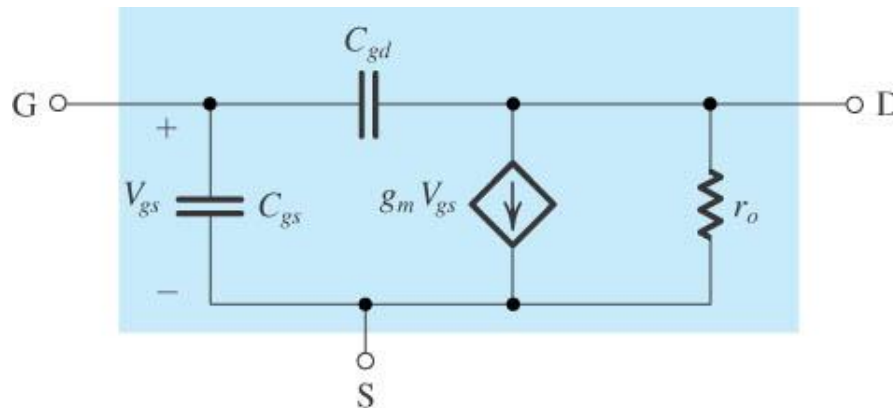
These parasitic capacitances will affect the high-frequency response of the transistor (and transistor circuit) !

High-frequency models for MOSFET

Small-signal HF model

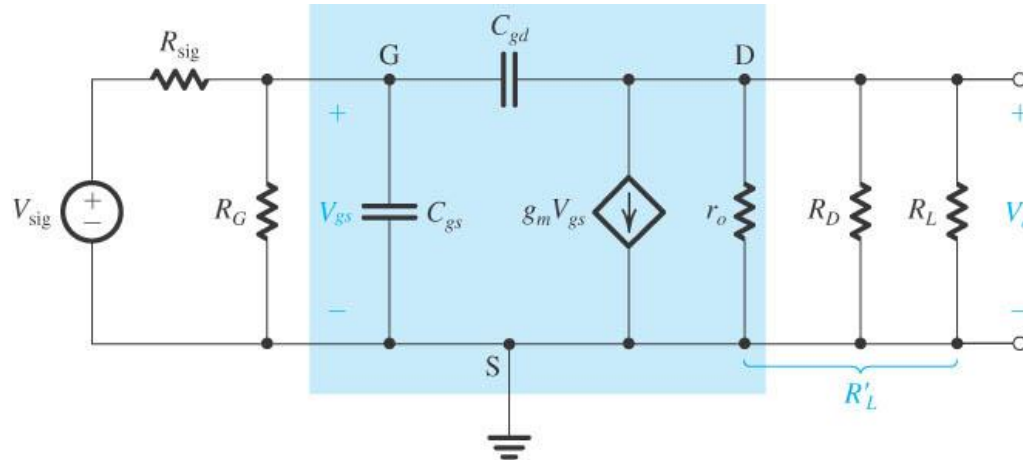


- C_{gd} small but plays significant role
- C_{db} often neglected to simplify analysis

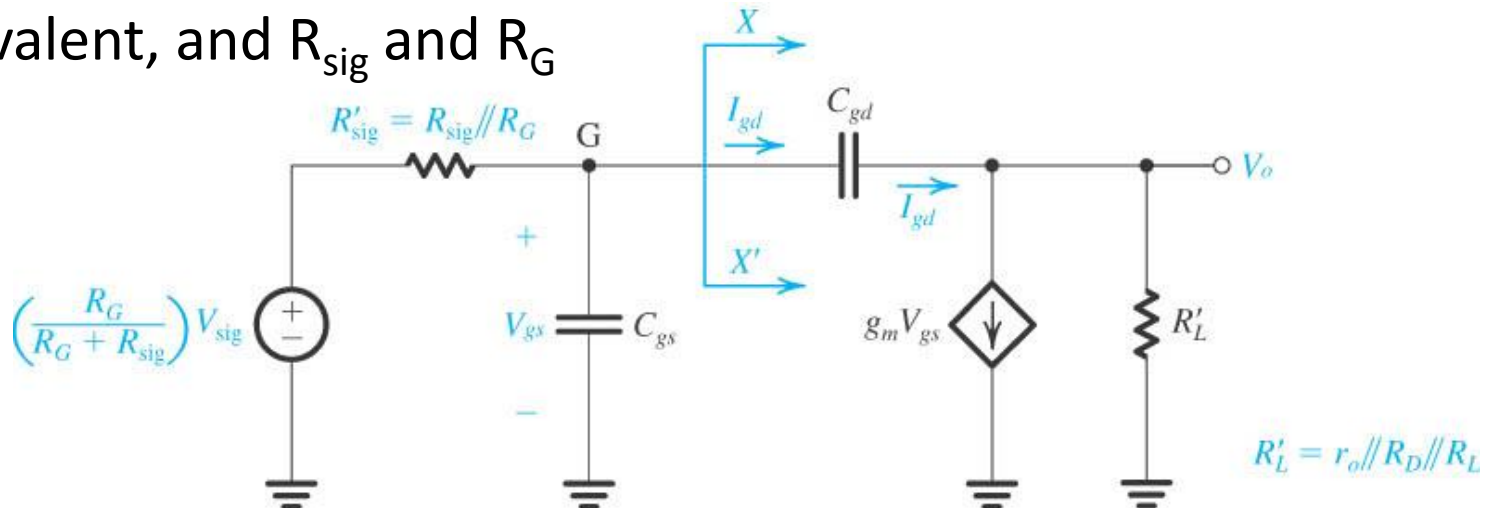


HF response of common-source amplifier

Replace NMOS with HF small-signal model

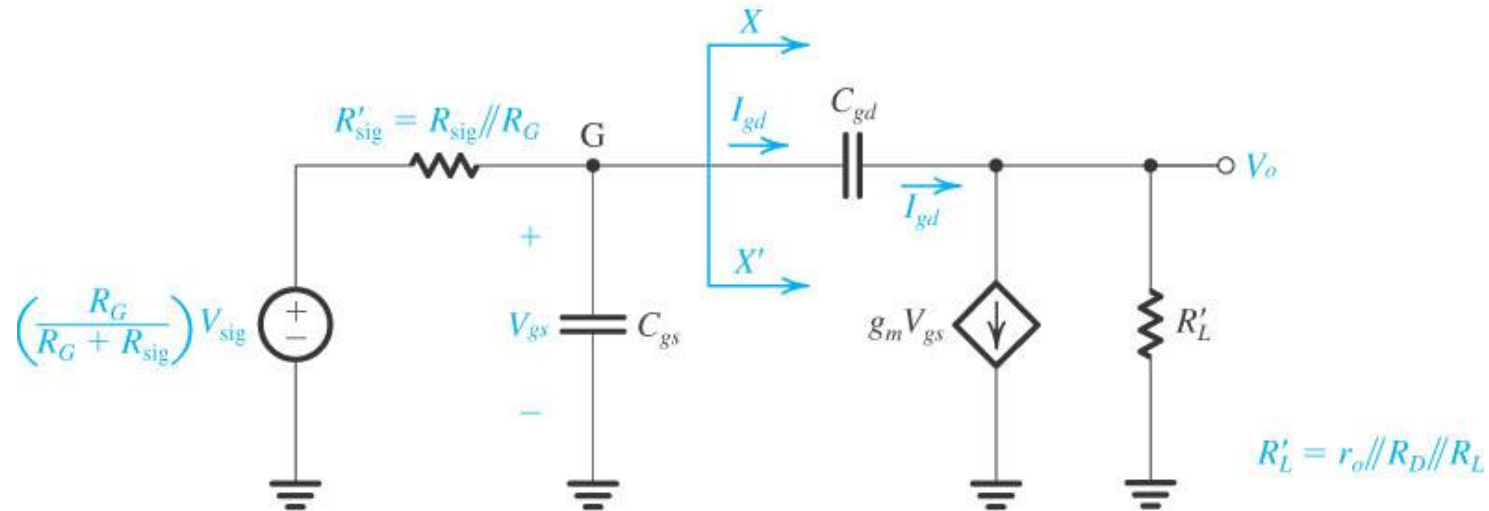


- Group r_o , R_D and R_L , and replace source and R_G with Thevenin equivalent, and R_{sig} and R_G



HF response of common-source amplifier

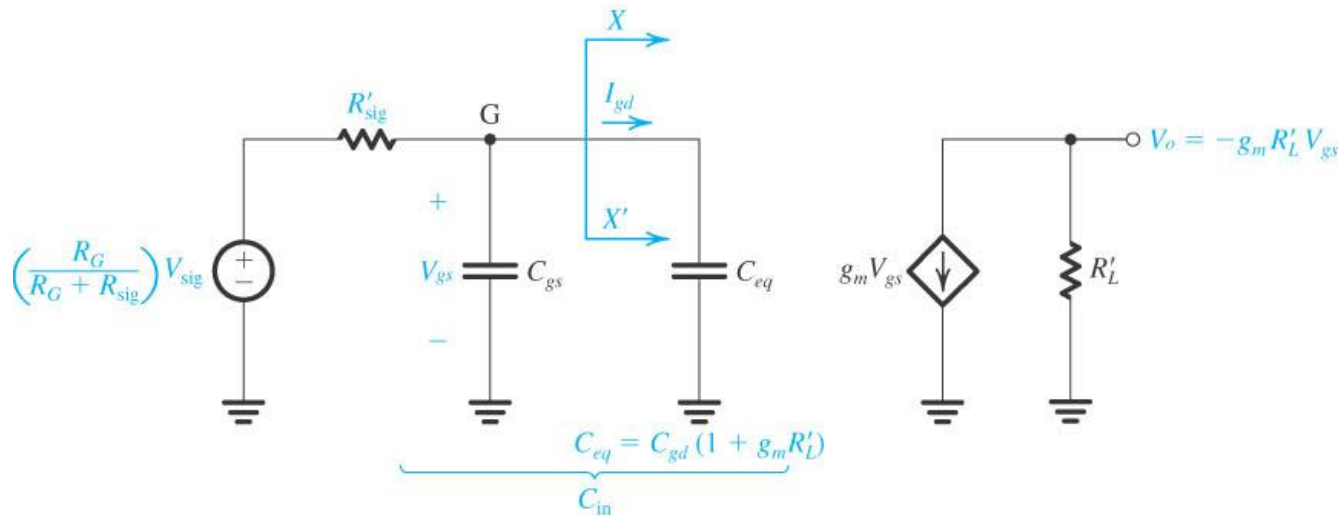
HF analysis



- Consider I_{gd} small w.r.t. $g_m V_{gs}$
 $\Rightarrow V_o \approx -g_m R'_L V_{gs}$
- $$I_{gd} = j\omega C_{gd} (V_{gs} - V_o)$$
$$= j\omega C_{gd} (V_{gs} + g_m R'_L V_{gs})$$
$$= j\omega C_{gd} (1 + g_m R'_L) V_{gs}$$

HF response of common-source amplifier

HF analysis (cont'd)



- $\Rightarrow C_{eq} = C_{gd} (1 + g_m R'_L)$
- $\Rightarrow C_{in} = C_{gs} + C_{eq} = C_{gs} + C_{gd} (1 + g_m R'_L)$
- $V_{gs} = \frac{1/(j\omega C_{in})}{1/(j\omega C_{in}) + R'_{sig}} V'_{sig}$

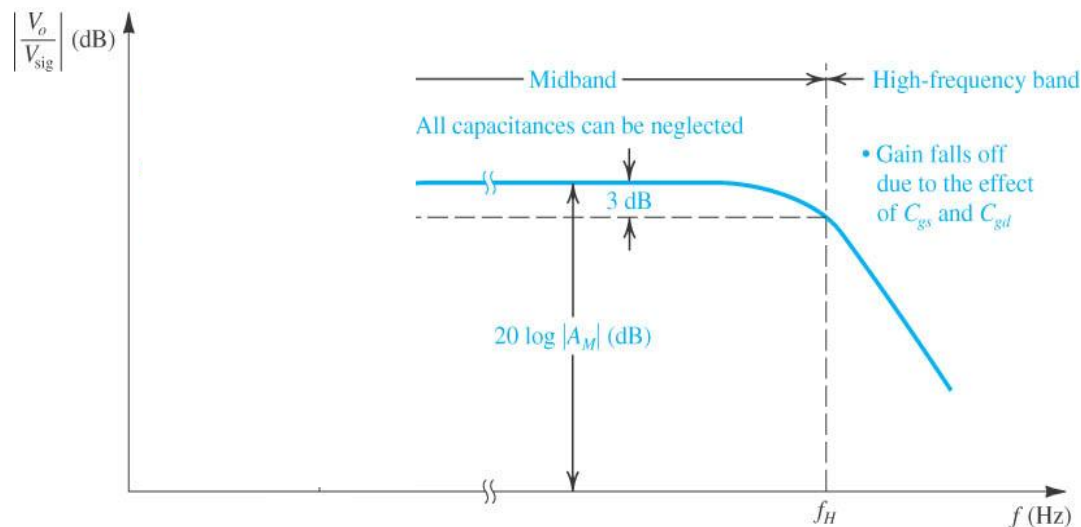
$$= \frac{1}{1 + j\omega C_{in} R'_{sig}} \left(\frac{R_G}{R_G + R_{sig}} \right) V_{sig}$$

$$\omega_0 = 1/(C_{in} R'_{sig})$$

HF response of common-source amplifier

HF analysis (cont'd)

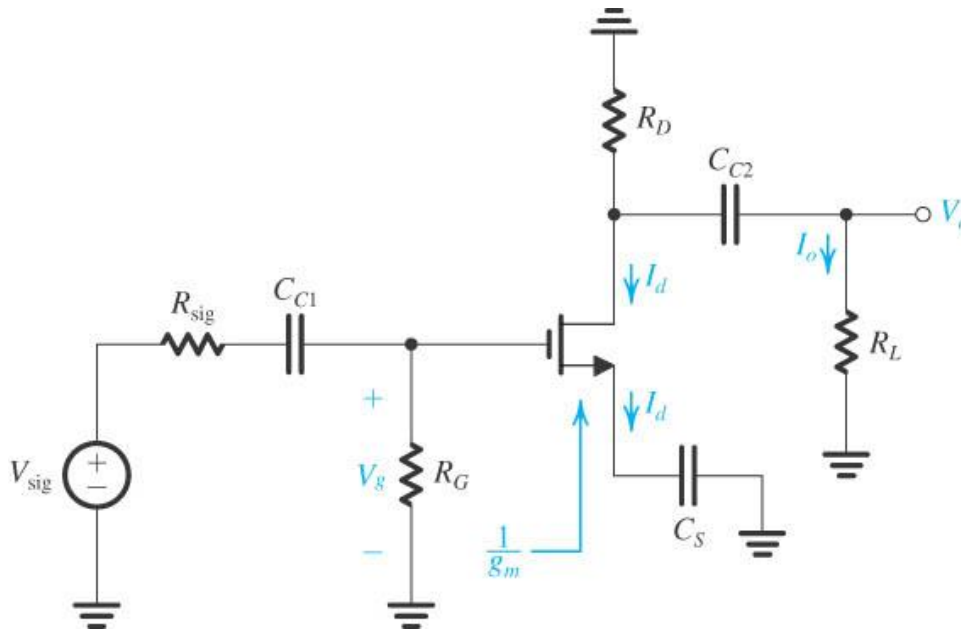
- $$\frac{V_o}{V_{sig}} = - \left(\frac{R_G}{R_G + R_{sig}} \right) (g_m R'_L) \frac{1}{1 + j \frac{\omega}{\omega_0}}$$
- At high frequencies, gain tends to zero
- Cut-off frequency: $f_H = \frac{\omega_0}{2\pi} = \frac{1}{2\pi C_{in} R'_{sig}}$



LF response of common-source amplifier

each coupling capacitor creates a high-pass filter

- Effect of C_{C1}



$$V_g = \frac{R_G}{R_G + R_{sig} + \frac{1}{j\omega C_{C1}}} V_{sig}$$
$$= \frac{R_G}{R_G + R_{sig}} \left[\frac{1}{1 + \frac{1}{j\omega C_{C1} (R_G + R_{sig})}} \right] V_{sig}$$



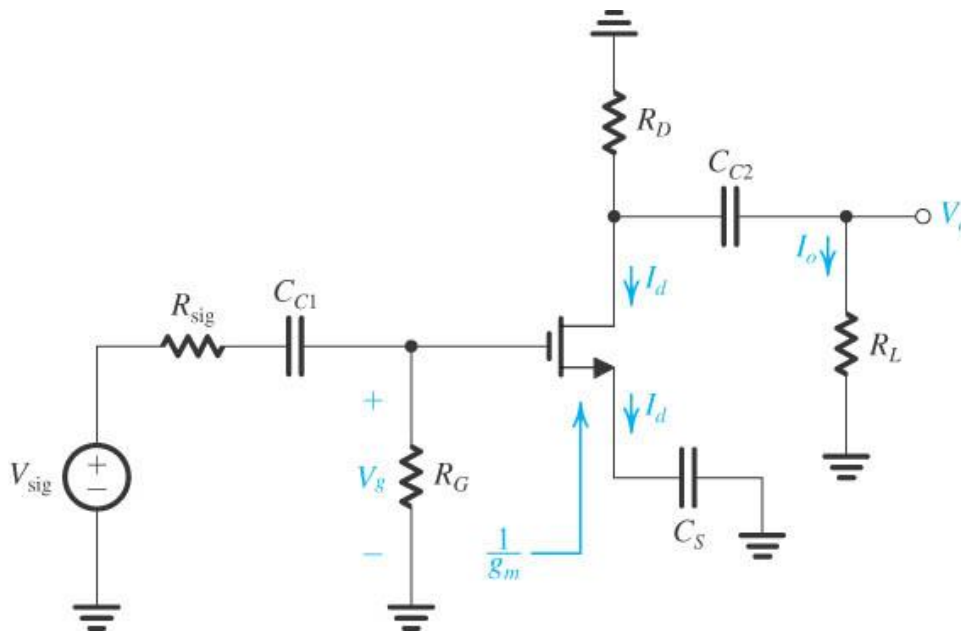
High-pass with cut-off frequency:

$$f_{P1} = \frac{1}{2\pi C_{C1} (R_G + R_{sig})}$$

LF response of common-source amplifier

each coupling capacitor creates a high-pass filter

- Effect of C_S (use T-model)



$$I_d = \frac{V_g}{\frac{1}{g_m} + \frac{1}{j\omega C_S}}$$
$$= \frac{1}{1 + \frac{1}{j\omega \frac{1}{g_m} C_S}} g_m V_g$$



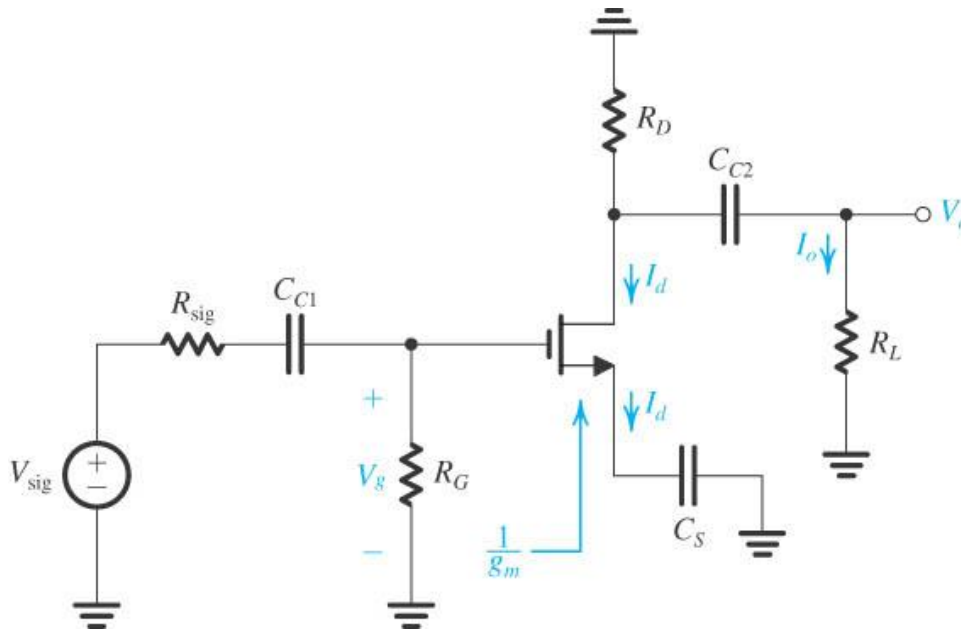
High-pass with cut-off frequency:

$$f_{P2} = \frac{1}{2\pi C_S (1/g_m)}$$

LF response of common-source amplifier

each coupling capacitor creates a high-pass filter

- Effect of C_{C2} (current divider)



$$I_o = -I_d \frac{R_D}{R_D + \frac{1}{j\omega C_{C2}} + R_L}$$
$$= -I_d \frac{R_D}{R_D + R_L} \boxed{\frac{1}{1 + \frac{1}{j\omega C_{C2} (R_D + R_L)}}}$$



High-pass with cut-off frequency:

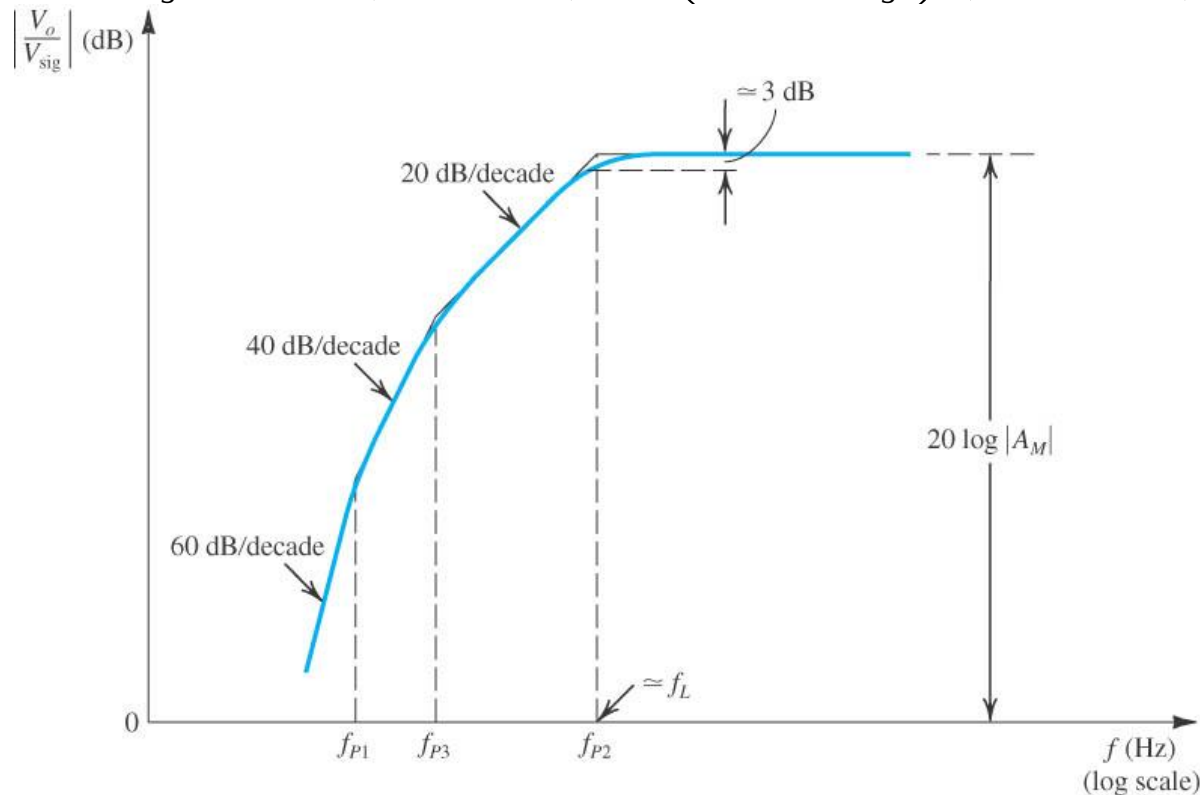
$$f_{P2} = \frac{1}{2\pi C_{C2} (R_D + R_L)}$$

LF response of common-source amplifier

Total low-frequency response

$$V_o = R_L I_o$$

$$\Rightarrow \frac{V_o}{V_{sig}} = -R_L \left(\frac{R_D}{R_D + R_L} \right) g_m \left(\frac{R_G}{R_G + R_{sig}} \right) \left(\frac{j\omega}{j\omega + \omega_{P1}} \right) \left(\frac{j\omega}{j\omega + \omega_{P2}} \right) \left(\frac{j\omega}{j\omega + \omega_{P3}} \right)$$



Usually, one cut-off frequency is higher than the two others

Frequency response of common-source amplifier

Total LF and HF response

